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Linear instability of Nanofluid with magnetic effect

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Abstract: - The investigation focuses onset of convection in a horizontal layer saturated nanofuid with magnetic field. The non-dimensional governing equations is solved using the normal mode technique, resulting in an eigenvalue problem. The eigenvalue problem for linear instability is solved using the one-term Galerkin approach which gives the analytical expression for Rayleigh number. Neutral curves are drawn for both steady and oscillatory instability for all physical parameters.

Keywords: *Thermal instability, Nanofluid, Eigenvalue Problem.*

1. Introduction

Due to its practical uses in a range of geological processes, including liquid re-injection, mi-gration, subsurface nuclear waste disposal, and drying processes, double-diffusive convective phenomena is a topic of great interest for researchers [1-2]. The double-diffusive instability of a Newtonian fluid in a horizontal layer has also been studied in great detail. Likewise, numerous materials exhibit non-Newtonian behaviour, such as paints, mud, clay, honey, blood, and hair gel, making non-Newtonian fluid convective events intriguing [3]. Numerous models, including the power-law model, the Maxwell model, the Jeffrey model, and other viscoplastic fluid models, are available in the literature to analyse the characteristics of non-Newtonian fluids [3].

The beginning of convection in a porous layer saturated with Oldroyd fluid was covered by Malashetty and Swamy [4]. It was possible to derive the analytical conditions for finite amplitude, steady, and stable convections. Using the thermal non-equilibrium effect, Malashetty et al. [5] and Kumar and Bhadauria [6] extended the same problem. The double-diffusive convective motion of a Maxwell liquid was studied by Awad et al. [7]. They come to the conclusion that the critical Rayleigh number falls by using the Maxwell parameter as a stand-in. Wang and Tan [8] investigated convective instability for non-Newtonian liquid in a porous layer using a modified Maxwell-Darcy model. Internally heated double diffusive instability in a non-Newtonian form of coupled stress fluid flooded porous layer was examined by Gaikwad and Kouser [9]. They found that the internal Rayleigh number stabilises the system.

Gaikwad and Dhanraj [10] investigated the effects of anisotropy and internal heating on the binary Maxwell liquid in a permeable layer. on a recent work, Yadav et al. [11] discovered the chemical reaction effect on the thermosolutal internally heated convection of a Maxwell fluid in a porous layer. They found that the Damkohler number has distinct effects on oscillatory and steady convection.

The current paper examines the beginning of magneto convection in a nanofluid. We write fundamental equations in section 2. Linear instability is covered in Section 3. The following parts provide the results and conclusions.

2.Mathematical formulation

Consider a heated, infinitely thin, horizontal layer of nanofluid with thickness 'd' that is con fined by the planes z = 0 and z = d. It is assumed that each boundary wall is impermeable and has perfect heat conductivity. The volumetric fraction φ and temperature T of nanoparticles are assumed to be T_0 and φ_0 at z = 0 and T1 and φ_1 at z = d, respectively ($T_0 > T_1$). The assumed reference temperature is T_1. The governing equations are:

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\rho_0\left(\frac{\partial \mathrm{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right) = -\nabla P + \sigma_1\left(\mathbf{V}\times B_0\,\hat{e}_z\right)\times B_0\,\hat{e}_z$$
$$+\left[\phi\rho_p + (1-\phi)\rho_0\left(1-\beta(T-T_1)\right)\right]\mathbf{g}, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + (\mathbf{V}\cdot\nabla)\right)\phi = D_B\nabla^2\phi + \frac{D_T}{T_1}\nabla^2 T, \tag{3}$$

$$\rho c\left(\frac{\partial}{\partial t} + (\mathbf{V}\cdot\nabla)\right)T = \kappa_T\nabla^2 T + \rho_p c_p\left[D_B\nabla\phi\cdot\nabla T + \frac{D_T}{T_1}\nabla T\cdot\nabla T\right]. \tag{4}$$

