Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 4 (2024)

Constraints Identification using a Simple Example Graphical Approach

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Abstract: Constraints represents an important aspect in control systems. Constraints are also inherent partof any dynamical system. Mathematical and graphical representations of constraints are common in literature. A real phenomenon of walk from chair to door and opening of door is considered and graphically represented in this article. Further a simple experiment is conductedto carry weight by right hand and opening door by left hand. This experiment considers different weights from 1 kg to 23 kg and constraint identification accordingly.

Key words: constraint, state variables, force, weight handlingIntroduction

Constraints in control systems are expressed in terms of upper or/ and lower bounds through equations. Equations are powerful tools to represent a system, condition, working set or a set of conditions (alternatively called equalities or inequalities). Actuators are inherent partof control systems. Actuators limit output variables and hence protects system as well. In caseof exponentially unstable linear systems with actuator constraints (saturation), only local stabilization with large domain of attraction is possible [8]. Using a Multi-Input-Single-Output system, constraints expressions from system dynamics may be used to form objective functions to formulate a nonlinear optimization problem with no constraints (with bounds on decision variables) [10].

Pole of the low pass filter is specified as twice as fast as the pole of the bare airframe yaw dynamics [1] as a part of model identification. Constraint is expressed in terms of constant: relation between bare airframe yaw damping coefficient and pole of low pass filter. Choice ofa saturation vector and desired position trajectory constraint associated with helicopter part- dynamics is expressed in [2]. In [4] it is explained that saturation of a device (such as transduceror actuator) is a hard constraint. Further constraint is modelled by a nonlinear operator that captures magnitude and rate constraint. A specific property of such operator is also expressed there in. These constraint sets are usually bounded and convex [4], have hypercube characteristics (another example as ellipsoid); however some class of convex sets may be not necessarily bounded. Definitions such as minimum-phase constraints non-minimum phase constraints and their variants, constraint invariant zeros are also given this reference. In a velocity estimation using encoder constraints used to solve an optimization problem [11]. Constraints were: difference between true angular position and the measured signal is boundedby an expression, further to obtain a smooth signal an estimator is used by discarding two conditions that limits reconstruction accuracy. The results are supported by experimental validation. Three-way Catalytic Converters state estimation involves systematic considerationof state constraints [5]. It is pointed in [7] Control methods emphasizing system stability, load disturbances, tracking performance, load adaptation, does not completely reflectpractical applications on actuator constraints. It is further mentioned that traditional back-stepping (control) approach cannot be applied directly to deal with actuator constraints. Hencea nonlinear feedback action is added to feed-water flow controller to compensate the actuator constraint. In another application of vanadium redox flow batteries (VRFB) a couple of constraints are mentioned: one is vanadium concentration must be greater than zero and secondtotal number of moles of vanadium in the system is constant [9]. Authors used Extended Kalman Filter (EKF) for unconstrained state estimate and then projected it to constrained surface with simulation results. Reduction of inequality constraint to an equality constraint byfinding maximum value of controller output is shown in [3].

While projecting/ estimating, if the estimated states violation occurs, then correction in the error variable may be added in associated estimator/ observer dynamics equations. In a spacecraft control application [6] constraints such as orbital control input force, constraints onthrust equipment, fuel quality, engine power are mentioned with simulation results stating the control inputs remains bounded. In context to Real Time Simulation, authors in [14] mentionsimportance of constraints on fast computations so as to match the pace with real-world time. For development of real time systems with small microcontrollers, cost of hardware development is critical constraint in short time.

Constraints are expressed as set of welding points, welding crossing constraint macro-regions,more than 15 detailed constraints expressed as parameter tuple sets, access-wise interference constraint, occurrence constraints (sum of all welding points performed by robot equals total welding points in that region), a constraint that prevents the accessing of same region from twodifferent regions, adaptation of resource constraints, access exclusion constraint [12].

Authors in [13] explains as a part of robot force-impedance control in a specific environment: cases of impedances constraints: softest case when the interaction force is zero (without obstacle), hardest case while environment is with infinite rigid obstacle. Simulation as well asexperiment validations are mentioned in [13]. In [18] a characteristic such as: orthogonal to a non-zero vector constraint is expressed. Graphical illustration of a vector, the orthogonal constraint vector and a characteristic angle is expressed in [18] as well. The physical interpretation of greatest lower bound associated with constrained vector is also discussed therein.

