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Fermatean Uncertainty Soft Sub Algebra in terms of Ideal Structures

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***Abstract*:** Ideal concepts are discussed in many mathematical applications. Various author has been studied and analytical in different ways. In this article, the idea of bipolar fermatean uncertainty sub algebra's in terms of R-ideals is planned. Also the correlation among bipolar fermatean uncertainty soft ideal and bipolar fermatean uncertainty soft R-ideals is expressed some interesting ideas also analyzed.

Keywords: *Fuzzy set, Bipolar fuzzy, Fermatean fuzzy set, Algebra, R-ideals, BCI-algebra, BCK-algebra, associative, cut set.*

1. **Introduction:** Later the idea of uncertainty collections of Zadeh [21], Lee [10] presented another trend of uncertainty collections called bipolar valued uncertainty sets (BVUS). Bipolar valued uncertainty set defined over the interval [-1, 1] which was to be extended from the ordinary fuzzy set interval [0, 1]. The idea of bipolar parameterized collections and several identification of bipolar parameterized collection were presented by Shabir and Naz [16]. Abdulla et al. [1] studied the idea of bipolar uncertainty parameterized collections by combining parameterized collections and bipolar uncertainty collections sponsored by Zhang [19, 20], and given parametrical ideal identifications of bipolar uncertainty parameterized collections. Akram et al. [3] explained an idea of positive and negative uncertainty soft sub semi group and positive and negative uncertainty soft-ideals in a semi group. The minus membership function and the plus membership function defined in [-1, 0] and [0, 1] in bipolar uncertainty setting. In this bipolar uncertainty setting '0' refers that the elements are subjected to irrelevant. They are familiar representation and down word representation. The familiar forms of positive and negative uncertainty collections are used in their representations. In 2011, positive and negative fuzzy K−sub algebras are analyzed by Farhat Nisar [5]. Stimulated by the notions in recent times, the result of bipolar valued fuzzy sub algebras/ideals of a BF-algebra [4] has discussed by applying the notion of bipolar valued uncertainty collection (BVUS) in BF-algebras [4]. Fermatean uncertainty bipolar model as a combination of uncertainty bipolar model and Pythagorean uncertainty bipolar. Group symmetry analyzes a moral character to molecule structures. The author [18] coined the Fermatean uncertainty set (FUS) with its relational measures. Collections data between parameterized collections were studied by Maji et al. [12]. Some author [2] explained various identifications on the parameterized collections and Sezgin and Atagun [17] investigated on parameterized set identifications as well. In this view, we analyze various domine of ideals and investigate some two axes fermatean uncertainty collections and its properties.

2. Preliminaries

Definition 2.1: [K. Lee, 2009] As per BCI-algebra we focus algebra $(X, *, 0)$ of type (2, 0) fulfills under some points, $for\,all\ \ell, m, n \in X$:

$$B(I1): (\ell * m) * (\ell * n) * (n * m) = 0$$

$B\,(I2): (\ell*(\ell*m)*m)=0$

$B\,(I3): (\ell*\ell)=0$

$B\,(I4): (\ell*m)=0 \ and \ (m*\ell)=0 \ imply \ \ell=m$

We represent a partial relation $\leq$ by $\ell \leq m \ iff \ \ell*m=0.$

If a BCI-algebra, X fulfills $0*\ell=0 \ for all \ \ell \in X$, then we can show X is BCK-algebra. All BCK-algebra X satisfying the given conditions $for \ all \ \ell, m, n \in X.$

$(BCK-1): (\ell*m)*n=(\ell*n)*m$

$(BCK-2): (\ell*n)*(m*n)*(\ell*m)=0$

$(BCK-3): (\ell*\ell)=0$

$(BCK-4): (\ell*m)=0=(m*\ell),$

$(\ell*n)*(m*n)=0,$

$(n*m)*(n*\ell)=0.$

Definition 2.2: [L.A. Zadeh, 1965] Let 'X' be a collection of all elements. An uncertainty collection 'A' falls from X is expressed as $A=\{(x:\mu A(x))/x\in X\}$, where $\mu A: A\to[0,1]$ is the grade mapping of the uncertainty collections A.

Definition 2.3: [K. Lee, 2009] Let 'X' is a Universe. Then a bipolar uncertainty collection A on X is represented by plus membership map μA^{+}, that is, $\mu A^{+}: X\to[0,1]$ and a negative membership map μA^{-}, (i, e), $\mu A^{-}: X\to[-1,0]$. For the state of easy way, we always utilize the symbol $A=\{(x, \mu A^{+}, \mu A^{-})/x\in X\}$.