were

$\mathbf{V}$ – velocity of nanofluid; c_p – nanofluid specific heat ;
μ – Coefficient of fluid viscosity; ρ_f – density of base fluid ;
T – temperature of nanofluid; ρ – Density ;
T_1 – reference temperature; P – Pressure ;
ρ_p – nanoparticle density; ρ_0 – reference density;
ϕ – volumetric fraction of nanoparticles; t – time ;
h_p – specific enthalpy of the nanoparticle; $\mathbf{g}$ – the accelaration due to gravity
$\hat{e_z} = (0,0,1)$ – unit vector along the vertical axis ;
$.\beta$ – thermal expansion coefficient ;
κ_T – thermal conductivity of nanofluid;
D_T – thermophoretic diffusion coefficient of nanoparticles;
D_B – Brownian diffusion coefficient of nanoparticles.

Subject to the boundary conditions

$$\left.\begin{array}{l}\mathbf{V}=0,\ \ T=1,\ \ \phi=0\ \ \text{at}\ \ z=0,\\ \mathbf{V}=0,\ \ T=0,\ \ \phi=1\ \ \text{at}\ \ z=1.\end{array}\right\} \tag{5}$$

The following non-dimensional parameters are introduced:

$$(x',y',z') = \frac{1}{d}(x,y,z), \quad t' = \frac{\alpha t}{d^2}, \quad P' = \frac{d^2 P}{\mu\alpha},$$
$$(u',v',w') = \frac{d}{\alpha}(u,v,w), \quad T' = \frac{T-T_1}{T_0-T_1}, \quad \phi' = \frac{\phi-\phi_0}{\phi_1-\phi_0}.$$

where $\alpha = \frac{1}{\rho C}$.

The non-dimensional form of Eqs. (1), (2), (3) and (4) are

$$\nabla'\cdot\mathbf{V}' = 0, \tag{6}$$

$$\frac{1}{\Pr}\left(\frac{\partial \mathbf{V}'}{\partial t'} + (\mathbf{V}'\cdot\nabla')\mathbf{V}'\right) = \nabla'P' + \nabla'^2\mathbf{V}' - R_m\,\hat{e_z} + R_T T'\,\hat{e_z} - R_n\phi'\,\hat{e_z}$$
$$+Ha^2[(\mathbf{V}'\times\hat{e}_z)\times\hat{e}_z], \tag{7}$$

$$\left(\frac{\partial}{\partial t'} + (\mathbf{V}'\cdot\nabla')\right)\phi' = \frac{N_A}{Le}\nabla'^2 T' + \frac{1}{Le}\nabla'^2\phi', \tag{8}$$

$$\left(\frac{\partial}{\partial t'} + (\mathbf{V}'\cdot\nabla')\right)T' = \nabla'^2 T' + \frac{N_B}{Le}(\nabla'T'\cdot\nabla'\phi') + \frac{N_A N_B}{Le}(\nabla'T'\cdot\nabla'T'). \tag{9}$$

where

$$\begin{cases} R_T = \dfrac{\rho_0 g\beta d^3(T_0 - T_1)}{\mu\alpha} - \text{Rayleigh number;} \\ R_n = \dfrac{(\rho_p - \rho_{f_0})(\phi_1 - \phi_0)gd^3}{\mu\alpha} - \text{Concentration Rayleigh number;} \\ R_m = \dfrac{[\rho_p\phi_0 + \rho_{f_0}(1-\phi_0)]gd^3}{\mu\alpha} - \text{Basic density Rayleigh number ;} \\ N_A = \dfrac{D_T(T_0 - T_1)}{D_B T_1(\phi_1 - \phi_0)} - \text{Modified diffusivity ratio;} \\ N_B = \dfrac{\rho_p C_P}{\rho C}(\phi_1 - \phi_0) - \text{Modified particle density increment ;} \\ Ha^2 = \dfrac{\sigma_1 B_0^2 d^2}{\mu} - \text{Hartmann number ;} \\ Le = \dfrac{\alpha}{D_B} - \text{ Lewis number ;} \qquad Pr = \dfrac{\nu}{\alpha} - \text{Prandtl number.} \end{cases}$$