Constraints are also expressed in graphical way as in [16]. In [16] constraint is graphically interpreted as product of three terms cantered around transfer function as a function of real andimaginary parts. Interpretation of such constraints is however complex to understand in general view of system responses or stability is. Active equality constraints and inequality constraints may or may not be active [17]. Also generalization of constraints is possible, and optimization code has linear algebra routines to utilize sparsity of constraints matrices [17]. State dependent constraints exists in Linear Quadratic dynamic games [19]. Controls for whichthe state-dependent constraints are considered do not influence the state variables. Authors in

[19] considers linear state-dependent constraints on decisions. The purity of oxygen in oxygenseparation process in lo capacity oxygen generator is sensitive to difference between adsorptionpressure (like 2.5 bar to 6.0 bars) and the pressure in the oxygen tank [15]. Authors in [15] experimentally found that too large difference between operating pressure of the adsorption bed and the oxygen product pressure in tank causes excessive purging and reduction in oxygenpurity. The approximate complex conjugate (ACC) control solutions are not available for systems with constraints (such as position, velocity and force constraints [20]). Hard constraintsand soft constraints are mentioned in [22]. Authors in [21], box constraint specifies the possiblevalues of each element in *a specific matrix*. The projections onto the constraints are explicitlymentioned. Terminology of probabilistic constraints (chance constraints) is discussed in [23]. Probability level of constrain violation within a range of 0 to 0.5 for example is mentioned. Chance constraints on state and control inputs are also considered. Transformation of one typeof (chance) constraint into another (deterministic) with the help of Gaussian cumulative distribution function is further discussed. Constraints linear growth is expressed therein. Finite dimensional semi-definite program approximating P (a linear program) can be obtained by replacing constraints on measures with constraints on moments [24].

Constraints in Mathematical aspects

Here are some examples of expressions:

L2 norm: $\|x\|_2 \leq \emptyset$ where x is a state vector and $\emptyset$ may be positive value within a specific range [25]. $\langle \boldsymbol{g}, \boldsymbol{\rho} \rangle = \sum^{m} \boldsymbol{j}_i \langle \mathsf{a}, \boldsymbol{\rho} \rangle = \mathbf{0}$ for nonzero g and other constants or set of constant numbers [18]. See inequality 12 and 13 from [12]. Similarly $x_{low} \leq x_t \leq x_{\mathrm{high}}$; $t = 1, \ldots, N$ [26]. Constraint $s_i \leq (1 + \xi)\, x$ corresponds to the possibility of an act satisfying a given condition[19].

Identification or verification of constraints

A set of examples is presented here for real constraints.

Example 1: A person has a walk from chair to cabin door. Next the person opens the door by lefthand and moves out. This can be represented by following set of figures:

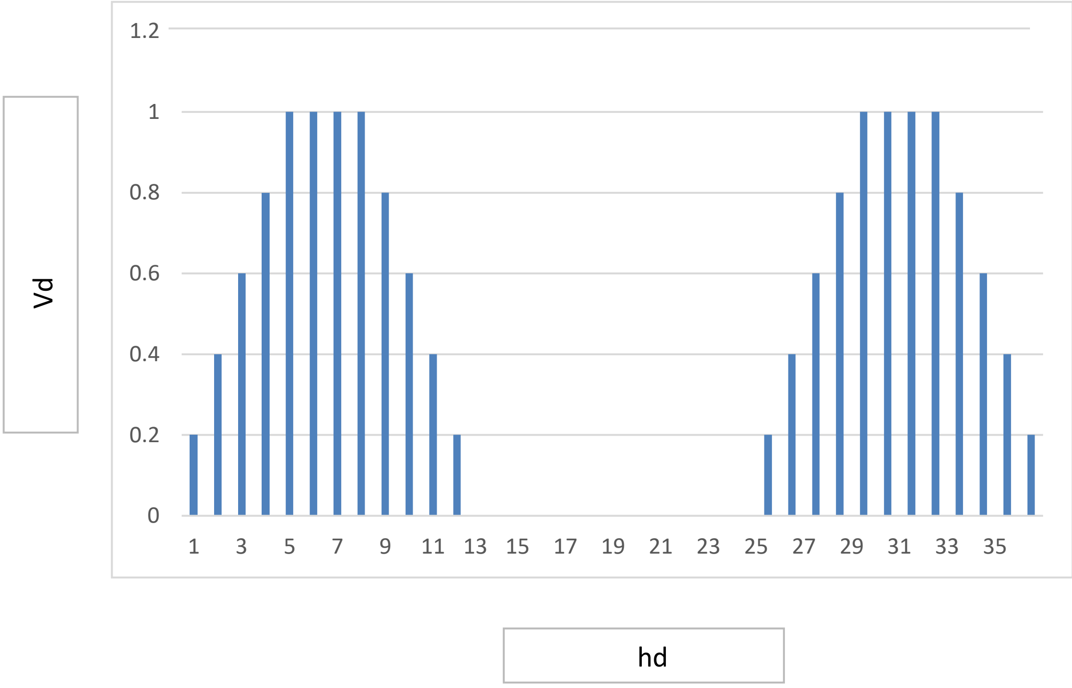

Figure: Upper-horizontal foot displacement (hd) vs. vertical foot displacement.