Definition 2.4: [Senapati and Yager, 2019] Let 'X' is Universe of discovers. A fermatean uncertainty set (FUS) F in X is a domain has the formulation as $F=\{(x, m_F(x), n_F(x))/x\in X\}$, where $m_F(x): X\to[0,1] \ and \ n_F(x): X\to[-1,0]$, which includes the result $0\leq(m_F(x))^3+(n_F(x))^3\leq 1, \forall x\in X$, the numbers $m_F(x)$ denotes the power of elements and $n_F(x)$ denotes the power of non-membership of the element $x\in F$, all fermatean uncertainty set 'F' and $x\in F$. $\prod F(x)=\sqrt[3]{1-(m_F(x))^3-(n_F(x))^3}$ is defined as the degree of middle of x to F. For convince, Senapati and Yager called $(m_F(x), n_F(x))$ fermatean fuzzy number (FFN) denoted by $F=(m_F, n_F)$.

Definition 2.5: [Moldtsov 1999] Let U is an initial Universe, P(U) is the power set of U and E is collection of all notations and A⊆E. A parameterized collections (δ_A, E) on the Universe 'U' is explained by the collections of order pairs $(\delta_A, E)=\{(e, \delta_A(e)): e\in E, \ \delta_A\in P(U)\}$, where $\delta_A: E\to P(U)$ such that $\delta_A(e)=\phi \ if \ e\notin A$. Here δ_A is known as tentative mapping of the parameterized collections.

Example 2.6: Let $U=\{v_1, v_2, v_3, v_4\}$ be a collection of four pants and $E=\{white(e_1), red(e_2), blue(e_3)\}$ be a collection of objects. If $A=\{e_1, e_2\}\subseteq E$. Let $\delta_A(e_1)=\{v_1, v_2, v_3, v_4\}$ and $\delta_A(e_2)=\{v_1, v_2, v_3\}$ then we form the parameterized set

$(\delta_A, E) = \{(e_1, \{v_1, v_2, v_3, v_4\}), (e_2, \{v_1, v_2, v_3\})\}$ over 'U' which symbolized the "color of the pants" which Mr. A is going to buy. This can be represented the soft set in the given format.

Σ	e_1	e_2	e_3
v_1	1	1	0
v_2	1	1	0
v_3	1	1	0
v_4	1	0	0

Definition 2.7: [Bipolar fermatean uncertainty soft set] Let 'X' is a collection of all elements. A bipolar fermatean uncertainty soft set (BPFUSS). $F = \{(u, m_F{}^P, n_F{}^P, m_F{}^N, n_F{}^N / u \in X)\}$, Where $m_F{}^P : X \to [0,1]$, $n_F{}^P : X \to [0,1]$, $m_F{}^N : X \to [0,1]$, $n_F{}^N : X \to [0,1]$ that are the mappings such that $0 \le (m_F)^3 + (n_F)^3 \le 1$ and $-1 \le (m_F)^3 + (n_F)^3 \le 0$ *and* $m_F{}^P(u)$ denotes positive membership degree, $n_F{}^P(u)$ represents positive non-membership degree, $n_F{}^N(u)$represents negative membership degree, $n_F{}^N(u)$ represents negative non-membership degree. The degree of indeterminacy.

$$\prod F^P(u) = \sqrt[3]{1 - (m_F{}^P(u))^3 - (n_F{}^P(u))^3} \ \ and \ \ \prod F^N(u) = \sqrt[3]{1 - (m_F{}^N(u))^3 - (n_F{}^N(u))^3}.$$

Definition2.8:Let $F_1 = \{(u, m_{F_1}{}^P, n_{F_1}{}^P, m_{F_1}{}^N, m_{F_1}{}^N / u \in X)\}$ and

$F_2 = \{(u, m_{F_2}{}^P, n_{F_2}{}^P, m_{F_2}{}^N, m_{F_2}{}^N / u \in X)\}$ be BPFUSS sets then,

(i) $F_1 \cup F_2 = \{(u, \max(m_{F_1}{}^P, m_{F_2}{}^P), \min(n_{F_1}{}^P, n_{F_2}{}^P), \min(m_{F_1}{}^N, m_{F_2}{}^N), \max(n_{F_1}{}^N, n_{F_2}{}^N)) / u \in X\}$

(ii) $F_1 \cap F_2 = \{(u, \min(m_{F_1}{}^P, m_{F_2}{}^P), \max(n_{F_1}{}^P, n_{F_2}{}^P), \max(m_{F_1}{}^N, m_{F_2}{}^N), \min(n_{F_1}{}^N, n_{F_2}{}^N)) / u \in X\}$

(iii) $F_1{}^C = \{(u, m_F{}^P, n_F{}^P, m_F{}^N, n_F{}^N / u \in X)\}$

(iv) $F_1 \subset F_2 = iff\ m_{F_1}{}^P(u) \le m_{F_2}{}^P(u), n_{F_1}{}^P(u) \ge n_{F_2}{}^P(u), m_{F_1}{}^N(u) \ge m_{F_2}{}^N(u), n_{F_1}{}^N(u) \le n_{F_2}{}^N(u)$.