2.1 Basic State

It is assumed that the basic state of the nanofluid is time independent and is described by

$$\mathbf{V}_b = 0,\ \phi_b = 0,\ T_b = 1 - z. \tag{10}$$

For small disturbances onto the basic state, we assume that

$$\mathbf{V}' = \mathbf{V}_b + \mathbf{V}, \qquad P' = P_b + P, \qquad T' = T_b + T, \qquad \phi' = \phi_b + \phi. \tag{11}$$

3. Linear stability analysis

By substituting Eq. (11) into Eqs. (14)-(17), we obtain

$$\nabla \cdot \mathbf{V} = 0, \tag{12}$$

$$\frac{1}{Pr}\frac{\partial \mathbf{V}}{\partial t} = \nabla P + \nabla^2\mathbf{V} - R_m\,\hat{e_z} + R_T T\,\hat{e_z} - R_n\phi\,\hat{e_z} + Ha^2\left[\left(\mathbf{V}\times\hat{e}_z\right)\times\hat{e}_z\right], \tag{13}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T, \tag{14}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Le}\nabla^2\phi + \frac{N_A}{Le}\nabla^2 T. \tag{15}$$

By Taking the third components of curl of (13) and curl of curl of (13), we obtain,

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2 + Ha^2\right)\omega_z = 0,$$

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2 - \nabla^4 + +Ha^2\frac{\partial^2}{\partial z^2}\right)\omega - R_T\nabla_h^2 T + Rn\nabla_h^2\phi = 0, \tag{17}$$

$$\omega - \left(\frac{\partial}{\partial t} - \nabla^2\right) = 0, \tag{18}$$

$$\frac{N_A}{Le}\nabla^2 T + \left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)\phi = 0. \tag{19}$$

Where $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

And $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Let us introduce the normal modes by writing the perturbations in the form of

$$(\omega, T, \phi) = (\omega, T, \phi)\mathrm{Sin}(n\pi z)e^{i(lx+my)+\sigma t}. \tag{20}$$

Substituting the above normal mode solution into the equations (17)- (19), then we get

$$\left(\frac{\sigma}{Pr}\delta^2 - \delta^4 + Ha^2\pi^2\right)\omega - R_T q^2 T + Rnq^2\phi = 0, \tag{21}$$

$$\omega - (\sigma + \delta^2)T = 0, \tag{22}$$

$$\frac{N_A}{Le}\delta^2 T + (\sigma + \frac{1}{Le}\delta^2)T = 0. \tag{23}$$

Where $\begin{cases} q^2 = l^2 + m^2 \text{ is the wave number,} \\ \sigma = \iota\omega, \\ \delta^2 = \pi^2 + q^2. \end{cases}$

Requiring zero determinant of the above system, one obtains,

$$R_T = \frac{(\delta^2 + i\omega)(Pr\delta^4 + \iota i\omega + \pi^2 PrHa^2)}{Prq^2} - \frac{Rn\delta^2 N_A}{\delta^2 + iLe\omega} \tag{24}$$

3.1. stationary convection

Substituting ω= 0 in Eq. (24), then we get

$$R_{T_{sc}} = \frac{\delta^6 + \pi^2\delta^2 Ha^2}{q^2} - RnN_A \tag{25}$$

For Newtonian liquids, in the absence of magnetic effect, the above formula becomes

$$R_{T_{sc}} = \frac{\delta^6}{q^2} \tag{26}$$

which is well agreed with Chandrasekhar [1].

