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Effect of Viscous Dissipation on Magneto Hydrodynamics Free Convection Flow from an Isothermal Truncated Cone in the Presence of Pressure Work with Temperature Dependent Viscosity

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Abstract:- The purpose of the existing study is to analyse how viscous dissipation effects on variable viscosity and pressure work together to produce constant magneto hydrodynamic free convection flow around an isothermal truncated cone. Using similarity solution of partial differential equations, further conforming to the energy and momentum equations are reformed into non-linear ordinary differential equations. Using a quasilinearization method in accumulation to an implicit finite difference technique, the outcomes are produced for the flow. Along with fixed magneto hydrodynamics, pressure work, and variable viscosity, the outcomes of some physical parameters on skin friction, the transfer of heat quantities, velocity, and temperature are given for altered data of viscous dissipation. It is presented that viscous dissipation has a substantial influence on the thickness of the thermal boundary layer.

Keywords*: viscous dissipation, MHD, isothermal truncated cone, pressure work, variable viscosity*.

1. Introduction

In fluid mechanics, one of the most fundamental flows is the measurement of natural convective flow. Because natural convection flows under the impact of gravitational force occur frequently in science and engineering applications, as well as in nature, they have undergone the most extensive measurement. Free convection is induced when a hot surface comes into interaction with a fluid due to the buoyancy force affected by the temperature differential. In many technical applications, such as steam generators, solar energy collectors, spaceship design, power modifiers and facets of flow and heat transmission are critical.

Free convective flows are explored by a few writers, especially in light of the unequal surface temperature. A vertical cone flow was presented by the authors Mark and Prins [1, 2] as a common relation for isothermal axisymmetric structures. Research on natural convection similarities flows on two-dimensional axisymmetric structures has been conducted by Braun et al. [3]. Roy [5] expanded on Hering and Grosh's [4] study in search of a high Prandtl number. Hering and Grosh, as mentioned in reference [4], investigated correlations between their results concerning natural convection stemming from a non-uniformly heated vertical cone. Elsewhere, Na and Chiou [6] delved into the aspect of free convection traversing the cone's frustum, when the surface is uniformly heated and there's no impact from a transverse curvature. Alamgir [7] has examined the complete process of heat transmission from a vertical cone using the fundamental scheme for laminar free convection. Pantokratoras [8] looked into the effects of pressure work function and viscous dissipation on natural convective flow along a vertical isothermal plate. Alam et al. [9] studied the typical convective flow of a vertical porous round cone that was maintained under pressure during research. Elbashbeshy et al. [10] are investigating how pressure work affects free convection flow around a truncated cone. The fluid's temperature-dependent viscosity was anticipated to be fixed during the overhead testing. However, it is found that viscosity varies significantly with temperature. Authors Gary et al. [11] and Mehta et al. [12] have noted that the flow parameters connected to the constant variable viscosity may alter significantly when this outcome is present. Several authors have described the outcomes in free convection from a vertical cone [13–15]. A recent study by the author Hossain et al. [16] examined how a viscous fluid normally constricts around a viscosity-truncated cone.

Viscous dissipation is the irreversible process by which a fluid acts on neighboring layers to perform work through the conversion of heat from shear stresses. When the induced kinetic energy becomes noticeable in relation to the heat transported, viscous dissipation in free convection has a noticeable influence. This happens when the convection region is large or when the corresponding body force is high. In naturally occurring devices, convection leads to viscous dissipation. Such dissipation effects can also be seen in processes with very vast scales, such as those occurring on big planets, in large amounts of gas in space, in geological processes, and in fluids inside different bodies, as well as in the incidence of strong gravitational fields. The characteristic of viscous mechanical dissipation effects is typically indicated by the Eckert number. The dissipation number, an independent metric, can be used to quantify the consequences of viscous dissipation. In light of this, viscous dissipative heat is comprised in the energy equation. The heat produced by viscous dissipation is small and usually ignored in the energy equation. However, the viscous dissipative effects cannot be disregarded in cases of intense gravitational force or high fluid Prandtl number.

