

Effect of Rotation on Linear Double Diffusive Convection of a Binary Liquid in a Metal Foam Porous Rectangular Enclosure Using Lapwood - Brinkman Model with Cross Diffusion Effect

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Abstract: In this paper, the cross-diffusion effects on double diffusive linear convection of binary liquid in a horizontal rectangular metal channel saturated by anisotropic porous media, which is heated and salted from below are studied. The critical Rayleigh number for the onset of convection in terms of anisotropy parameter, solute Rayleigh number, Soret and DuFour parameters are determined numerically using Newton-Raphson method and the effects are shown graphically

Keywords: Double diffusive convection, Anisotropy, Fourier series, Rayleigh number, Soret/DuFour parameter.

1. Introduction

Double diffusive convection is a fluid flow phenomenon in which two different components of a fluid go through separate molecular diffusion processes, resulting in distinct density fluctuations. Temperature and concentration of a solute, such as salt, are often involved in the two components. The study of double diffusive convection is significant in a variety of scientific and engineering applications, including oceanography, astronomy, geophysics, and industrial processes requiring heat and mass movement. To understand the behaviour of fluids under these conditions, researchers employ mathematical models, numerical simulations, and experimental studies.

The double diffusive convection in porous media has become important because of its applications in geophysics, particularly in saline geothermal fields where hot brines remain beneath less saline, cooler ground water. In a system where two diffusing properties are present, instabilities can occur only if one of the components are destabilizing. When heat and mass transfer occur simultaneously in a moving fluid, the relation between the fluxes and the driving potentials are of more intricate in nature. If the cross-diffusion terms are included in the species transport equations, then the situation will be quite different. Due to cross-diffusion effects, each property gradient has a significant influence on the flux of the other property. There are many studies available on the effect of cross diffusions on the onset of double diffusive convection in a porous medium.

Heat and mass transfer by natural convection at a stagnation point in a porous medium considering Soret and DuFour effects, was studied by Postelincu [1]. Gaikwad et al. [2] was discussed linear and non-linear double diffusive convection in a fluid-saturated anisotropic porous layer with cross diffusion effects. Afify [3] was investigated effects of temperature-dependent viscosity with Soret and DuFour numbers on non-Darcy MHD free convective heat and mass transfer past a vertical surface embedded in a porous medium. El-Arabawy [4] was studied Soret and DuFour effects on natural convection flow past a vertical surface in a porous medium with variable surface temperature. Mojtabi[5] was discussed the effects of double diffusive convection in porous media. Asogwa et al. [6] was studied the Double diffusive convection and cross diffusion effects on Casson fluid over a

Lorentz force driven Riga plate in a porous medium with heat sink. Noreldin et al. [7] was studied by the cross diffusion effect on double diffusive convection in a rotating vertical porous cylinder with vertical through flow. Noreen et al. [8] studied the double-diffusive convection in flow of viscous fluid is investigated inside a horizontal channel. Lakshmi et al. [9] was studied the natural convection of a binary liquid in cylindrical porous annuli/rectangular porous enclosures with cross-diffusion effects under local thermal non-equilibrium state. Aghighi M et al. [10] study the double-diffusive natural convection of Casson fluids in an enclosure. Shilpa et al. [11] studied the Soret and DuFour effects on MHD double-diffusive mixed convective heat and mass transfer of couple stress fluid in a channel formed by electrically conducting and non-conducting walls. Tayebi et al. [12] studied the double-diffusive natural convection with Soret/DuFour effects and energy optimization of Nano-encapsulated phase change material in a novel form of a wavy-walled I-shaped domain. Malashety and Biradar [13] studied the onset of double diffusive convection in a binary Maxwell fluid saturated porous layer with cross-diffusion effects. Bhadauria [14] study the double-diffusive convection in a saturated anisotropic porous layer with internal heat source. The impact of the Soret and DuFour effects on double-diffusive convection in a vertical porous layer filled with a binary mixture and exposed to horizontal heat and solute gradients was investigated analytically and numerically by Bouachir [15]. An analytical investigation has been conducted into the impact of "Cross Diffusion" on the linear and nonlinear stability of double diffusive convection within a fluid-saturated porous media by Rudraiah [16] The effect of the through flow and gravity fluctuation on thermosolutal convection in an anisotropic porous medium with the Darcy–Brinkman effect was numerically by Honnappa et al. [17]. Double diffusive convection in a rectangular container with thermal radiation and a porous lining is studied analytically by Sasirekha et al. [18] Numerical Study of Double-Diffusive Convection Using Finite Element Method in an Irregular Porous Cavity Under Inclined Magnetic Field by Chuhan [19] LTNE Effects and Symmetric Properties of Nonlinear Stability and Linear Instability of Double-Diffusive Convection in a Rotating Forchheimer-Brinkmann Model was studied by Abed et al. [20] In a porous material with horizontal through flow, the implications of coupled horizontal and vertical heterogeneity on the onset of convection by Nield [21]. In a vertical layer of Darcy porous medium, the stability of double-diffusive buoyant flow is studied, with boundaries maintained at varying constant temperatures and solute concentration by Shankar [22]. Double-diffusive convection of a revolving circular cylinder suspended by phase-change materials encapsulated in nano particles in a porous cavity was studied by Raizah [23]. Variations in the solute concentration and boundary temperature vertically lead to the instability of double diffusion natural convection in porous layers was studied Lu S and Jia [24]. Narayana [25, 26, 27] was examined the effects of Soret and Dufour in a doubly layered Darcy porous material.