3. Bipolar Fermatean Fuzzy Soft Algebra

Definition 3.1: A bipolar fermatean uncertainty parameterized collections F in X called bipolar fermatean uncertainty soft sub algebra of X if it fulfills,

(i) $m_F{}^P(u * v) \ge T\{m_F{}^P(u), m_F{}^P(v)\}$

(ii) $n_F{}^P(u * v) \le S\{n_F{}^P(u), n_F{}^P(v)\}$

(iii) $m_F{}^N(u * v) \le S\{m_F{}^N(u), m_F{}^N(v)\}$

(iv) $n_F{}^N(u * v) \ge T\{n_F{}^N(u), n_F{}^N(v)\}$, $for all\ u, v \in X$.

Definition 3.2: A Bipolar fermatean uncertainty parameterized collections 'F' of a BCK-algebra X is known to be a bipolar fermatean uncertainty soft ideal (BPFUSI) of X, if the subsequent results are satisfied.

(i) $m_F{}^P(0) \ge m_F{}^P(u)$ *and* $n_F{}^P(0) \le n_F{}^P(u)$

(ii) $m_F^{\ N}(0) \le m_F^{\ N}(u)$ and $n_F^{\ N}(0) \ge n_F^{\ N}(u)$

(iii) $m_F^{\ P}(u) \ge T\{m_F^{\ P}(u*), m_F^{\ P}(v)\}$ and $n_F^{\ P}(u) \le S\{n_F^{\ P}(u*v), n_F^{\ P}(v)\}$

(iv) $m_F^{\ N}(u) \le S\{m_F^{\ N}(u*v), m_F^{\ N}(v)\}$ and $n_F^{\ N}(u) \ge T\{n_F^{\ N}(u*v), n_F^{\ N}(v)\}$, if $u, v \in X$.

Definition 3.3: A bipolar uncertainty soft set F in X is known as a bipolar fermatean uncertainty soft R-ideal (BPFUSRI) of X if it fulfills,

(i) $m_F^{\ P}(0) \ge m_F^{\ P}(u)$ and $n_F^{\ P}(0) \le n_F^{\ P}(u)$

(ii) $m_F^{\ N}(0) \le m_F^{\ N}(u)$ and $n_F^{\ N}(0) \ge n_F^{\ N}(u)$

(iii) $m_F^{\ P}(v*u) \ge T\{m_F^{\ P}(u*w)*(0*v), m_F^{\ P}(w)\}$ and
$n_F^{\ P}(v*u) \le S\{n_F^{\ P}(u*w)*(0*v), n_F^{\ P}(w)\}$

(iv) $m_F^{\ N}(v*u) \le S\{m_F^{\ N}(u*w)*(0*v), m_F^{\ N}(w)\}$ and
$n_F^{\ N}(v*u) \ge T\{n_F^{\ N}(u*w)*(0*v), n_F^{\ N}(w)\}$, for all $u, v, w \in X$.

Example 3.4: We have a BCK-algebra $X = \{\ell, m, n, p\}$ with the following Cayley table.

$*$	l	m	n	p
l	l	m	n	p
m	m	l	p	n
n	n	p	l	m
p	p	n	m	l

Define a BPFUSS 'F' in X by

x	l	m	n	p
$(m_F^{\ P}, n_F^{\ P})$	$[0.2, 0.5]$	$[0.4, 0.6]$	$[0.5, 0.7]$	$[0.2, 0.9]$
$(m_F^{\ N}, n_F^{\ N})$	$[-0.7, -0.1]$	$[-0.9, -0]$	$[-0.4, -0]$	$[-0.7, -0]$

Then, 'F' is BPFUSRI of X.
The consecutive results are the standard results with relevant results.

Theorem 3.5: If 'F' is a BPFUSRI of X, then
$m_F^{\ P}(u) = m_F^{\ P}(0*u), n_F^{\ P}(u) = n_F^{\ P}(0*u), m_F^{\ N}(u) = m_F^{\ N}(0*u)$, and $n_F^{\ N}(u) = n_F^{\ N}(0*u)$, for all $u \in X$.

Proof: Let 'F' be a BPFUSRI of X.
Taking v = w = 0 in definition 3.3 and 2.1 (iii) and (ii), we get,

$$m_F^{\ N}(0*u) \le m_F^{\ N}(u),\ n_F^{\ N}(0*u) \ge n_F^{\ N}(u)$$
$$m_F^{\ P}(0*u) \ge m_F^{\ P}(u),\ n_F^{\ P}(0*u) \le n_F^{\ P}(u)$$

By setting u = w = 0 in definition 3.3 and 2.1 (iii) and (ii). We get,

$$m_F^{\ N}(v) = m_F^{\ N}(v*0) \le m_F^{\ N}(0*(v*0)) \le m_F^{\ N}(0*v)$$
$$n_F^{\ N}(v) = n_F^{\ N}(v*0) \ge n_F^{\ N}(0*(v*0)) \ge n_F^{\ N}(0*v)$$
$$m_F^{\ P}(v) = m_F^{\ P}(v*0) \ge m_F^{\ P}(0*(v*0)) \ge m_F^{\ P}(0*v)$$

$$n_F{}^P(v)=n_F{}^P(v*0)\leq n_F{}^P(0*(v*0))\leq n_F{}^P(0*v),\ for all v\in X.$$

Hence, $m_F{}^P(u)=m_F{}^P(0*u),\ n_F{}^P(u)=n_F{}^P(0*u)$

$$m_F{}^N(u)=m_F{}^N(0*u),\ n_F{}^N(u)=n_F{}^N(0*u),\ for all u\in X.$$

Theorem 3.6: Every BPFUSRI of X is both a BPFUSA of X and BPFUSI of X.