The phrase "viscous dissipation" employed in [17] makes the supposition that the fluid is more viscous than elastic. The influences of time-periodic boundary conditions and viscous dissipation on free convection flow between parallel plates were examined by Jha [18]. M Ajaykumar et al. [19] observed the effects on flow and transfer of heat in a flowing fluid across a moving flat surface of viscous dissipation, an applied magnetic field, and the Prandtl number. Chen [20] investigated the MHD flow and heat transmission analytic solution over a stretching sheet for two different kinds of viscoelastic fluids with thermal radiation, energy dissipation, and an internal heat source.

Examining MHD flow and heat transfer in diverse media has been a significant field of study for boundary layer flow. Because of its several applications, including crystal growth for nuclear reactor cooling, magnetohydrodynamic generators, and plasma research, this type of flow has drawn the interest of numerous researchers [21–26]. The impacts of pressure function and temperature dependent viscosity and viscous dissipation on MHD are the main topics of discussion.

2. Mathematical Formulation

The steady free convection flow of a viscous incompressible fluid in two dimensions. Figure 1 depicts the physical coordinate structure.

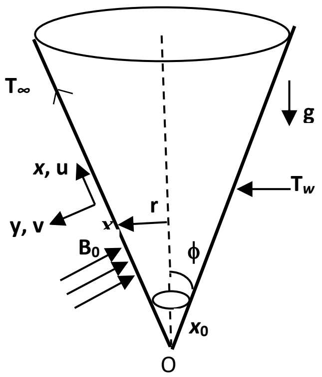

Figure. 1 Co-ordinate scheme and flow configuration

The electrically conducting fluid across an isothermal truncated cone in two dimensions using magnetohydrodynamics in the domain $x_0 \leq x \leq \infty$, are given by

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta cos\phi(T-T_\infty)+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{T\beta u}{\rho C_p}\frac{\partial p}{\partial x} \tag{3}$$

The boundary conditions are known by:

$$\begin{aligned} &u=0, \quad v=0, \quad T=T_w \;\; at \quad y=0 \\ &u\to 0, \quad T\to T_\infty \quad as \quad y\to\infty \end{aligned} \tag{4}$$

A semi-empirical formula's viscosity is determined by

$$\frac{\mu}{\mu_\infty}=\frac{1}{1+\gamma\left(T-T_\infty\right)} \tag{5}$$

And as known by Ling and Dybbs [27], have persisted recognized, here μ_∞ be the viscosity.
By combining the stream function ψ well defined by

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y} \;\text{ and }\; v=-\frac{1}{r}\frac{\partial\psi}{\partial x} \tag{6}$$

It is estimated that the boundary layer is sufficiently thin. The local radius $r = xsin\phi$ can be changed to a position in the boundary layer using the isothermal truncated cone radius. Where the cone's semi-vertical angle is ϕ.

Relating the subsequent transformations:

$$\xi=\frac{x^*}{x_0}=\frac{x-x_0}{x_0}, \quad \eta=\frac{y}{x^*}(Gr_{x^*})^{1/4}, \; Gr_{x^*}=\frac{g\beta cos\phi(T_w-T_\infty)x^{*3}}{\vartheta^2}$$

$$\psi=\upsilon r(Gr_{x^*})^{1/4}f(\xi,\eta), T-T_\infty=(T_w-T_\infty)G(\xi,\eta), \; f'=F=\frac{\partial f}{\partial\eta}$$

$$u=\frac{\upsilon(Gr_{x^*})^{1/2}}{x^*}F=U_rF\,, \; v=-\frac{\upsilon(Gr_{x^*})^{1/4}}{x^*}\left[\left(\frac{\xi}{\xi+1}+\frac{3}{4}\right)F+\xi\frac{\partial f}{\partial\xi}-\frac{1}{4}\eta F\right] \tag{7}$$

The continuity equation (1) is found to be equally satisfied with equations from (1) to (4), while equations (2) and (3) are reduced to.

$$F''+(1+\varepsilon G)\left[\left(\frac{\xi}{\xi+1}+\frac{3}{4}\right)fF'-\frac{1}{2}F^2+G-MF\right]-\left(\frac{\varepsilon}{1+\varepsilon G}\right)G'F'-(1+\varepsilon G)\xi\left[FF_\xi-F'f_\xi\right]=0 \tag{8}$$

$$G''+\left(\frac{\xi}{\xi+1}+\frac{3}{4}\right)fG'P_r-\xi\left(FG_\xi-G'f_\xi\right)P_r-\epsilon GFP_r-P_rE_c(F')^2=0 \tag{9}$$