In this chapter we study the effect of rotation on double diffusive convection in a rectangular channel filled with anisotropic porous media is considered. Walls of the channel are non-uniformly heated to establish linear temperature gradient and they are assumed to be impermeable and perfectly heat conducting, heated and salted from below in the presence of Soret and DuFour effects. This is studied numerically using linear stability analysis. The generalized Darcy model without time derivative is employed for the momentum equation. The effects of anisotropy parameter, diffusivity ratio, solute Rayleigh number and Soret and DuFour effects, Taylor number are discussed in detail.

2. Mathematical Formulation:

In a two-dimensional porous rectangular enclosure with height 'd' and breadth 'a' we examine the cross diffusion effect on natural convection of binary liquid with the channel wall being salted and heated from below. we assumed that the cross diffusion effect of Du Four and Soret effects are within the system. A constant temperature gradient (ΔT) and a constant concentration gradient (ΔS) are maintained between the boundaries. To describe fluid flow in porous media we use Lapwood- brinkman model in the momentum equation. Since the channel is a rectangle, we consider the vertical direction as the z -axis and the horizontal direction as the x -axis. Figure 1

shows a vertical wall from $x = -\frac{a}{2}$ to $x = \frac{a}{2}$, and horizontal walls are shown from $z = 0$ to $z = d$

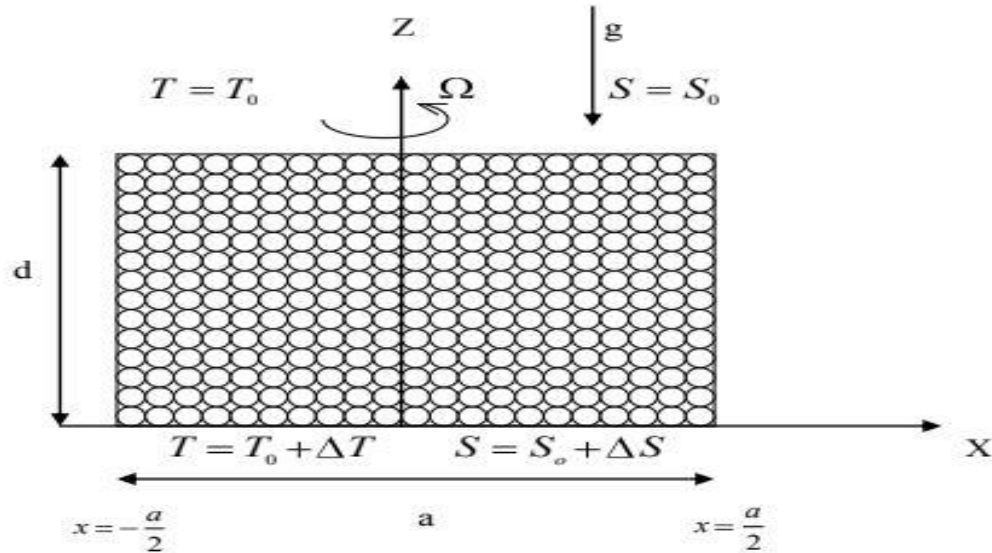


Figure1; Schematic diagram of the flow configuration

Basic Governing Equations are:

Equation of Continuity

$$\nabla \cdot \vec{q} = 0, \quad (1)$$

Momentam Equation

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \rho \vec{g} - \frac{\mu_f}{K} \vec{q} + \mu_e \nabla^2 \vec{q} + 2\Omega \times \vec{q}, \quad (2)$$

Energe Equation

$$\gamma \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = K_{11} \nabla^2 T + K_{12} \nabla^2 S, \quad (3)$$

Concentration Equation

$$\varepsilon \frac{\partial S}{\partial t} + (\vec{q} \cdot \nabla) S = K_{22} \nabla^2 T + K_{21} \nabla^2 S, \quad (4)$$

Equation of State

$$\rho = \rho_0 [1 - \beta_T (T - T_0) + \beta_S (S - S_0)]. \quad (5)$$

We neglect the inertia terms by assuming that the Darcy- Prandtl number is large (because the inertia effect is smaller than the Darcy effect, so we neglect the inertia term and this Darcy effect help to simplify the equation) and also introduce the Boussinesq approximation (because to simplify the modeling buoyancy- driven flow and easy to solve

numerically so we use Boussinesq approximation) then the two dimensional form of the Darcy- Boussinesq equations given by

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad (6)$$

$$\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{g}{K_x} u - \nu \frac{\partial^2 u}{\partial x^2} - 2\nu \Omega = 0, \quad (7)$$

$$\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\nu}{K_y} v - \nu \frac{\partial^2 u}{\partial y^2} + 2u\Omega = 0, \quad (8)$$

$$\frac{1}{\rho_0} \frac{\partial p}{\partial z} + \frac{\nu}{K_z} w - \nu \frac{\partial^2 w}{\partial z^2} + \frac{\rho}{\rho_0} g = 0, \quad (9)$$

$$\varepsilon \frac{\partial T}{\partial t} + \left(u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} \right) = K_{11x} \frac{\partial^2 T_f}{\partial x^2} + K_{11z} \frac{\partial^2 T_f}{\partial z^2} + K_{12} \left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial z^2} \right), \quad (10)$$

$$\varepsilon \frac{\partial S}{\partial t} + \left(u \frac{\partial S}{\partial x} + w \frac{\partial S}{\partial z} \right) = K_{22x} \frac{\partial^2 T_s}{\partial x^2} + K_{22z} \frac{\partial^2 T_s}{\partial z^2} + K_{21} \left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial z^2} \right), \quad (11)$$

$$\rho = \rho_0 [1 - \beta_T (T - T_0) + \beta_S (S - S_0)]. \quad (12)$$

Basic State:

