Pendant Domination of Line Graph of N - Sunlet Graph

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***Abstract*:-** The Line graph of a graph $G = (V, E)$ is a graph in which the vertices are correspond to the edges of G and any two vertices are adjacent if and only if the corresponding edges of G are incident with same vertex of G. Let $G = (V, E)$ be any graph. A dominating set S in G is said to be a pendant dominating set if induced sub graph of $< S >$ contains at least one pendant vertex. The least cardinality of a pendant dominating set in G is said to be a pendant domination number of G and is denoted by $\gamma_{pe}(G)$. In this article, we determine the pendant domination number and split pendant domination number of line graph of $n -$ sunlet graph S_n.

***Keywords**: Pendant Dominating Set, Pendant Domination Number, Line Graph, Sunlet Graph and Split Pendant Dominating Set.*

1. Introduction

Let $G = (V, E)$ be any graph. A dominating set S in G is said to be a pendant dominating set if the induced sub graph of $< S >$ contains at least one pendant vertex. The least cardinality of a pendant dominating set in G is said to be a pendant domination number of G and is denoted by $\gamma_{pe}(G)$.

A dominating set S in G is called a pendant dominating set if the induced sub graph of $< S >$ contains at least one pendant vertex. The concept of pendant domination in graphs is defined and studied in [8]. The least cardinality of a pendant dominating set in G is called the pendant domination number of G and is denoted by $\gamma_{pe}(G)$. In [8], authors have obtained fundamental results related to pendant domination parameter including exact values for standard graphs and bounds in terms of order and pendant domination number. Domination is the most important and vastly growing research area in the field of graph theory. The study of domination in graph theory is fastest growing area and it came as a result of study of games such as game of chess where the goal is to dominate various squares of a chessboard by certain chess pieces. The concept of domination was used by De Jaenisch in 1862 while studying the problems of determining the minimum number of queens to dominate chessboard. Berge defined the concept of domination number of graph. But the fastest growth in study of dominating set in graph theory began in 1960. Later the concept of dominating set and domination number was used by Ore in 1962, Cockayne and Hedetniemi in 1977.

The concept of domination has wide range of application in Graph theory. The concept of domination enables us to find the shortest or longest distance between any two points or places. Domination is also used in the field of Land surveying, Electrical networks, Networking, Routing problems, nuclear plants problem, Modeling problems, Coding theory etc. In this article, we discuss about the pendant domination number, split pendant domination number and connected domination number of n-Sunlet graph S_n. In the entire paper we consider the finite, undirected, simple and non-trivial graph. The n-sunlet graph of vertices 2n is obtained by attaching n - pendant edges to the Cycle graph C_n and it is denoted by S_n.

2. Definitions

Definition 2.1. A subset S of the set of vertices $V(G)$ is said to be dominating set of G if every vertex in $V - S$ is adjacent to at least one vertex in S. The number of vertices of a minimum dominating set of a graph G is called domination number of G and it is denoted by $\gamma(G)$.

Definition 2.2. Let $G = (V, E)$ be any graph. A dominating set S in G is said to be a pendant dominating set if induced sub graph of $< S >$ contains at least one pendant vertex. The least cardinality of a pendant dominating set in G is said to be a pendant domination number of G and is denoted by $\gamma_{pe}(G)$.

Definition 2.3. The Line graph of a graph G is denoted by $L(G)$ and is defined as the graph G which has its vertices as the edges and two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent.

Definition 2.4. The $n -$ sunlet graph is a graph with $2n$ vertices are obtained by attaching n - pendant edges to the cycle graph C_n and it is denoted by S_n.

Definition 2.5. The pendant dominating set S of a graph G is said to be split pendant dominating set of G if induced sub graph $< V - S >$ is disconnected. The minimum cardinality of a split pendant dominating set is called split pendant dominating number and it is denoted by $\gamma_{spe}(G)$.