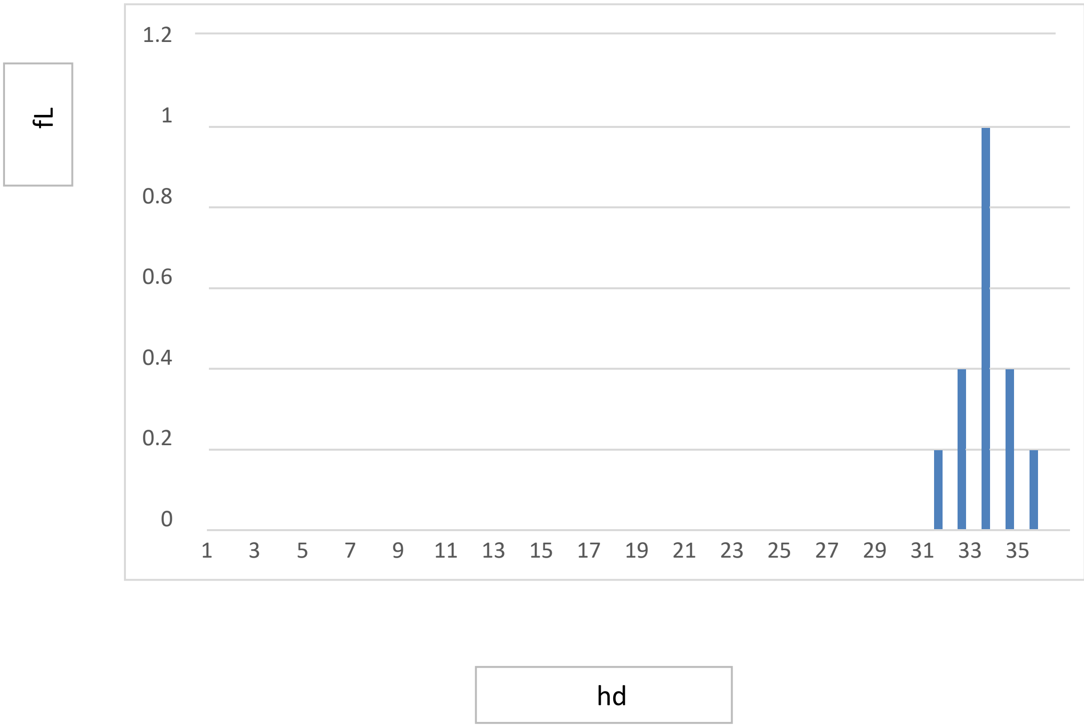

Figure: Middle-horizontal displacement (hd) vs. left-hand-force

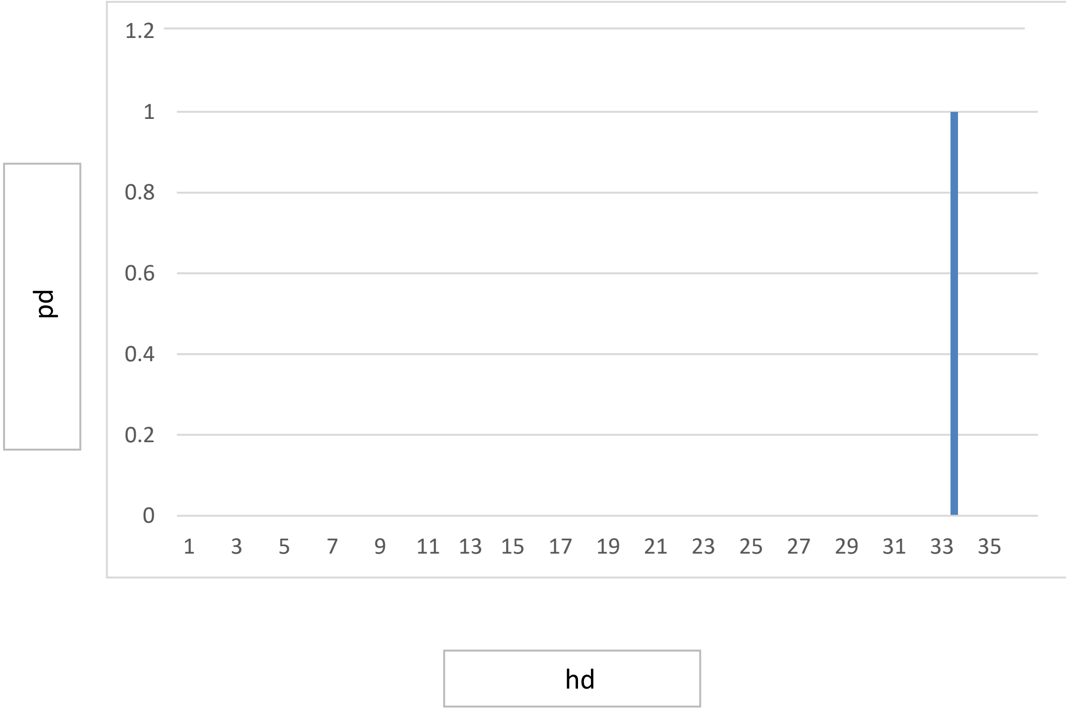

Figure: Lower-horizontal foot displacement vs. door position-angle in degrees

Upper figure (which are generated in Excel spreadsheet) shows movement of left foot steps. X axis is horizontal distance (hd) units. Y axis is Vd- vertical distance travelled by left-foot. Middle figure shows force applied by left hand to cabin door handle when person reaches neardoor. Lower figure shows deflection of door in degrees on Y axis. Now constraint can be considered like this one: when a person holds say 1 kg wait in right hand and does the same steps as shown in figures; there exists a constraint on weight handled by right hand with door opened and person moved out without touching the door to body. The complexity cases for such constraint can be increased while door to be opened successfully and person moves out of cabin without door touching. Once the left hand applies force to cabin handle, immediatelyhand is removed from handle and person is supposed to quit from the cabin door outside. Thedoor also gets closed as it has hydraulic closing-spring-mass-damper system. Verification of this constrain is an easy thing and does not require any advanced sensors; manual time calculations are represented here. Horizontal distance is about 1.3 meters. Vertical distance travelled by feet is about 0.05 meters. Force applied to door is about 10 N. Angular displacement of door is 60 degrees.

Example 2: If the right hand is carrying a container such that additional constraint is – the doorshall not touch the container. Assumption: the container in right hand is easily held by right hand fingers, and force applied by left hand is perpendicular to vertical plane of door. Expectedactions further are:

a) the door shall be opened by left hand

b) the person shall quickly get out of the door

Note: When person is carrying a container in right hand, there exists a limit on weightcarried by right hand such that condition (a) and (condition (b) are fulfilled.

Real results are obtained here for this experiment:

Weight carried by right hand (kg)	Distance carried by feet (m)	Constraint handled successfully? (Yes/No)	Time required to dothe task (seconds)
1	1.52	Yes	5
2	1.52	Yes	5

5	1.52	Yes	5
10	1.52	Yes	6
15*	1.52	Yes	6.5
20*	1.52	Yes	7
23*	1.52	No	10

*These weights are not possible to carry in hand, hence carried in a bag with left hand.