In this state, fluid is appearing to be inactive. Hence, we have

$$(u, v, w) = 0, T_f = T_{fb}(z), \rho = \rho_b(z), T_s = T_{sb}(z), S = S_b(z), p = p_b(z). \quad (13)$$

Using the equation (13) to (7) - (12) we get

$$\frac{dp_b(z)}{dz} = -\rho_b(z)g, \quad \frac{d^2 T_{fb}(z)}{dz^2} = 0, \quad \frac{d^2 T_{sb}(z)}{dz^2} = 0, \quad \frac{d^2 S_b(z)}{dz^2} = 0$$

$$\rho = \rho_0 [1 - \beta_T (T_b(z) - T_0) + \beta_S (S_b(z) - S_0)] \quad (14)$$

Because the two-dimensional motion, we brought the stream function ψ as follows

$$u = \frac{\partial \psi}{\partial z}, \quad w = -\frac{\partial \psi}{\partial x}$$

(15)

Also we brought the variables. They were not dimensional, and they were described as follows;

$$\begin{aligned} x &= a x^*, \quad z = d z^*, \quad u = \frac{K_{11} a}{d^2} u^*, \quad w = \frac{K_{11}}{d} w^*, \quad t = \frac{d^2}{K_{11}} t^* \\ p &= \frac{K_{11} \nu \rho_0}{k_x} p^*, \quad \phi = (\Delta T) \phi^*, \quad S = (\Delta S) S^*, \quad T_0 = (\Delta T) T^*, \quad \psi = \frac{K_{11} a}{d} \psi^* \\ T_{fb}(z) &= \left\{ (T_0)^* + \Delta T \left(1 - \frac{z}{d} \right) \right\} + \phi^*, \quad S_b = \left\{ (S_0)^* + \Delta S \left(1 - \frac{z}{d} \right) \right\} + S^*. \end{aligned} \quad (16)$$

Remove the pressure and the density from (7) to (13) and by using stream function we get

$$\xi \frac{\partial^2 \psi}{\partial x^2} + (1 + Ta) \frac{\partial^2 \psi}{\partial z^2} + \xi Ra_T \frac{\partial \phi}{\partial x} - \xi Ra_S \frac{\partial S}{\partial x} - Da \frac{\partial^4 \psi}{\partial x^2 \partial z^2} = 0, \quad (17)$$

$$\left(\gamma \frac{\partial}{\partial t} - \eta_f \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2} \right) \phi = -\frac{\partial \psi}{\partial x} + \frac{\partial(\psi, \phi)}{\partial(x, z)} + Du \frac{Ra_S}{Ra_T} \nabla^2 S, \quad (18)$$

$$\left(\varepsilon \frac{\partial}{\partial t} - \tau \nabla^2 \right) S = -\frac{\partial \psi}{\partial x} + \frac{\partial(\psi, S)}{\partial(x, z)} + Sr \frac{Ra_T}{Ra_S} \nabla^2 \phi, \quad (19)$$

3 NUMERICAL SOLUTION AND LINEAR STABILITY ANALYSIS;

The equations (17) - (19) in linear forms are

$$\xi \frac{\partial^2 \psi}{\partial x^2} + (1 + Ta) \frac{\partial^2 \psi}{\partial z^2} + \xi Ra_T \frac{\partial \phi}{\partial x} - \xi Ra_S \frac{\partial S}{\partial x} - Da \frac{\partial^4 \psi}{\partial x^2 \partial z^2} = 0, \quad (20)$$

$$\left(\gamma \frac{\partial}{\partial t} - \eta_f \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2} \right) \phi = -\frac{\partial \psi}{\partial x} + Du \frac{Ra_S}{Ra_T} \nabla^2 S, \quad (21)$$

$$\left(\varepsilon \frac{\partial}{\partial t} - \tau \nabla^2 \right) S = -\frac{\partial \psi}{\partial x} + Sr \frac{Ra_T}{Ra_S} \nabla^2 \phi. \quad (22)$$

Equations (20) to (22) are subject to the following boundary condition:

$$\psi = \phi = S = 0 \quad \text{at} \quad \begin{cases} x = -\frac{1}{2}, & x = \frac{1}{2}, & 0 < z < 1 \\ z = 0, & z = 1, & -\frac{1}{2} < x < \frac{1}{2} \end{cases} \quad (23)$$

The onset of convection is described by the equation (20) to (22) and a series of Fourier coefficient representing the solution for ψ , ϕ , and S that satisfy the equation (23) is as follows:

$$\psi = e^{\sigma} \left[\frac{1}{2} C_0 + \sum_{i=1}^{\infty} C_i(x) \cos i\pi z + \sum_{i=1}^{\infty} D_i(x) \sin i\pi z \right], \quad (24)$$

$$\phi = e^{\sigma} \left[\frac{1}{2} F_0 + \sum_{i=1}^{\infty} F_i(x) \cos i\pi z + \sum_{i=1}^{\infty} G_i(x) \sin i\pi z \right], \quad (25)$$

$$S = e^{\sigma} \left[\frac{1}{2} H_0 + \sum_{i=1}^{\infty} H_i(x) \cos i\pi z + \sum_{i=1}^{\infty} I_i(x) \sin i\pi z \right]. \quad (26)$$