Definition 2.6. The pendant dominating set S of a graph G is said to be non- split pendant dominating set of G if induced sub graph $< V - S >$ is connected. The minimum cardinality of a non-split pendant dominating set is called non-split pendant dominating number and it is denoted by $\gamma_{nspe}(G)$.

Remarks:

(i) Number of vertices in n - sunlet graph S_n is $p = 2n$
(ii) Number of edges in n - sunlet graph S_n is $q = 2n$
(iii) Maximum degree in n - sunlet graph S_n is $\Delta = 3$
(iv) Minimum degree in n-sunlet graph S_n is $\delta = 1$
(v) Maximum degree in the line graph of S_n is $\Delta = 4$
(vi) Minimum degree in the line graph of S_n is $\delta = 2$

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3. Pendant Domination of Line graph of n - Sunlet graph S_n

Proposition 3.1. [10] The domination number of line graph of $3 -$ sunlet graph is 2

i.e. $\gamma[L(S_3)] = 2$

Theorem 3.1. The pendant domination number of line graph of $3 -$ sunlet graph is 2

$$\gamma_{pe}[L(S_3)] = 2$$

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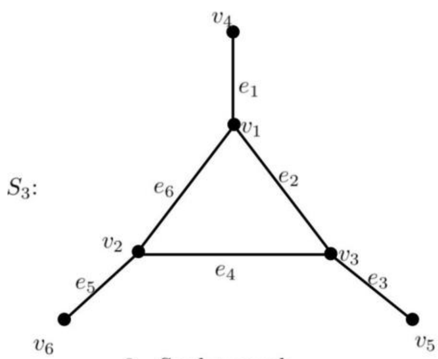
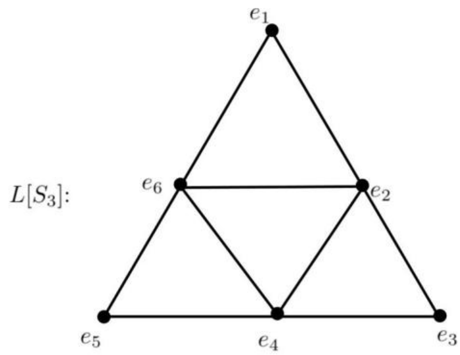


3 - Sunlet graph



Line graph of 3 - Sunlet graph

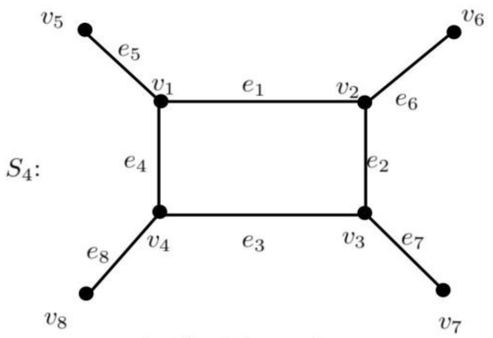
Proof. Let $S = \{e_2, e_6\}$ be a vertex set. The vertices $\{e_2\}$ and $\{e_6\}$ are adjacent to each other. The vertex $\{e_2\}$ dominates the vertices $\{e_1, e_3, e_4\}$ where as the vertex $\{e_6\}$ dominates the vertices $\{e_1, e_5\}$.Therefore S is the minimal pendant dominating set of $L(S_3)$. If we remove the vertex $\{e_2\}$ from S then $\{e_2\}$ is not adjacent to $\{e_6\}$ and also not dominated by $\{e_6\}$. Thus S will not be a pendant dominating set of $L(S_3)$. Similarly, if we remove the vertex $\{e_6\}$ from S then $\{e_6\}$ is not adjacent to $\{e_2\}$ and also not dominated by $\{e_2\}$. Thus S will not be a pendant dominating set of $L(S_3)$. From both the cases we conclude that S is the minimal pendant dominating set and thus the pendant domination number of line graph of $3-$ sunlet graph is 2. i.e. $\gamma_{pe}[L(S_3)] = 2$