Proof: Let 'F' be BPFUSRI of X. Using set definition- 3.3 and theorem- 3.5, we have,

$$\begin{aligned}m_F{}^N(u)&=m_F{}^N(0*u)\\&\leq S\left\{m_F{}^N(u*w)*(0*0),m_F{}^N(w)\right\}\\&=S\left\{m_F{}^N(u*w),m_F{}^N(w)\right\}\end{aligned}$$

$$\begin{aligned}n_F{}^N(u)&=n_F{}^N(0*u)\\&\geq T\left\{n_F{}^N(u*w)*(0*0),n_F{}^N(w)\right\}\\&=T\left\{n_F{}^N(u*w),n_F{}^N(w)\right\}\end{aligned}$$

$$\begin{aligned}m_F{}^P(u)&=m_F{}^P(0*u)\\&\geq T\left\{m_F{}^P(u*w)*(0*0),m_F{}^P(w)\right\}\\&=T\left\{m_F{}^P(u*w),m_F{}^P(w)\right\}\end{aligned}$$

$$\begin{aligned}n_F{}^P(u)&=n_F{}^P(0*u)\\&\leq S\left\{n_F{}^P(u*w)*(0*0),n_F{}^P(w)\right\}\\&=S\left\{n_F{}^P(u*w),n_F{}^P(w)\right\},\ for all u,v,w\in X.\end{aligned}$$

Hence, 'A' is BPFUSI of X.

Now for any $u,v\in X$, then

$$\begin{aligned}m_F{}^N(u*v)&\leq S\left\{\left(m_F{}^N(u*v)*u\right),m_F{}^N(u)\right\}\\&=S\left\{m_F{}^N(0*v),m_F{}^N(u)\right\}\\&=S\left\{m_F{}^N(u),m_F{}^N(v)\right\}\end{aligned}$$

$$\begin{aligned}n_F{}^N(u*v)&\geq T\left\{\left(n_F{}^N(u*v)*u\right),n_F{}^N(u)\right\}\\&=T\left\{n_F{}^N(0*v),n_F{}^N(u)\right\}\\&=T\left\{n_F{}^N(u),n_F{}^N(v)\right\}\end{aligned}$$

$$\begin{aligned}m_F{}^P(u*v)&\geq T\left\{\left(m_F{}^P(u*v)*u\right),m_F{}^P(u)\right\}\\&=T\left\{m_F{}^P(0*v),m_F{}^P(u)\right\}\\&=T\left\{m_F{}^P(u),m_F{}^P(v)\right\}\end{aligned}$$

$$\begin{aligned}n_F{}^P(u*v)&\leq S\left\{\left(n_F{}^P(u*v)*u\right),n_F{}^P(u)\right\}\\&=S\left\{n_F{}^P(0*v),n_F{}^P(u)\right\}\\&=S\left\{n_F{}^P(u),n_F{}^P(v)\right\}\end{aligned}$$

Therefore 'A' is BPFUSA of X. The example given below express that the reverse of theorem– 3.6 need not be true.

Example 3.7: Let $X = \{\ell, m, n\}$ be a BCK-algebra with the following clayey table.

$*$	l	m	n
l	l	m	n
m	m	l	n
n	n	m	l

Define a BPFUS 'F' in X by

a	l	m	n
$(m_F{}^P, n_F{}^P)$	$[0.6, 0.9]$	$[0.2, 0.6]$	$[0.2, 0.6]$
$(m_F{}^N, n_F{}^N)$	$[-0.2, -0.4]$	$[-0.3, -0.5]$	$[-0.5, -0.8]$

Then, 'F' is both a BPFUSI and a BPFUSA of X, but not BPFUSRI of X.

Theorem 3.8: Let 'F' be a BPFUSI of X. If the equation $u * v \le w$ holds in X, then,

(i) $m_F{}^N(u) \le S\{m_F{}^N(v), m_F{}^N(w)\}$ *and* $n_F{}^N(u) \ge T\{n_F{}^N(v), n_F{}^N(w)\}$

(ii) $m_F{}^P(u) \ge T\{m_F{}^P(v), m_F{}^P(w)\}$ *and* $n_F{}^P(u) \le S\{n_F{}^P(v), n_F{}^P(w)\}$

Proof: Let $u, v, w \in X$ *and* $u * v \le w,$ *then* $(u * v) * w = 0$ *and so*