When $\sigma = 0$ for marginal stability. The boundary condition (23) is satisfied by the equation (24) to (26). For every value of x , $C_i = F_i = H_i = 0$ and ψ, ϕ and S are replaced by a single term then equation (24) to (26) are reduced to form

$$\psi = D_1(x) \sin \pi z \quad \phi = G(x) \sin \pi z \quad S = I(x) \sin \pi z \quad (27)$$

The boundary condition (23) on the horizontal boundary is satisfied by these equations

The following are the Boundary conditions for D , G and I are

$$D_1\left(\pm\frac{1}{2}\right) = 0, \quad G\left(\pm\frac{1}{2}\right) = 0, \quad I\left(\pm\frac{1}{2}\right) = 0, \quad (28)$$

Using the equation (27) in equations (20) to (23), we get

$$\left(\xi \frac{d^2}{dx^2} - (1 + Ta)\pi^2 + Da\pi^2 \frac{d^2}{dx^2} \right) D_1(x) + \xi Ra_T \frac{dG(x)}{dx} - \xi Ra_S \frac{dI(x)}{dx} = 0, \quad (29)$$

$$\left(\eta_f \frac{d^2}{dx^2} - \pi^2 \right) G(x) = \frac{dD_1(x)}{dx} - S_1 \left(\frac{d^2}{dx^2} - \pi^2 \right) I(x), \quad (30)$$

$$\tau \left(\eta_s \frac{d^2}{dx^2} - \pi^2 \right) I(x) = \frac{dD_1(x)}{dx} - S_2 \left(\frac{d^2}{dx^2} - \pi^2 \right) G(x).$$

(31)

When $D_1(x)$ and $I(x)$ are removed from the equation (29) to (31) we obtain an six order differential equation in the following form:

$$\left[\delta_1^2 D^6 - \delta_2^2 D^4 + \delta_3^2 D^2 - \delta_4^2 \right] G(x) = 0 \quad (32)$$

Where:

$$\delta_1^2 = \xi \tau + Da \tau \pi^2 - \xi S_1 S_2 - Da S_1 S_2 \pi^2$$

$$\delta_2^2 = 2\xi \tau \pi^2 \xi + 2Da \tau \pi^4 + \tau \pi^2 + \tau Ta \pi^2 - \xi Ra_T \tau + \xi Ra_S - \xi Ra_S S_2 - 2\xi S_1 S_2 \pi^2 - S_1 S_2 \pi^2 - S_1 S_2 Ta \pi^2 - 2S_1 S_2 Da \pi^4 + \xi Ra_T S_1$$

$$\delta_3^2 = \xi \tau \pi^2 + 2\tau \pi^4 + 2\tau Ta \pi^4 + Da \tau \pi^6 - \xi Ra_T \tau \pi^2 + \xi Ra_T S_1 \pi^2 + \xi Ra_S \pi^2 - \xi Ra_S S_2 \pi^2 - \xi S_1 S_2 \pi^4 - 2S_1 S_2 \pi^4 - 2S_1 S_2 Ta \pi^4 - S_1 S_2 Da \pi^6$$

$$\delta_4^2 = \tau \pi^6 + \tau Ta \pi^4 - S_1 S_2 \pi^4 - S_1 S_2 Ta \pi^6.$$

Where, $D = \frac{d}{dx}$, $S_1 = Du \frac{Ra_S}{Ra_T}$, $S_2 = Sr \frac{Ra_T}{Ra_S}$

The boundary condition on G which can be obtained from equation (28) to (31) in the form

$$G\left(\pm \frac{1}{2}\right) = DG\left(\pm \frac{1}{2}\right) = D^2 G\left(\pm \frac{1}{2}\right) = 0$$

(33)

Equation (32) has the following general solution

$$G(x) = c_1 e^{(m_1 x)} + c_2 e^{(m_2 x)} + c_3 e^{(m_3 x)} + c_4 e^{(m_4 x)} + c_5 e^{(m_5 x)} + c_6 e^{(m_6 x)}. \quad (34)$$

Where, c_i 's represents arbitrary constants and m_i 's represents the roots of the auxiliary equation (32). Since the auxiliary equation involves cubic in D^2 , we can put

$$m_2 = -m_1, \quad m_4 = -m_3, \quad m_6 = -m_5.$$

Where,

$$m_1 = \frac{1}{6\delta_1^2} \left[2\delta_2^2 + \left(2^{\frac{4}{3}} \frac{K_1}{K_3} \right) + \left(2^{\frac{2}{3}} K_3 \right) \right],$$

$$m_3 = \frac{1}{12\delta_1^2} \left[4\delta_2^2 - \left(2^{\frac{4}{3}} (1 + \sqrt{-3}) \frac{K_1}{K_3} \right) + \left(2^{\frac{2}{3}} (-1 + \sqrt{-3}) K_3 \right) \right],$$

$$m_5 = \frac{1}{12\delta_1^2} \left[4\delta_2^2 + \left(2^{\frac{4}{3}} (-1 + \sqrt{-3}) \frac{K_1}{K_3} \right) - \left(2^{\frac{2}{3}} (1 + \sqrt{-3}) K_3 \right) \right].$$