Proposition 3.2. [10] The domination number of line graph of $4-$ sunlet graph is 2

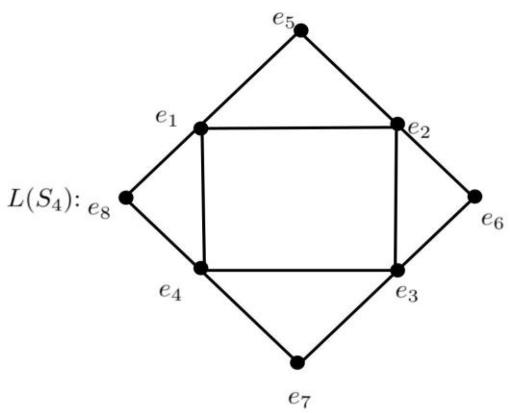
i.e. $\gamma[L(S_4)] = 2$

Theorem 3.2. The pendant domination number of line graph of $4-$ sunlet graph is 3

$$\gamma_{pe}[L(S_4)] = 3$$



4 - Sunlet graph

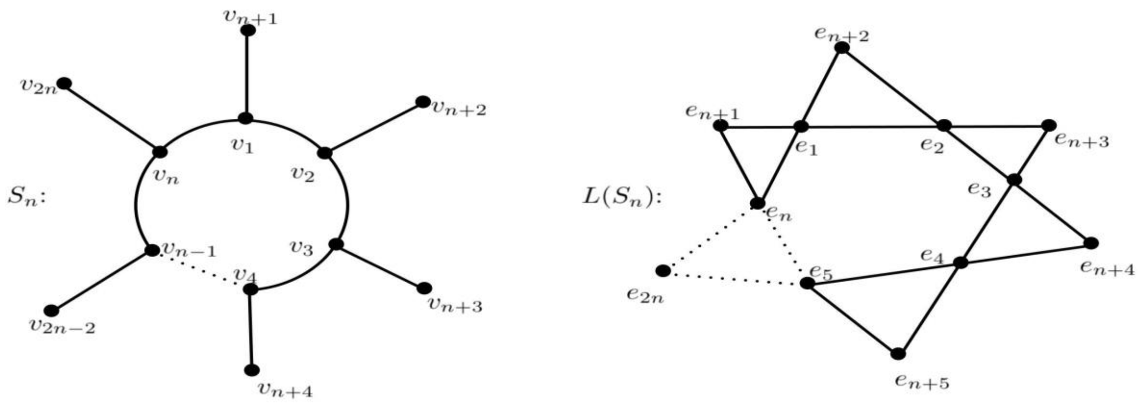


Line graph of 4 - Sunlet graph

Proof. Let $S = \{e_1, e_2, e_7\}$ be a vertex set. The vertices $\{e_1\}$ and $\{e_2\}$ are adjacent to each other. The vertex $\{e_1\}$ dominates the vertices $\{e_4, e_5, e_8\}$ where as the vertex $\{e_2\}$ dominates the vertices $\{e_3, e_5, e_6\}$. The vertex $\{e_7\}$ remains isolated and considered in the pendant dominating set. Therefore S is the minimal pendant dominating set of $L(S_4)$. If we remove the vertex $\{e_1\}$ from S then $\{e_1\}$ is not adjacent to $\{e_2\}$ and also not dominated by $\{e_2\}$. Thus S will not be a pendant dominating set of $L(S_4)$. Similarly, if we remove the vertex $\{e_2\}$ from S then $\{e_2\}$ is not adjacent to $\{e_1\}$ and also not dominated by $\{e_1\}$. Thus S will not be a pendant dominating set of $L(S_4)$. From both the cases we conclude that S is the minimal pendant dominating set and thus the pendant domination number of line graph of $4-$ sunlet graph is 3. i.e. $\gamma_{pe}[L(S_4)] = 3$

Proposition 3.3. [10] The domination number of line graph of $n-$sunlet graph is $\gamma[L(S_n)] = \left\lceil\frac{n}{2}\right\rceil$ where $n \geq 3$