Now the walking signal is represented by A sin (2*pi*f*t); where A is peak vertical distance travelled by foot in meters, f is frequency of walking (reciprocal of time to travel 2 feet/i.e. 0.0609 m), t is real time in seconds. As weight to be lifted increases, the constraints on peak vertical foot displacement increases. Also minimum deflection of door that will allow a personto pass through is assumed as 45 degrees.

The standard spring-mass-damper system can be represented by:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-k}{m} & \frac{-c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} u$$

$$\text{and } y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$\dot{x_1} = x_2, \dot{x_2} = x_3$, $\boldsymbol{x}$ is state vector, $\boldsymbol{y}$ is output vector, $\boldsymbol{u}$ is control input vector, $k, c, and\ M\ are\ spring\ constant, damping$ coefficient $and\ door{-}\ mass\ respectively.$

Also x represents $(displacement\ and\ velocity\ of\ the\ door\ respectively)\ and\ u\ is\ control\ input\ (force\ applied\ to\ door).$

Further the force applied to the door handle by left hand and weight carried by right hand are correlated with each other. This dynamics is however very complex. The relation between thisforce (left hand) and weight carried by right hand is nonlinear in nature and can be representedby :

force applied by left hand

= f(weight carried by right hand, velocity of walking, additional parameters)

The velocity of walk gets affected nonlinearly by weight carried by left hand. The additional parameters includes balancing force to handle weight, vision input to control weight.

Discussion

The challenge is to develop mathematical relation between force applied to open the door, therelation of weight-constraint to open door satisfying condition of passing human quickly through the door such that door does not touch the container/weight.

Conclusion

An interesting case of constraint is identified and is represented in graphical way. A small phenomenon of a walk from chair to door is considered. Its foot movements, horizontal distance travelled, vertical distance travelled, door movement, force applied to door-handle areconsidered as system parameters. In future-scope the problem statement can be created in termsof control systems and solutions can be developed / understood in different ways.

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