By, the overhead non-dimensional equations (8)–(9) have known boundary conditions

$$\begin{aligned} &f'=F=0, \quad G=1 \text{ at } \quad \eta=0 \\ &F=0, \; G=0 \text{ as } \eta\to\infty \end{aligned} \tag{10}$$

Gebhart [28] is the person presented first pressure work $\epsilon = g\beta x^* / C_p$ first and $E_c=\frac{U^2}{C_p(T_w-T_\infty)}$ is the Eckert number.
The skin friction (C_f) and transferring of heat aspect interns of Nusselt number (Nu) can be indicated as

$$C_f=\frac{2\tau_w}{\rho U_r^2} \text{ and } \quad Nu=-\frac{q_w x^*}{k(T_w-T_\infty)}, \text{ Where } \tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \text{and}$$

$$q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

By means of the transformation (7), then C_f and Nu take the form:

$$(Gr_{x^*})^{1/4} C_f = \left(\frac{2}{1+\varepsilon}\right) F'(\xi, 0) \quad (11)$$

$$\frac{Nu}{(Gr_{x^*})^{1/4}} = -G'(\xi, 0) \quad (12)$$

3. Outcomes and Confab

Combined with the implicit finite difference system and quasilinearization method, can provides an answer to the coupled nonlinear ordinary differential equations (8) and (9) in addition to the boundary conditions (10). In directive to be concise, this technique description is left out here as it is covered in [29, 30]. To verify our results when ϵ=0, ε=0, and Ec=0 for heat transfer and skin friction, for the persistent values of Pr = 0.7, 1.0, and 10.0 at the foremost edge ξ=0, we have related our steady state conclusions with those of Na & Chiou [6], as represented in Table 1.

The several data of viscous dissipation and magneto hydrodynamic in the incidence of pressure work, the heat transfer, skin friction, temperature, velocity, variable viscosity and Prandtl number discoveries are shown as the stability in Figures 2 through 5.

Table 1. Represents the $F'(0,0)$ and – G' (0, 0) for few data's of (Prandtl number Pr = 0.7, 1.0 and 10.0 at $\epsilon = 0, \varepsilon = 0, \mathrm{Ec} = 0$ *and* $\xi = 0$) with [6]

$\epsilon = 0$, ε=0, Ec=0				
	$F'(0,0)$		$-G'(0,0)$	
Pr	Ref.[6]	Present results	Ref.[6]	Present results
0.7	0.9584	0.9588	0.3532	0.3539
1.0	0.9081	0.9080	0.4010	0.4014
10.0	0.5930	0.5932	0.8269	0.8261

Figure. 2 illustrations that the skin friction $[c_f(Gr_{x^*})^{1/4}]$ and transfer of heat aspect $[(N_u(Gr_{x^*})^{-1/4})]$ for Eckert number (Ec) at the stream wise location $\xi = 1.0$, for $Pr = 0.72, \epsilon = 0.5, \varepsilon = 0.5$ is present in figure 2(a) & 2(b). It is supposed that $[c_f(Gr_{x^*})^{1/4}]$ originate to drop and $[(N_u(Gr_{x^*})^{-1/4}]$ upsurges with surge of dimensionless distance (ξ) in the range $(0 \leq \xi \leq 1.0)$. The proportion of drop in $[c_f(Gr_{x^*})^{1/4}]$ is 4.26% whereas the proportion of upsurge in $[(N_u(Gr_{x^*})^{-1/4})]$ is 12.22% near $\xi = 1.0$