(i) $m_F{}^N(u) \le S\{m_F{}^N(u * v), m_F{}^N(v)\}$

$$\le S\{S\{m_F{}^N(u * v) * w, m_F{}^N(w), m_F{}^N(v)\}\}$$

$$= S\{S\{m_F{}^N(0), m_F{}^N(w), m_F{}^N(v)\}\} = S\{m_F{}^N(v), m_F{}^N(w)\}$$

$$n_F{}^N(u) \ge T\{n_F{}^N(u * v), n_F{}^N(v)\}$$

$$\ge T\{T\{n_F{}^N(u * v) * w, n_F{}^N(w), n_F{}^N(v)\}\}$$

$$= T\{T\{n_F{}^N(0), n_F{}^N(w), n_F{}^N(v)\}\} = T\{n_F{}^N(v), n_F{}^N(w)\}$$

Also,

(ii) $m_F{}^P(u) \ge T\{m_F{}^P(u * v), m_F{}^P(v)\}$

$$\ge T\{T\{m_F{}^P(u * v) * w, m_F{}^P(w), m_F{}^P(v)\}\}$$

$$= T\{T\{m_F{}^P(0), m_F{}^P(w), m_F{}^P(v)\}\} = T\{m_F{}^P(v), m_F{}^P(w)\}$$

$$n_F{}^P(u) \le S\{n_F{}^P(u * v), n_F{}^P(v)\}$$

$$\le S\{S\{n_F{}^P(u * v) * w, n_F{}^P(w), n_F{}^P(v)\}\}$$

$$= S\{S\{n_F{}^P(0), n_F{}^P(w), n_F{}^P(v)\}\} = S\{n_F{}^P(u), n_F{}^P(w)\}$$

Hence the proof.

Theorem 3.9: Let 'F' be a BPFUSI of X. The given results are same.

(i) F is a BPFUSRI of X

(ii) F satisfies the following results,

$$m_F{}^N(v * (u * w)) \le m_F{}^N((u * w) * (0 * v))$$

$$n_F{}^N(v*(u*w)) \geq n_F{}^N((u*w)*(0*v))$$
$$m_F{}^P(v*(u*w)) \geq m_F{}^P((u*w)*(0*v))$$
$$n_F{}^P(v*(u*w)) \leq n_F{}^P((u*w)*(0*v)),\ if\ u,v,w \in X.$$

(iii) F satisfies the following results.

$$m_F{}^N(v*u) \leq m_F{}^N(u*(0*v)),\ \ n_F{}^N(v*u) \geq n_F{}^N(u*(0*v))$$
$$m_F{}^P(v*u) \geq m_F{}^P(u*(0*v)),\ \ n_F{}^P(v*u) \leq n_F{}^P(u*(0*v))$$

Proof: (i) → (ii)

Let us see 'A' is a BPFUSRI of X and let u, v, w ϵ X by the definition- 3.3, we get,

$$m_F{}^N(v*(u*w)) \leq S\{m_F{}^N(((u*w)*0)*(0*v)), m_F{}^N(0)\}$$
$$= m_F{}^N((u*w)*(0*v))\ \ and$$
$$n_F{}^N(v*(u*w)) \geq T\{n_F{}^N(((u*w)*0)*(0*v)), n_F{}^N(0)\}$$
$$= n_F{}^N((u*w)*(0*v))$$
$$m_F{}^P(v*(u*w)) \geq T\{m_F{}^P(((u*w)*0)*(0*v)), m_F{}^P(0)\}$$
$$= m_F{}^P((u*w)*(0*v))\ \ and$$
$$n_F{}^P(v*(u*w)) \leq S\{n_F{}^P(((u*w)*0)*(0*v)), n_F{}^P(0)\}$$
$$= n_F{}^P((u*w)*(0*v))$$

(ii) → (iii) taking w = 0 in (ii) using (i) induce (iii)

(iv) → (i) Note that $(u*(0*v))*((u*w)*(0*v)) \leq w,\ if\ u,v,w \in X.$

It gives from (iii) and previous result – 3.8 that,

$$m_F{}^N(v*u) \leq m_F{}^N(u*(0*v))$$
$$\leq S\{m_F{}^N((u*w)*(0*v)), m_F{}^N(w)\}$$
$$n_F{}^N(v*u) \geq n_F{}^N(u*(0*v))$$
$$\geq T\{n_F{}^N((u*w)*(0*v)), n_F{}^N(w)\}$$
$$m_F{}^P(v*u) \geq m_F{}^P(u*(0*v))$$
$$\geq T\{m_F{}^P((u*w)*(0*v)), m_F{}^P(w)\}$$
$$n_F{}^P(v*u) \leq n_F{}^P(u*(0*v))$$
$$\leq S\{n_F{}^P((u*v)*(0*v)), n_F{}^P(w)\}$$

Hence, 'F' is a BPFUSI of X.

Theorem 3.10: Every BPFUSI of X is a BPFUSRI of X if X is associative.

Proof: Let 'F' be a BPFUSI of X, since $0*u = u\ for\,all\,u \in X,$ that is,

$$v*u = (0*v)*u$$
$$= (0*u)*v$$
$$= u*v$$
$$= u*(0*v),\ \ for\,all\,u,v \in X.$$

Therefore,

$$m_F{}^N(v*u)=m_F{}^N(u*(0*v))$$
$$n_F{}^N(v*u)=n_F{}^N(u*(0*v))$$
$$m_F{}^P(v*u)=m_F{}^P(u*(0*v))$$
$$n_F{}^P(v*u)=n_F{}^P(u*(0*v)),\ by theorem-3.9.$$

We conclude that 'F' is BPFURI of X.