A system of six equations are arising from equations (32) – (34) and we get a non- trivial solution for this homogeneous system of equations, we require

$$\begin{vmatrix} e^{\frac{1}{2}m_1} & e^{-\frac{1}{2}m_1} & e^{\frac{1}{2}m_3} & e^{-\frac{1}{2}m_3} & e^{\frac{1}{2}m_5} & e^{-\frac{1}{2}m_5} \\ e^{-\frac{1}{2}m_1} & e^{\frac{1}{2}m_1} & e^{-\frac{1}{2}m_3} & e^{\frac{1}{2}m_3} & e^{-\frac{1}{2}m_5} & e^{\frac{1}{2}m_5} \\ m_1 e^{\frac{1}{2}m_1} & -m_1 e^{-\frac{1}{2}m_1} & m_3 e^{\frac{1}{2}m_3} & -m_3 e^{-\frac{1}{2}m_3} & m_5 e^{\frac{1}{2}m_5} & -m_5 e^{-\frac{1}{2}m_5} \\ m_1 e^{-\frac{1}{2}m_1} & -m_1 e^{\frac{1}{2}m_1} & m_3 e^{-\frac{1}{2}m_3} & -m_3 e^{\frac{1}{2}m_3} & m_5 e^{-\frac{1}{2}m_5} & -m_5 e^{\frac{1}{2}m_5} \\ m_1^2 e^{\frac{1}{2}m_1} & m_1^2 e^{-\frac{1}{2}m_1} & m_3^2 e^{\frac{1}{2}m_3} & m_3^2 e^{-\frac{1}{2}m_3} & m_5^2 e^{\frac{1}{2}m_5} & m_5^2 e^{-\frac{1}{2}m_5} \\ m_1^2 e^{-\frac{1}{2}m_1} & m_1^2 e^{\frac{1}{2}m_1} & m_3^2 e^{-\frac{1}{2}m_3} & m_3^2 e^{\frac{1}{2}m_3} & m_5^2 e^{-\frac{1}{2}m_5} & m_5^2 e^{\frac{1}{2}m_5} \end{vmatrix} = 0 \quad (35)$$

The left hand side of above equation (35) is considered as function of Ra_C , say $f(Ra_C)$ with Ra_C depending on ξ, Ra_S, τ, Du and Sr . Hence, it is possible to write the equation (35) as $f(Ra_C) = 0$. Applying the Newton-Raphson technique for various values of ξ, Ra_S, τ, Du and Sr , critical Rayleigh number Ra_C is computed numerically by employing the following formula

$$(Ra_C)_{k+1} = (Ra_C)_k - \frac{f(Ra_C)_k}{f'(Ra_C)_k}. \quad (k = 0, 1, 2, 3, \dots) \quad (36)$$

Where,

$$f'((Ra_C)_k) = \lim_{\delta Ra_C \rightarrow 0} \left[\frac{f((Ra_C)_k + \delta Ra_C) - f((Ra_C)_k)}{\delta Ra_C} \right].$$

4.Results and Discussion;

Figure 2 shows the variation Ra_C with Ra_S for different values of mechanical anisotropy parameter ξ and for fixed values of other parameters. We observe that Ra_C decreases with increasing anisotropy parameter

ξ . Indicating that the effect of increasing ξ is to advance the onset of convection. The effect of increasing ξ on Ra_C diminishes as ξ becomes large.

Figure 3 and 4 shows the variation of Ra_C with Ra_S for different values of Du parameter and for fixed values of other parameters. We observed from these figures that for small values of Ra_S , the effect of increasing Du parameter is to decrease the Ra_C , while the trends revers for large value of Ra_S

Figure 5 and 6 shows the variation of Ra_C with Ra_S for different values of Sr parameter and for fixed values of other parameters. We find from these figure that as the Sr parameter increases positively Ra_C decreases. However, we find that the effect of increasing small Sr number is to increase the Ra_C . This is due to fact that, for small Sr number, the heavier component migrate towards the hotter region, thus counteracting the density gradient caused by temperature.

Figure 7 shows the variation of Ra_C with Ra_S for different values of diffusivity ratio τ and for fixed values of other parameters. We observe that when Ra_S is small, an increase in τ increases the Ra_C , indicating that the effect of increasing τ is to stabilize the system. However, for large value of the Ra_S , the trend reverses.

Figure 8 shows the variation of Ra_C with Ra_S for different values of Da and for fixed values of other parameters. We observe that Ra_C decreases with increasing Da , indicating that the effect of increasing Da is to advance the onset of convection.

Figure 9 shows the variation of Ra_C with Ra_S for different values of Ta and for fixed values of other parameters. We observed from the diagram that for small value of Ra_S and increasing the value of Ta decreases the Ra_C . While trends revers for large value Ra_S and also observed that increasing the Ta make the system stable.