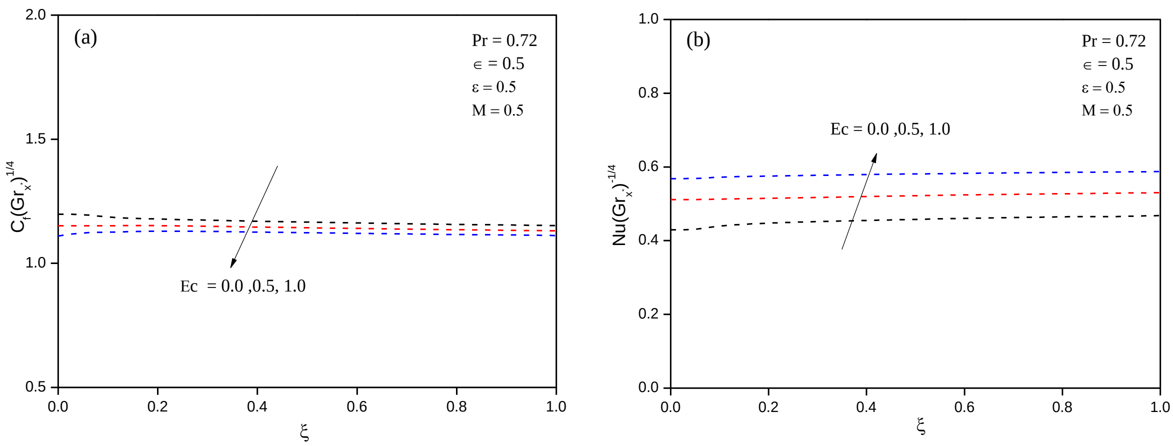

Figure 2. (a) Skin friction and (b) heat transfer aspect for different data of Eckert number (Ec) when $Pr = 0.72, \epsilon = 0.5, \varepsilon = 0.5$ *and* $M = 0.5$

Figures 3(a) and 3(b) show the resulting temperature (G) and velocity (F) in matching proportion. It is thought that when temperature drops and the Eckert number increases, velocity drops suddenly close to the wall. Based on these data, the conclusion is that at rising separations from the leading edge, momentum decreases and the thermal boundary thickness decreases in relation to η. The percentage of the thermal boundary layer thickness reduction is around 2.15 and the momentum boundary layer thickness fall is 1.54 at η = 1.2 when the Eckert number changes from 0.0 to 1.0.

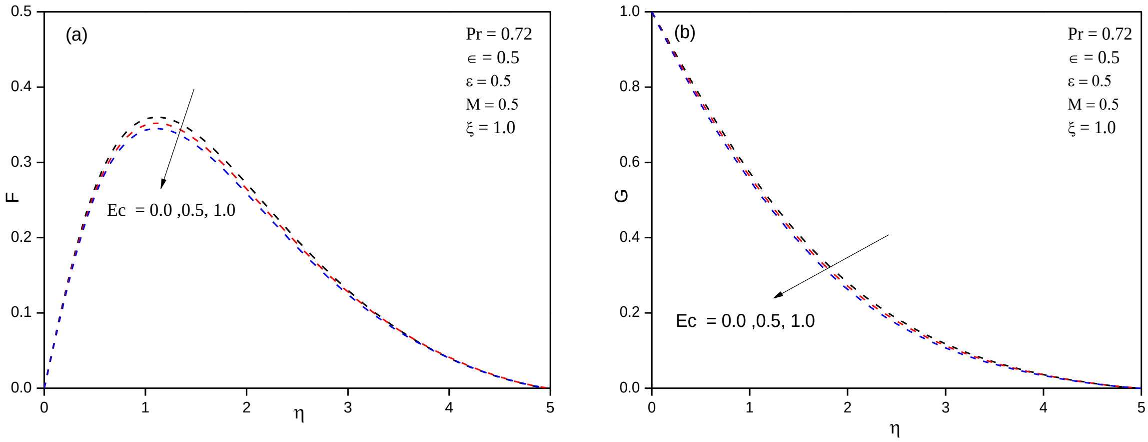

Figure 3. (a) Illustrations the Velocity and (b) Temperature outlines for different data of Eckert number (Ec) when Pr=0.72,ϵ=0.5 ,ε=0.5 and M=0.5

Figure. 4 displays the skin friction$[c_f(Gr_{x^*})^{1/4}]$ and transfer of heat $[(N_u(Gr_{x^*})^{-1/4})]$ for magnetic parameter (M) at the stream wise location $\xi = 1.0$, for $Pr = 0.72, \epsilon = 0.5, \varepsilon = 0.5$ is present in figure 4(a) & 4(b). It is supposed that $[c_f(Gr_{x^*})^{1/4}]$ originate to drop & $[(N_u(Gr_{x^*})^{-1/4}]$ declines with improvement of dimensionless distance (ξ) in the range $(0 \leq \xi \leq 1.0)$. The relation of drop in $[c_f(Gr_{x^*})^{1/4}]$ is 28.86% however the relation of drop in $[(N_u(Gr_{x^*})^{-1/4})]$ is 12.59% near $\xi = 1.0$. This is expected as an extreme value of M within the boundary layer actually upsurges the Lorentz force, which effectively limits the flow in the opposite direction. Thus, magnetic fields serve as the retarding force that actually lowers the local Nusselt number and skin friction.