3.2. Oscillatory

To find the R_T for oscillatory convection we find the roots of imaginary part of Rayleigh number. On substituting roots into the real part of Rayleigh number we get the R_T for oscillatory convection.

$$R_{T_{oc}} = \frac{\delta^2(\delta^4 + Le^2\omega^2)(Pr\delta^4 - \omega^2 + \pi^2 PrHa^2) - Prq^2 Rn\delta^4 N_A}{Prq^2(\delta^4 + Le^2\omega^2)} \tag{27}$$

where

$$\omega^2 = \frac{\delta^2\left(-\delta^2 - \frac{LePrq^2 RnN_A}{(1+Pr)\delta^4 + \pi^2 PrHa^2}\right)}{Le^2} \tag{28}$$

4. **Results and Conclusions**

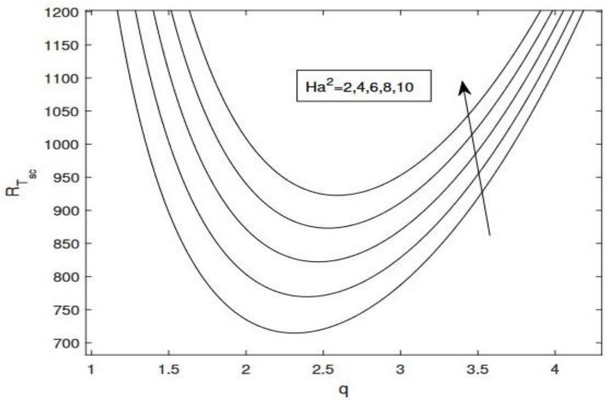

Figure 1: Neutral curves for the different values of Ha^2 and for the fixed values of $N_A = 2, Pr = 5, Rn = 0.2, Le = 10$ at the onset of stationary convection.

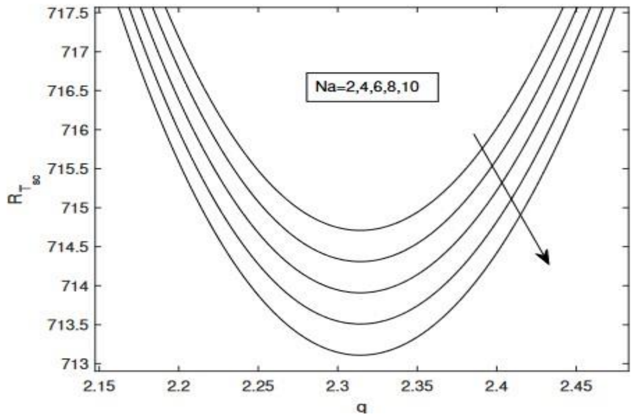

Figure 2: Neutral curves for the different values of N_A and for the fixed values of $Ha^2 = 2, Pr = 10, Rn = 0.2, Le = 10$ at the onset of stationary convection.

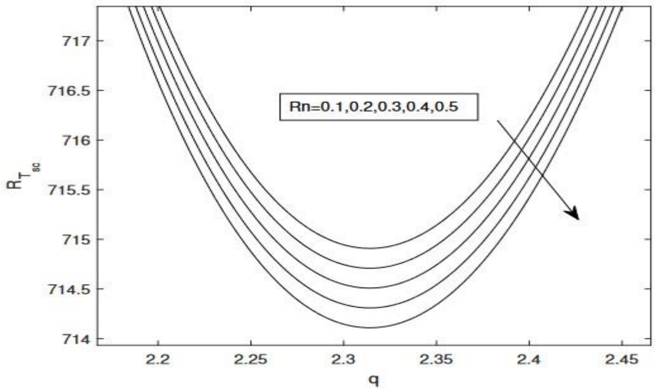

Figure 3: Neutral curves for the different values of Rn and for the fixed values of $N_A = 2, Pr = 8, Ha^2 = 2, Le = 10$ at the onset of stationary convection.