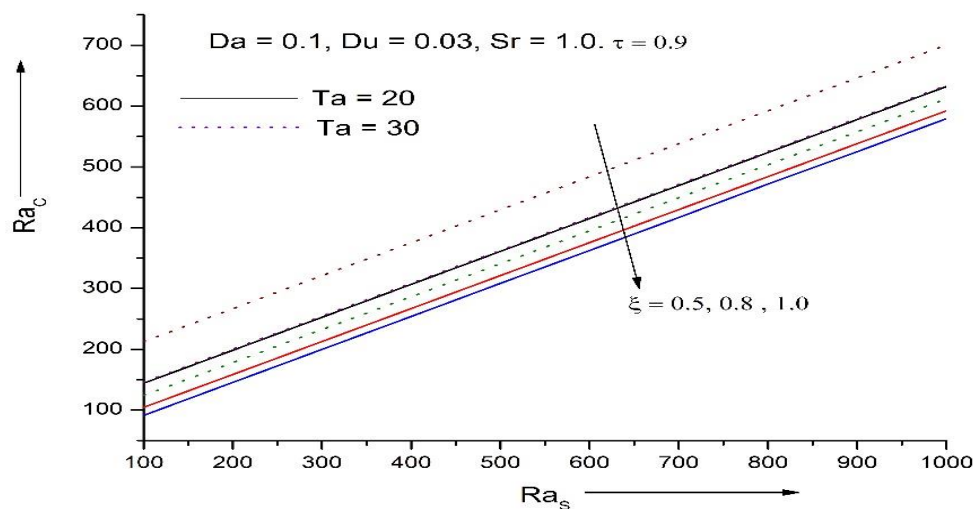
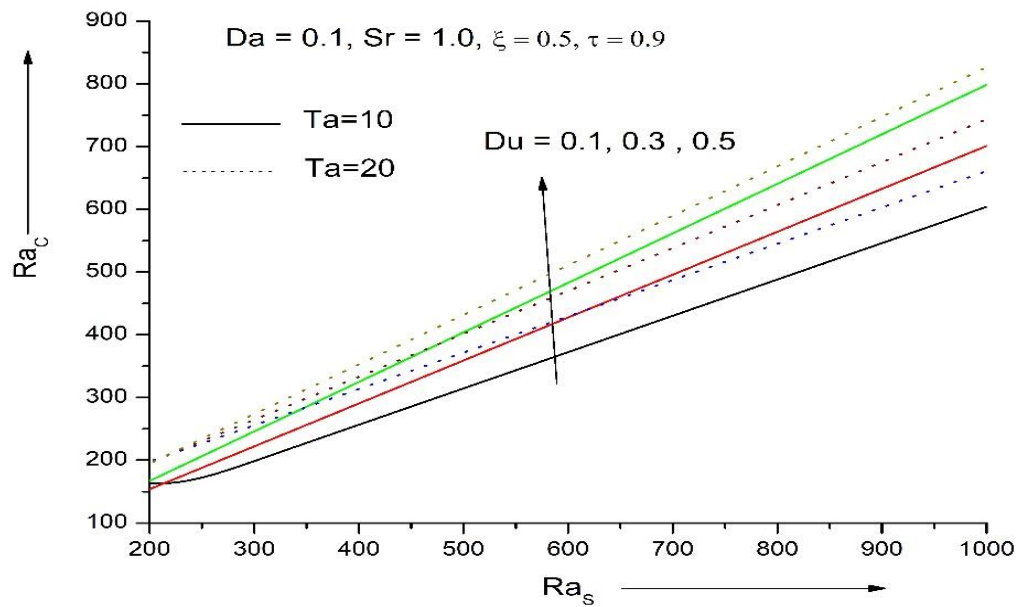
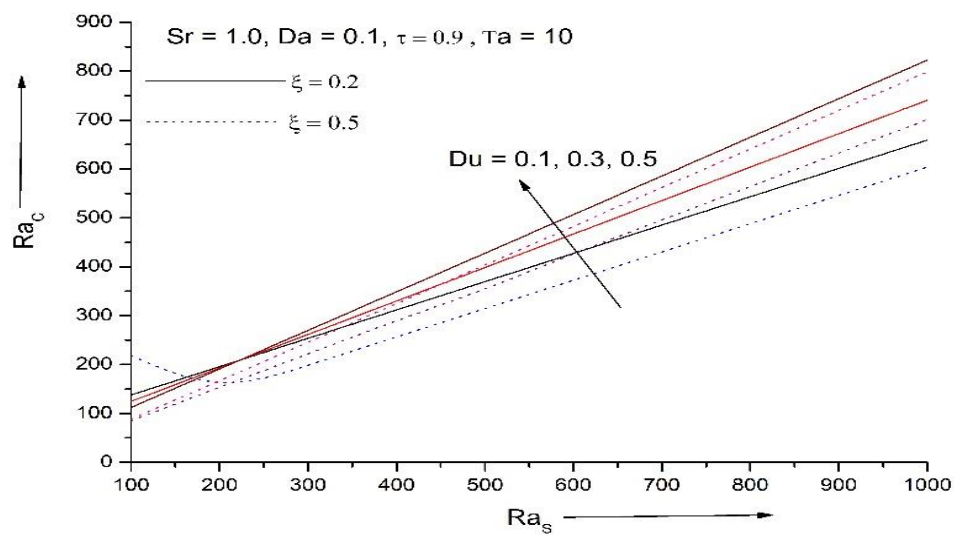


Figure 2; The change of Ra_C versus Ra_S for distinct values of Ta and ξ .Figure 3; The change of Ra_C versus Ra_S for distinct values of Ta and Du .Figure 4; The change of Ra_C versus Ra_S for distinct values of ξ and Du .