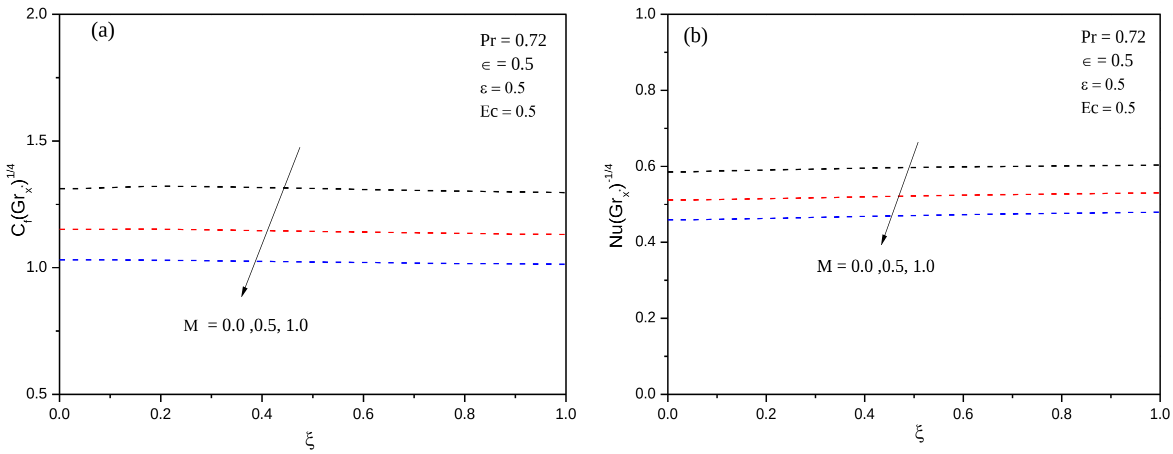

Figure 4. (a) The Skin friction and (b) heat transfer for different data of magnetic factor (M) when $Pr = 0.72, \epsilon = 0.5\ , \varepsilon = 0.5\ and\ Ec = 0.5$

Figures 5(a) and 5(b) show the resulting temperature (G) and velocity (F) in matching proportion. It is supposed that as temperature rises and the MHD factor increases, velocity abruptly reduces in the vicinity of the wall. The data show that as we move away from the leading edge, momentum decreases and the thickness of the thermal boundary layer increases, relative to η. The percentage of the thermal boundary layer thickness increases and the momentum boundary layer thickness reduction around η = 1.2 are about 8.29% when the magnetic parameter upsurges from 0.0 to 1.0. Larger M values are related with lower peak values in the velocity profile. In addition, applying a transverse magnetic field to a fluid that conducts electricity outcomes in the production of the resistive Lorentz force. The fluid's axial velocity tends to be slowed down by this force. Therefore, we conclude that the MHD parameter causes the boundary layer to rise.