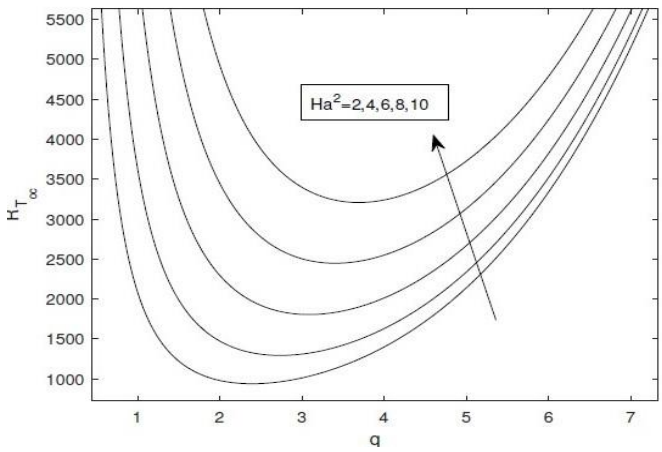

Figure 4: Neutral curves for the different values of Ha^2 and for the fixed values of $N_A = 5, Pr = 10, Rn = 0.2, Le = 5$ at the onset of stationary convection.

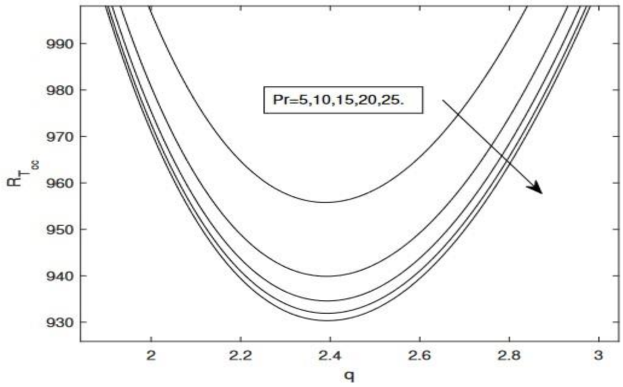

Figure 5: Neutral curves for the different values of Pr and for the fixed values of $N_A = 4, Ha^2 = 2, Rn = 0.2, Le = 5$ at the onset of stationary convection.

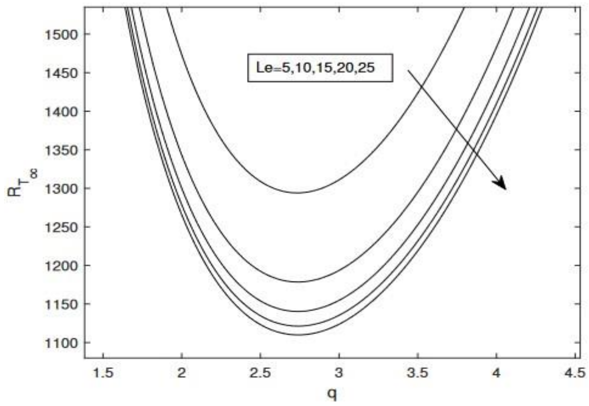

Figure 6: Neutral curves for the different values of Le and for the fixed values of $N_A = 5, Pr = 10, Rn = 0.2, Ha^2 = 4$ at the onset of stationary convection.

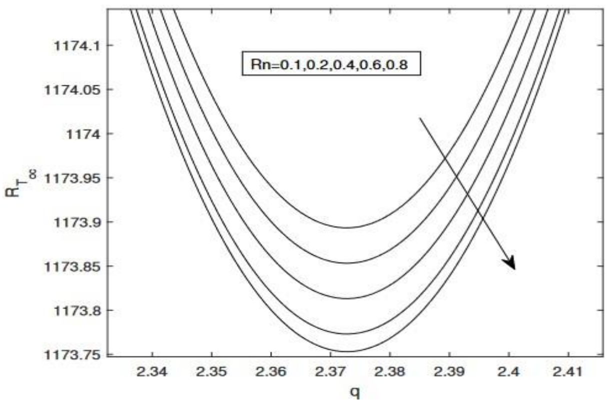

Figure 7: Neutral curves for the different values of Rn and for the fixed values of $N_A = 2, Pr = 2, Ha^2 = 2, Le = 3$ at the onset of stationary convection.