The following section implemented the bipolar characteristics of R-ideal structures.

4. BIPOLAR R-IDEAL STRUCTURES

Theorem 4.1: Let F be a BPFURI of X. Then the collection $\Delta=\left\{u\in X/m_F{}^N(u)=m_F{}^N(0), n_F{}^N(u)=n_F{}^N(0), m_F{}^P(u)=m_F{}^P(0), n_F{}^P(u)=n_F{}^P(0)\right\}$ is an R-ideal of X.

Proof: Clearly, $0\in\Delta$. Let u, v, w ϵ X be such that $((u*w)*(0*v))\in\Delta\ and\ w\in\Delta$. Then,

$$m_F{}^N(u)\le m_F{}^N(v*u)$$
$$\le S\left\{m_F{}^N(u*(0*v)), m_F{}^N(w)\right\}$$
$$=m_F{}^N(0)$$
$$n_F{}^N(u)\ge n_F{}^N(v*u)$$
$$\ge T\left\{n_F{}^N(u*(0*v)), n_F{}^N(w)\right\}$$
$$=n_F{}^N(0)$$
$$m_F{}^P(u)\ge m_F{}^P(v*u)$$
$$\ge T\left\{m_F{}^P(u*(0*v)), m_F{}^P(w)\right\}$$
$$=m_F{}^P(0)$$
$$n_F{}^P(u)\le n_F{}^P(v*u)$$
$$\le S\left\{n_F{}^P(u*(0*v)), n_F{}^P(w)\right\}$$
$$=n_F{}^P(0)$$

By using definition 2.1 then,

$$m_F{}^N(v*u)=m_F{}^N(0),\ n_F{}^N(v*u)=n_F{}^N(0).$$

That is $v*u\in\Delta$. Therefore Δ is R-ideal of X.

Theorem 4.2: If $F_1\ and\ F_2$ are a BPFUSRI of X, then $F_1\cap F_2$ is also BPFUSRI of X.

Proof: Now, $m_{F_1}{}^N(0)\le m_{F_1}{}^N(u),\ n_{F_1}{}^N(0)\ge n_{F_1}{}^N(u)\ and$

$$m_{F_2}{}^N(0)\le m_{F_2}{}^N(u),\ n_{F_2}{}^N(0)\ge n_{F_2}{}^N(u),\ for all u\in X.$$
$$S\left\{m_{F_1}{}^N(0),\ m_{F_2}{}^N(0)\right\}\le S\left\{m_{F_1}{}^N(u),\ m_{F_2}{}^N(u)\right\}=m_{F_1\cap F_2}{}^N(0)\le m_{F_1\cap F_2}{}^N(0)\ and$$
$$T\left\{m_{F_1}{}^N(0),\ m_{F_2}{}^N(0)\right\}\ge T\left\{m_{F_1}{}^N(u),\ m_{F_2}{}^N(u)\right\}=m_{F_1\cap F_2}{}^N(0)\ge m_{F_1\cap F_2}{}^N(0),\ for all u\in X.$$

Also,

$$m_{F_1}{}^N(v*u)\le S\left\{m_{F_1}{}^N((u*w)*(0*v)),\ m_{F_1}{}^N(w)\right\}$$

$$m_{F_2}{}^N(v*u)\le S\{m_{F_2}{}^N((u*w)*(0*v)), m_{F_2}{}^N(w)\}$$
$$n_{F_1}{}^N(v*u)\ge T\{n_{F_1}{}^N((u*w)*(0*v)), n_{F_1}{}^N(w)\}$$
$$n_{F_2}{}^N(v*u)\ge T\{n_{F_2}{}^N((u*w)*(0*v)), n_{F_2}{}^N(w)\}$$
$$S\{m_{F_1}{}^N(v*u), m_{F_2}{}^N(v*u)\}\le S\{m_{F_1}{}^N((u*w)*(0*v)),\ m_{F_1}{}^N(w)\}, S\{m_{F_2}{}^N((u*w)*(0*v)),\ m_{F_2}{}^N(w)\}$$

$$T\{n_{F_1}{}^N(v*u),\ n_{F_2}{}^N(v*u)\}\ge T\{n_{F_1}{}^N((u*w)*(0*v)),\ n_{F_1}{}^N(w)\}, T\{n_{F_2}{}^N((u*w)*(0*v)),\ n_{F_2}{}^N(w)\}$$

$$m_{F_1}{}^P(0)\ge m_{F_1}{}^P(u),\quad n_{F_1}{}^P(0)\le n_{F_1}{}^P(u)\ and$$
$$m_{F_2}{}^P(0)\ge m_{F_2}{}^P(u),\quad n_{F_2}{}^P(0)\le n_{F_2}{}^P(u),\ for\,all\,u\in X.$$
$$T\{m_{F_1}{}^P(0),\ m_{F_2}{}^P(0)\}\ge T\{m_{F_1}{}^P(u),\ m_{F_2}{}^P(u)\}$$
$$= m_{F_1\cap F_2}{}^P(0)\ge m_{F_1\cap F_2}{}^P(u)\ and$$
$$S\{n_{F_1}{}^P(0),\ n_{F_2}{}^P(0)\}\le S\{n_{F_1}{}^P(u),\ n_{F_2}{}^P(u)\}$$
$$= n_{F_1\cap F_2}{}^P(0)\le n_{F_1\cap F_2}{}^P(u),\ for\,all\,u\in X.$$