Theorem 3.3. The pendant domination number of line graph of $n-$sunlet graph is $\gamma_{pe}[L(S_n)] = \left\lceil\frac{n+1}{2}\right\rceil$ where $n \geq 3$



n - Sunlet graph   Line graph of n - sunlet graph

Proof. Let n be the number of vertices of $n-$sunlet graph with 2n edges. Let $S_n = \{e_1, e_2, \ldots, e_{n-1}, e_n, e_{n+1}, \ldots, e_{2n-1}, e_{2n}\}$ be the vertices of a line graph of $n-$sunlet graph. Let $S = \{e_{2n}, e_{2n-2}, e_{2n-4}, \ldots, e_n\}$ be a vertex set. The vertex $\{e_{2n}\}$ is adjacent to the vertices $\{e_n, e_{2n-2}\}$ so these vertices are dominated by the vertex $\{e_{2n}\}$. Since the degree of each vertex of the set $\{e_1, e_2, \ldots, e_{n-1}, e_n\}$ is dominated by atleast four vertices. Similarly, all the vertices of $L(S_n)$ is dominated by atleast one vertex in S resulting that S is the minimal pendant dominating set. If any vertex $\{v\} \in S$ is removed from S then S will not be a pendant dominating set. Therefore S is the minimal pendant dominating set and hence the pendant domination number of line graph of $n-$sunlet graph is $\gamma_{pe}[L(S_n)] = \left\lceil\frac{n+1}{2}\right\rceil$ where $n \geq 3$

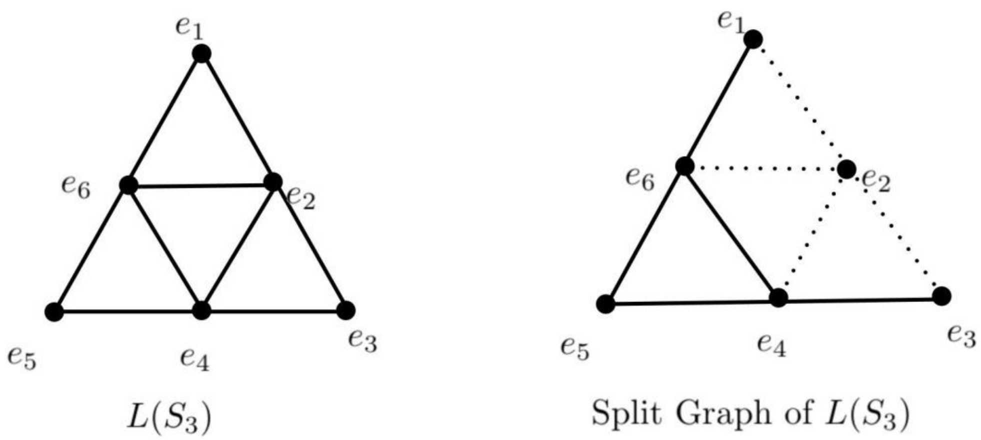
4. Split Pendant Domination of Line graph of n - Sunlet graph S_n

Proposition 4.1. **[10]** The split domination number of line graph of $3-$sunlet graph is 2

i.e. $\gamma_s[L(S_3)] = 2$

Theorem 4.1. The split pendant domination number of line graph of $3-$sunlet graph is 3