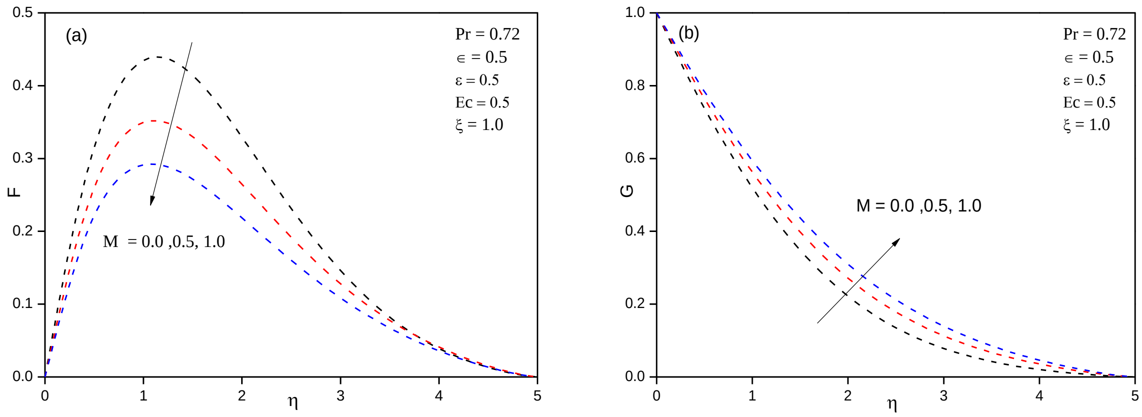

Figure 5. (a) Velocity and (b) Temperature aspect for different data magnetic parameter (M) when Pr=0.72, ϵ=0.5, ε=0.5 and Ec=0.5

4. Conclusions

The present research has examined the consequences of viscous dissipation using the MHD parameter, leading to the resulting findings.

1. For stable pressure work (ϵ = 0.5), Prandtl number (Pr = 0.72), variable viscosity (ε = 0.5) and magnetic parameter (M = 0.5) the transfer of heat factor increases, but the skin friction factor reduces as the viscous dissipation parameter increases.
2. The temperature contours and velocity factor drop with an upsurge in the viscous dissipation parameter with the stable variable viscosity (ε = 0.5), Prandtl number (Pr = 0.72), MHD parameter (M = 0.5) and pressure work (ϵ = 0.5).
3. As the magnetic parameter upsurges, the skin friction aspect falls and the heat transmission factor drops with stable pressure work (ϵ = 0.5), Prandtl number (Pr = 0.72), (Ec = 0.5) Eckert number and variable viscosity (ε = 0.5)..
4. In the case of the stable variable viscosity (ε = 0.5), pressure work (ϵ = 0.5), Prandtl number(Pr = 0.72) and (Ec = 0.5) Eckert number as the magnetic parameter increases, velocity profiles get lower.

Conflict of Interest

The writers certify that there is no conflict between the subjects discussed in this work and their personal interests.

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Refrences

[1] Merk, E. J., Prinss, J. A., "Thermal Convection in Laminar Boundary I", Appl. Sci. Res., 4A (1954), 3, pp.11-24.
[2] Merk, E. J., Prinss, J. A., "Thermal Convection in Laminar Boundary II", App. Sci. Res., 4A (1954), 3,pp.195-206.