Again,

$$m_{F_1}{}^P(v*u)\ge T\{m_{F_1}{}^P((u*w)*(0*v)),\ m_{F_1}{}^P(w)\}$$
$$m_{F_2}{}^P(v*u)\ge T\{m_{F_2}{}^P((u*w)*(0*v)),\ m_{F_2}{}^P(w)\}$$
$$n_{F_1}{}^P(v*u)\le S\{n_{F_1}{}^P((u*w)*(0*v)),\ n_{F_1}{}^P(w)\}$$
$$n_{F_2}{}^P(v*u)\le S\{n_{F_2}{}^P((u*w)*(0*v)),\ n_{F_2}{}^P(w)\}$$
$$T\{m_{F_1}{}^N(v*u),\ m_{F_2}{}^N(v*u)\}\ge T\{T\{m_{F_1}{}^P((u*w)*(0*v)),\ m_{F_1}{}^P(w)\},\ T\ \{m_{F_2}{}^P((u*w)*(0*v)),\ m_{F_2}{}^P(w)\}\}$$

$$S\{n_{F_1}{}^N(v*u),\ n_{F_2}{}^N(v*u)\}\le S\{S\{n_{F_1}{}^P((u*w)*(0*v)),\ n_{F_1}{}^P(w)\},\ S\{n_{F_2}{}^P((u*w)*(0*v)),\ n_{F_2}{}^P(w)\}\}$$

$$m_{F_1\cap F_2}{}^P(0)\ge T\{m_{F_1\cap F_2}{}^P((u*w)*(0*v)),\ m_{F_1\cap F_2}{}^P(w)\}\ and$$
$$n_{F_1\cap F_2}{}^P(0)\le S\{n_{F_1\cap F_2}{}^P((u*w)*(0*v)),\ n_{F_1\cap F_2}{}^P(w)\},\ for\,all\,u,v,w\in X.$$

Hence, F1∩F2 is also BPFUSRI of X.

Definition 4.3: For a bipolar fermatean uncertainty soft set 'F' in X and $(\alpha,\ \beta)\in[0,\ 1]\ and$ $(\gamma,\ \sigma)\in[-1,\ 0]$, the positive $(\alpha,\beta)-cut$ and negative $(\gamma,\ \sigma)-cut$ are denoted by $F^P(\alpha,\ \beta)\ and$ $F^N(\gamma,\ \sigma)$ are expressed as follows:

$$F^P(\alpha,\beta)=\{a\in X/m_F{}^P(u)\ge\alpha\ and\ n_F{}^P(u)\le\beta\}\ and$$
$$F^N(\gamma,\sigma)=\{u\in X/m_F{}^N(u)\ge\gamma\ and\ n_F{}^N(u)\le\sigma\}\ with\ \alpha+\beta\le 1\ and\ \gamma+\sigma\ge -1\ respectively.$$

The bipolar fermatean uncertainty soft level cut of F denoted by F_{cut} is represented to be the collections $F_{cut}=(F^P(\alpha,\beta),\ F^N(\gamma,\sigma))$.

Theorem 4.4: A bipolar fermatean uncertainty soft set F in X is a BPFUSRI of X iff for all $(\alpha, \beta) \in [0, 1]$ and $(\gamma, \sigma) \in [-1, 0]$, the non-empty positive $(\alpha, \beta) - cut$ and the non-empty negative $(\gamma, \sigma) - cut$ are BPFUSRI of X.

Proof: Let 'A' be BPFUSRI of X and clear that $F^P(\alpha, \beta)$ *and* $F^N(\gamma, \sigma)$ are non-empty for $(\alpha, \beta) \in [0,1]$ *and* $(\gamma, \sigma) \in [-1, 0]$, obviously $0 \in F^P(\alpha, \beta) \cap F^N(\gamma, \sigma)$.