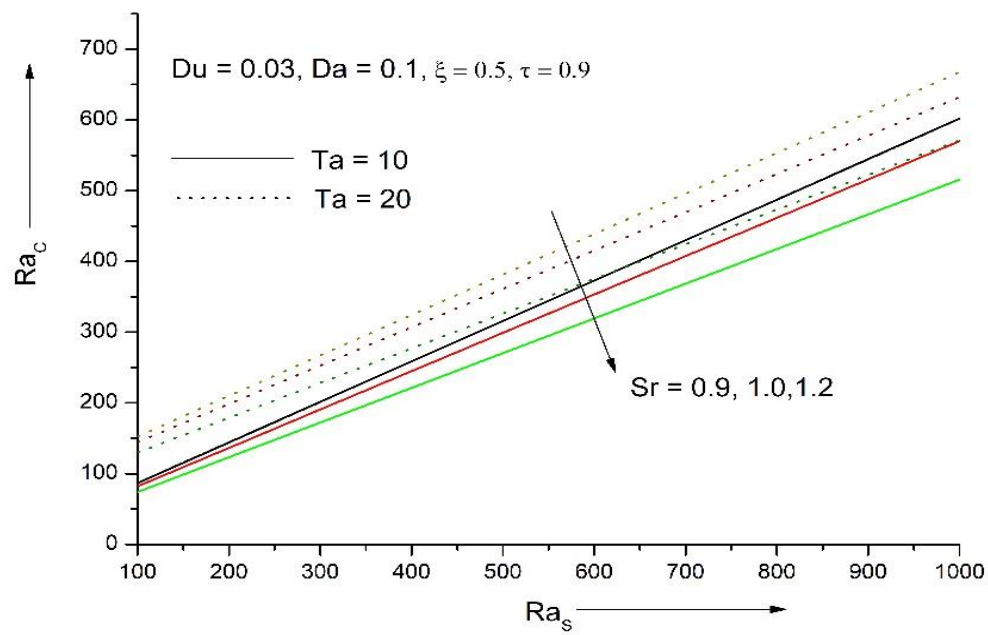


Figure 5; The change of Ra_C versus Ra_S for distinct values of Ta and Sr .

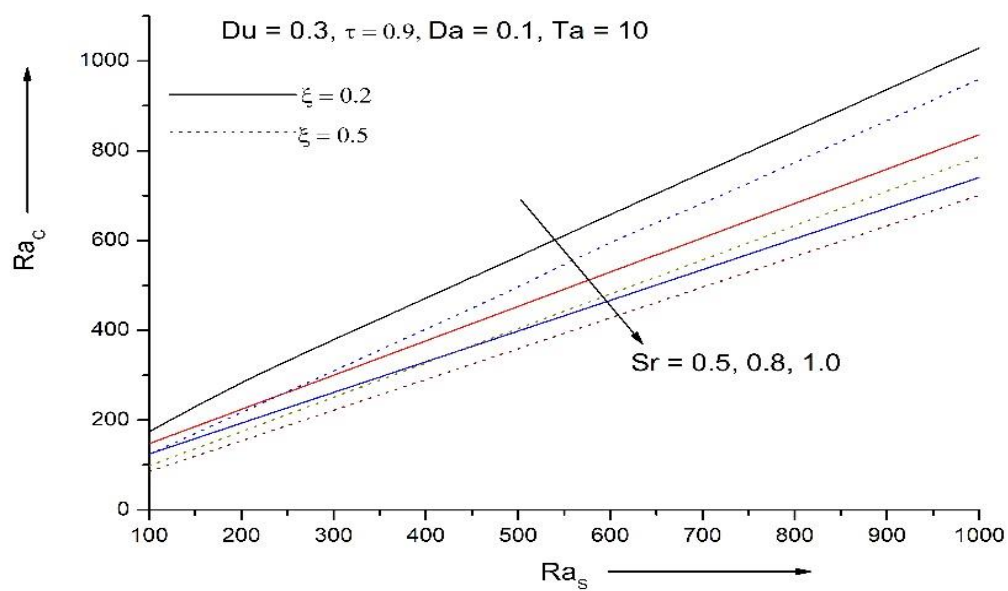


Figure 6; The change of Ra_C versus Ra_S for distinct values of ξ and Sr .