$$\gamma_{spe}[L(S_3)] = 3$$

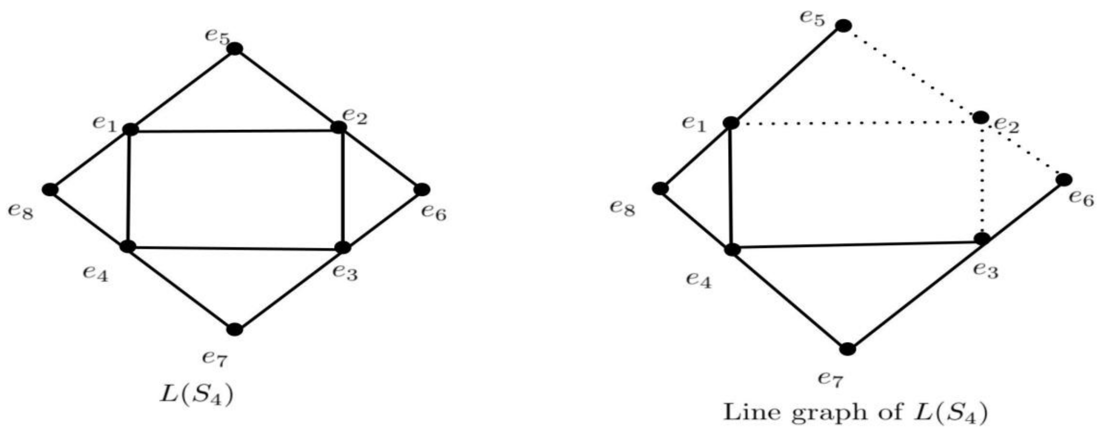


Proof. Let $S = \{e_2, e_6\}$ be a vertex set. By **Proposition 3.1**. we have, S is the pendant dominating set of line graph of S_3 as all of the vertices of S_3 is adjacent to atleast one vertex of S. If $S' = S - \{e_2\}$ then on removal of the vertex $\{e_2\}$ from S results in non-adjacency between the vertices of S then S' is not a pendent dominating set. Therefore, S is the minimal pendant dominating set. Let us consider $S_3' = S_3 - S$ and on removal of any of the vertices from S_3, the graph S_3 becomes disconnected as the vertex $\{e_2\}$ becomes isolated making the vertices $\{e_1, e_3\}$ as the pendant vertices. Therefore, the pendant dominating set $S = \{e_4, e_6, e_2\}$ is the split pendant dominating set as the vertex $\{e_2\}$ is isolated which disconnects the graph $L(S_3)$. Hence the split pendant domination number of a $3-$sunlet graph is 3

Theorem 4.2. The split pendant domination number of line graph $4-$sunlet graph is 4

$$\gamma_{spe}[L(S_4)] = 4$$

Proof. Followed by **Theorem 4.1**.



Proposition 4.2. **[10]** The split domination number of line graph of $n-$sunlet graph is

$\gamma_s[L(S_n)] = \left\lceil\frac{n}{2}\right\rceil$ where $n \geq 3$

Theorem 4.3. The split pendant domination number of line graph of $n-$ sunlet graph is

$$\gamma_{spe}[L(S_n)] = \left\lceil\frac{n+1}{2}\right\rceil + 1 \quad \text{where } n \geq 3$$

Proof. Let $S_n = \{e_1, e_2, \dots, e_{n-1}, e_n, e_{n+1}, \dots, e_{2n-1}, e_{2n}\}$ be a vertex set. Let S be a pendant dominating set of the line graph of S_n as all of the vertices are adjacent to atleast one vertex in S. If $S_n' = S_n - \{e_i\}$ where $1 \leq i \leq 2n$ then on removal of any of the vertex from S results in a non-adjacency between the vertices of S. Therefore, S is the minimal pendant dominating set. If we remove any of the vertex from S_n the graph becomes disconnected and isolated. Therefore, S is the split pendant dominating set as any of the vertex $\{e_i\}$ is isolated. Hence the split pendant domination number of line graph of $n-$ sunlet graph is $\left\lceil\frac{n+1}{2}\right\rceil + 1$ where $n \geq 3$

5. Conclusion

This paper makes a significant contribution by introducing two new forms of pendant domination in the line graph of the n-sunlet graph S_n, the pendant domination number and the split pendant domination number. we extend the classical domination number to these new concepts and provide results for the line graph of S_n offering a richer understanding of domination in this context. These results may have applications in various fields, such as network theory, where the structure of dominating sets plays a crucial role in network efficiency, coverage and robustness.

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