Let for all u, v, w ϵ X be such that

$$m_F{}^N((u*w)*(0*v)) \in F^N(\gamma, \sigma) \text{ and } m_F{}^N(w) \in F^N(\gamma, \sigma)$$

$$n_F{}^N((u*w)*(0*v)) \in F^N(\gamma, \sigma) \text{ and } n_F{}^N(w) \in F^N(\gamma, \sigma)$$

Then

$$m_F{}^N((u*w)*(0*v)) \le \gamma, \; m_F{}^N(w) \le \gamma$$

$$n_F{}^N((u*w)*(0*v)) \ge \sigma, \; n_F{}^N(w) \le \sigma.$$

It follows from definition 2.1 that

$$m_F{}^N(v*u) \le S\left\{m_F{}^N((u*w)*(0*v)), m_F{}^N(w)\right\} \le \gamma \text{ and}$$

$$n_F{}^N(v*u) \ge T\left\{n_F{}^N((u*w)*(0*v)), n_F{}^N(w)\right\} \ge \sigma.$$

So that, $v*u \in F^N(\gamma, \sigma)$.

Now let us see that,

$$m_F{}^P((u*w)*(0*v)) \in F^P(\alpha, \beta) \text{ and } m_F{}^P(w) \in F^P(\alpha, \beta) \text{ and}$$

$$n_F{}^P((u*w)*(0*v)) \in F^P(\alpha, \beta) \text{ and } n_F{}^P(w) \in F^P(\alpha, \beta)$$

Then

$$m_F{}^P((u*w)*(0*v)) \ge \alpha, m_F{}^P(w) \ge \alpha$$

$$n_F{}^P((u*w)*(0*v)) \le \beta, n_F{}^P(w) \le \beta.$$

If obeys from the definition 2.1 that

$$m_F{}^P(v*u) \ge T\left\{m_F{}^P((u*w)*(0*v)), m_F{}^P(w)\right\} \ge \alpha \text{ and}$$

$$n_F{}^P(v*u) \le S\left\{n_F{}^P((u*w)*(0*v)), n_F{}^P(w)\right\} \le \beta$$

So that, $v*u \in F^P(\alpha, \beta)$.

Therefore, $F^P(\alpha, \beta)$ *and* $F^N(\gamma, \sigma)$ are R-ideal of X. Reversely, suppose that the non-empty, negative $(\gamma, \sigma) - cut$ and the elements of positive $(\alpha, \beta) - cut$ are R-ideal of X for every $(\alpha, \beta) \in [0,1]$ *and* $(\gamma, \sigma) \in [-1, 0]$.

If $m_F{}^N(0) \ge m_F{}^N(u),\; n_F{}^N(0) \le n_F{}^N(u)$

$$m_F{}^P(0) \le m_F{}^P(u),\; n_F{}^P(0) \ge n_F{}^P(u), \text{ for } u \in X.$$

Then either $0 \notin F^N(m_F{}^N(u), n_F{}^N(u))$ or $0 \notin F^P(m_F{}^P(u), n_F{}^P(u))$.

This is a contradiction that $m_F{}^N(0) \le m_F{}^N(u),\; n_F{}^N(0) \ge n_F{}^N(u)$ and $m_F{}^P(0) \ge m_F{}^P(u),\; n_F{}^P(0) \ge n_F{}^P(u), \text{ for all } u \in X$.

Let us assume that,

$$m_F{}^N(v*u) \geq S\{m_F{}^N((u*w)*(0*v)), m_F{}^N(w)\} = \gamma \ and$$
$$n_F{}^N(v*u) \leq T\{n_F{}^N((u*w)*(0*v)), n_F{}^N(w)\} = \sigma \ for \ all \ u,v,w \in X.$$

Then, $((u*w)*(0*v)) \in F^N(\gamma,\sigma) \ and \ w \in F^N(\gamma,\sigma), \ but \ v*u \notin F^N(\gamma,\sigma).$

This is not possible and thus,

$$m_F{}^N(v*u) \leq S\{m_F{}^N((u*w)*(0*v)), m_F{}^N(w)\} = \gamma \ and$$
$$n_F{}^N(v*u) \geq T\{n_F{}^N((u*w)*(0*v)), n_F{}^N(w)\} = \sigma \ for \ all \ u,v,w \in X.$$

If $m_F{}^P(v*u) \leq T\{m_F{}^P((u*w)*(0*v)), m_F{}^P(w)\} = \alpha \ and$

$n_F{}^P(v*u) \geq S\{n_F{}^P((u*w)*(0*v)), n_F{}^N(w)\} = \beta \ for \ all \ u,v,w \in X.$

Then, $((u*w)*(0*v)) \in F^P(\alpha,\beta) \ and \ w \in F^P(\alpha,\beta), \ but \ v*u \notin F^P(\alpha,\beta).$

This is not possible and thus,

$$m_F{}^P(v*u) \geq T\{m_F{}^P((u*w)*(0*v)), m_F{}^P(w)\} = \alpha \ and$$
$$n_F{}^P(v*u) \leq S\{n_F{}^P((u*w)*(0*v)), n_F{}^N(w)\} = \beta \ for \ all \ u,v,w \in X.$$

Consequently, 'F' in BPFUSRI of X.

Conclusion: Here, the notion of bipolar fermatean uncertainty soft R-ideals in terms of BCK-algebra are introduced and their properties are investigated. Also relationships between bipolar fermatean uncertainty soft sub algebra, bipolar fermatean uncertainty soft ideals are analyzed.

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