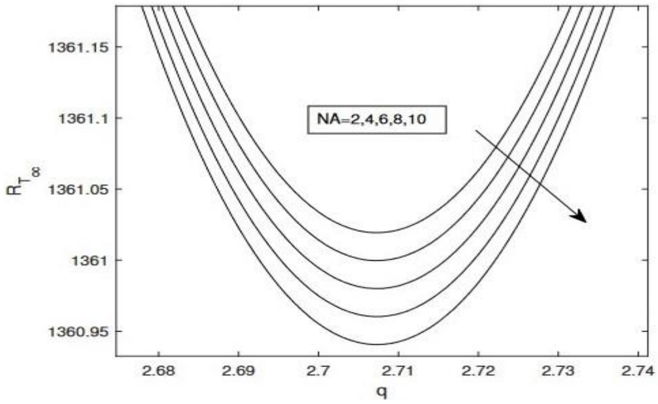

Figure 8: Neutral curves for the different values of N_A and for the fixed values of $Ha^2 = 4, Pr = 2, Rn = 0.2, Le = 5$ at the onset of stationary convection.

It is investigated how a magnetic field affects the convective instability of a Casson nanofluid. To our knowledge, no prior research has been done on the current issue. The Rayleigh number (R_T), Hartmann number (Ha^2), Lewis number (Le), Prandtl number (Pr), Modified diffusivity ratio (N_A)and Concentration Rayleigh number (Rn) are the non-dimension controlling parameters of the start of the convection. Using the Galerkin approach, the eigenvalue problem for linear stability analysis is resolved, providing the analytical equation for the Rayleigh number. For all physical parameters, neutral curves are constructed for both steady and oscillatory instability.

Figs. 1-3 shows the neutral curves in the plane $(R_{T_{sc}}, q)$. Lewis number and Prandtl number does not show any effect on the neutral curves in the plane $(R_{T_{sc}}, q)$, because, $R_{T_{sc}}$ is independent of Pr and Le. In Fig. 1, neutral curves have been shown for distinct values of Ha^2 at the onset of stationary convection. From Fig. 1, it is observed that as Ha^2 increases the neutral curves move upward monotonically and indicating that instability in the system. Hence, an enhance value of Ha^2 has a stabilizing effect in the system. Neutral curves for distinct values of the N_A and Rn in the plane $(R_{T_{sc}}, q)$ have been shown in Figs. 2 and 3 respectively. According to these, as N_A and Rn increases critical $R_{T_{sc}}$ decreases. Means that, N_A and Rn have a destabilizing effect in the system.

The neutral curves at the onset of oscillatory convection have shown in Figs. 4-8 for the distinct values of physical parameters. A stabilizing effect of Ha^2 on oscillatory convection can be observed in Fig. 4. The same behavior is observed in stationary convection. Effect of Pr on $R_{T_{oc}}$ has shown in Fig. 5. Critical $R_{T_{sc}}$ decreases as Pr increases, indicating that an enhance in the value of Pr advances the onset of convection. In Fig. 6, neutral curves have been shown for distinct values of Le at the onset of oscillatory convection. It is shown that the Critical $R_{T_{sc}}$ is a decreasing function of Le. Hence, Le has destabilizing effect on the flow. Neutral curves for distinct values of the Na and Rn in the plane $(R_{T_{oc}}, q)$ have been shown in Figs. 7 and 8 respectively. According to these, as Na and Rn increases critical $R_{T_{sc}}$ increases. Means that, N_A and Rn have the stabilizing effect in the system.

4.1. Conclusions

This work considers the convective instability problem of a nanofluid with magnetic effects with linear evaluations. The Galerkin method is used to study the linear theory. Remarkably, there is no appreciable effect of the Prandtl number Pr and Lewis number Le on stationary convection. However, the Hartmann number Ha^2 stabilize the flow whereas the, concentration Rayleigh number Rn and modified diffusivity ratio N_A destabilizes the flow. However, in oscillatory convection, it is discovered that the concentration Rayleigh number Rn, modified diffusivity ratio N_A, Lewis number Le and Prandtl Pr destabilize the flow while and Hartmann number Ha^2 stabilize it.

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