[3] Braun, W. H., et al., " Free Convection Similarity Flows about Two Dimensional and Axisymmetric Bodies with Closed Lower Ends", Int. J. Heat and Mass Transfer, 2 (1961), 1-2, pp.121-135.
[4] Hering, R. G., Grosh, R. J., "Laminar Free Convection from a Non-Isothermal Cone, Int. Heat and Mass Transfer", 5 (1962), 11, pp. 1059-1068.
[5] Roy, S., "Free Convection over a Slender Vertical Cone at High Prandtl Numbers, ASME J. Heat Mass Transfer", 101 (1974), Feb., pp. 174-176.
[6] Na, T. Y., Chiou, J. P., "Laminar Natural Convection over a Frustum of a Cone", Applied Scientific Research, 35(1979), 5, pp. 409-421.
[7] Alamgir, M., "Over-All Heat Transfer from Vertical Cones in Laminar Free Convection: An Approximate Method, Transactions of ASME Journal of Heat Transfer", 101 (1979), 1, pp. 174-176.
[8] Pantokratoras, A., "Effect of Viscous Dissipation and Pressure Stress Work in Natural Convection along a Vertical Isothermal Plate", Int. J. Heat Mass Transfer, 46 (2003), 25, pp. 4979-4983.
[9] Alam, Md. M., et al., "Free Convection from a Vertical Permeable Circular Cone with Pressure Work and Non-Uniform Surface Temperature", Nonlinear Analysis Modeling and Control, 12 (2007), 1, pp. 21-32.
[10] Elbashbeshy, E. M. A., et al., "Effect of pressure work on Free Convection flow about a truncated cone", Int. J.physical sciences, Year 2013, Vol. 1, No. 2, pp. 001-010.
[11] J. Gary, D.R. Kassory, H. Tadjeran, A. Zebib, "The effect of significant viscosity variation on convective heat transport in water- saturated porous media", J. Fluid Mech. 117 (1992) 33–249.
[12] K.N. Mehta, S. Stood, "Transient free convection flow with temperature dependent viscosity in a fluid saturated medium", Int. J. Eng. Sci. 30 (1992), 1083–1087.
[13] I. Pop, H.S. Takhar, "Compressibility effects in laminar free convection from a vertical cone", Appl. Scient. Res. 48 (1991), 71–82.
[14] T. Watanabe, "Free convection boundary layer flow with suction injection over a cone", Acta Mech. 151 (2001) 103–114.
[15] H.S. Takhar, A.J. Chamkha, G. Nath, "Effects of thermo-physical quantities on the natural convection flow of gases over a vertical cone", Int. J. Eng. Sci. 42 (2004), 243–256.
[16] M.A. Hossian, M.S. Munir, H.S. Takhar, "Natural convection flow of a viscous fluid about a truncated cone with temperature dependent viscosity", Acta Mech.140 (2000), 171–181.
[17] M. Subhas Abel, E. Sanjayanand, and M. M. Nandeppanavar, "Viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and ohmic dissipations," Communications in Nonlinear Science and Numerical Simulation, vol. 13, no. 9, pp.1808–1821, 2008.
[18]. B. K. Jha and A. O. Ajibade, "Effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary conditions," Communications in Nonlinear Science and Numerical Simulation, vol. 17, no. 4, pp. 1576– 1587, 2012.
[19] Ajaykumar M, Ajay C K, Srinivasa A H. Effects of viscous dissipation, internal heat source/sink and Prandtl number on flow and heat transfer in a moving fluid over a moving flat surface with an applied magnetic field. Materials Today: Proceedings. 2023 May 10.
[20]. C. Chen, "On the analytic solution of MHD flow and heat transfer for two types of viscoelastic fluid over a stretching sheet with energy dissipation, internal heat source and thermal radiation," International Journal of Heat and Mass Transfer, vol. 53, no. 19-20, pp. 4264–4273, 2010.
[21] K.B. Pavlov, "Magnetohydrodynamic flow of an incompressible viscous fluid caused by the deformation of a plane surface", Magn. Gidrondin 4 (1974), 146–152.
[22] A. Chakrabarthi, A.S. Gupta, "A note on MHD flow over a stretching permeable surface", Q. Appl. Math. 37 (1979), 73–78.
[23] T. Chiam, "Magneto hydrodynamic boundary layer flow due to a continuous moving flate plate", Comput. Math. Appl. 26 (1993), 1–8.
[24] V. Kumaran, A.K. Banerjee, A. Vanav Kumar, K. Vajravelu, "MHD flow past a stretching permeable sheet", Appl. Math. Comput. 210 (2009), 26–32.
[25] Ajaykumar, M., and A. H. Srinivasa. "Variable viscosity effects on unsteady MHD laminar boundary layer flow and heat transfer over a stretching sheet." AIP Conference Proceedings. Vol. 2277. No. 1. AIP Publishing, (2020).
[26] M. Sheikholeslami, D.D. Ganji, M.M. Rashidi, "Ferro fluid flow and heat transfer in a semi annulus enclosure in the presence of magnetic source considering thermal radiation", J. Taiwan Inst. Chem. Eng. 47 (2015), 6–17.
[27] J. X. Ling, A. Dybbs, "A forced convection over a flate plate submersed in a porous Medium", variable viscosity, case paper 87- WA/HT-23. New York, ASME, (1987).
[28] Gebhart, B., "Effects of Viscous Dissipation in Natural Convection", Journal of Fluid Mech., 14 (1962), 2, pp. 225-232.

[29] K. Inouye and A. Tate, “Finite difference version of quasilinearization applied to boundary later equations”, AIAAJ.12, (1974), pp. 558-560.
[30] M. Ajaykumar, A. H. Srinivasa, Effect of pressure work on free convection flow from an isothermal truncated cone with temperature dependent viscosity, J. Math. Comput. Sci., 10 (2020), 2422-2433.