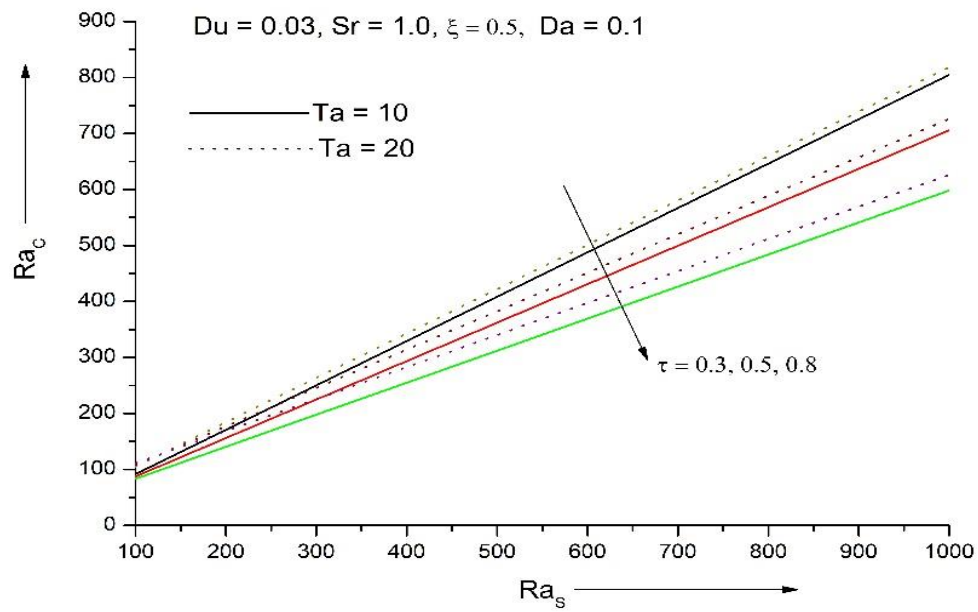


Figure 7; The change of Ra_C versus Ra_S for distinct values of Ta and τ .

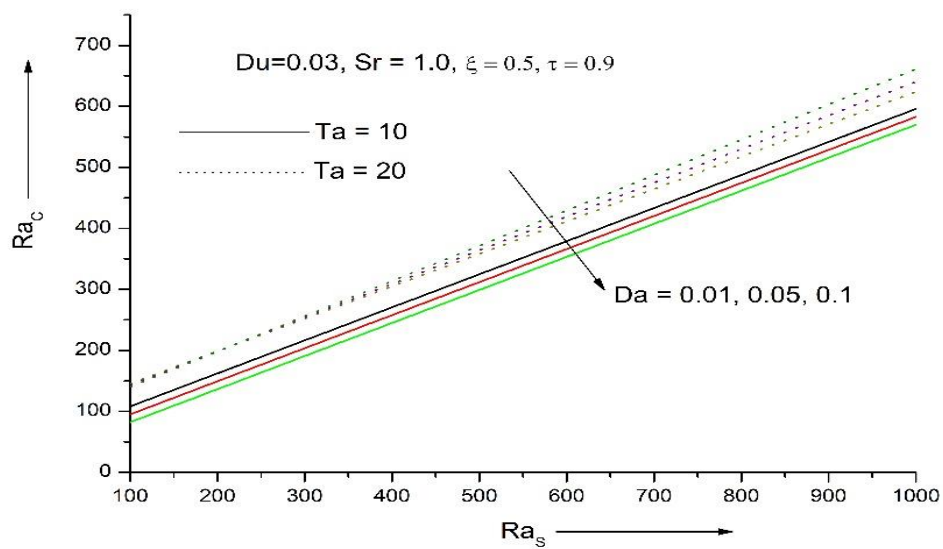


Figure 8; The change of Ra_C versus Ra_S for distinct values of Ta and Da .

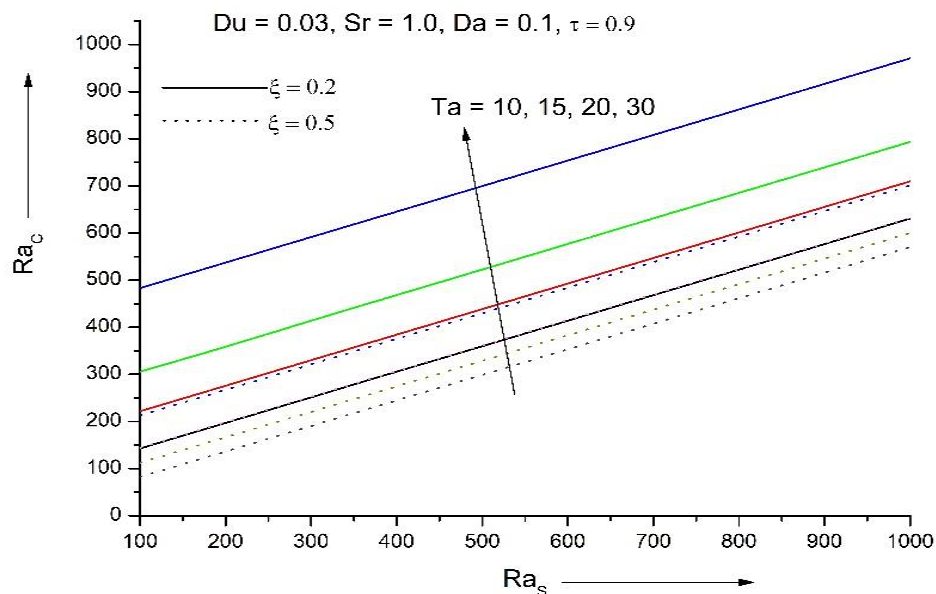


Figure 9; The change of Ra_C versus Ra_S for distinct values of ξ and Ta .

5. Conclusion:

The cross-diffusion effects and double diffusive convection in a horizontal rectangular channel saturated anisotropic porous media which is heated and salted below, is studied numerically. The effect of ξ , Ra_S , Sr and Du are shown graphically and the conclusions are, for the positive Sr parameter destabilizes the system, while the negative Sr parameter stabilizes system. The Du parameter destabilizes the system in stationary modes. The τ stabilizes the system for small values of Ra_S and it destabilizes for large value of Ra_S . The increase of Ta number, Ra_C is also increases and it indicate that Ta stabilizes the system.

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