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Finsler Geometry in the Analysis of Images

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Abstract: This work aims to provide a clear overview of the importance of Finsler Geometry in the analysis of images.

Key words: Finsler metric, Tensor, Image Analysis

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Diffusion tensor imaging (DTI) is recognized by authors [1] as a valuable technique for identifying microstructural alterations associated with neuropathology and therapeutic interventions. This imaging modality can effectively characterize the magnitude, degree of anisotropy, and orientation of directional diffusion. The present review explores the biological mechanisms underlying DTI, as well as the processes involved in the acquisition and analysis of DTI data. It summarizes the correlations between DTI metrics and pathological features of white matter, including ischemia, myelination, axonal injury, inflammation, and edema. Furthermore, the review examines the application of DTI in tissue characterization within neurotherapeutic contexts. Common DTI metrics, such as mean diffusivity (MD), fractional anisotropy (FA), radial diffusivity (Dr), and axial diffusivity (Da), are discussed in detail. Notably, while FA demonstrates high sensitivity to microstructural changes, it lacks specificity regarding the nature of these changes, whether radial or axial. To enhance specificity and provide a more comprehensive characterization of tissue microstructure, future research should incorporate multiple diffusion tensor metrics, such as MD and FA or Da and Dr.

According to the authors [2], functional magnetic resonance urography (fMRU) provides valuable morphological and functional data based on perfusion analysis. Complementing this, diffusion tensor imaging (DTI) assesses renal microstructure, offering insights into the interplay between renal structure and function. The primary aim of this study was to investigate the feasibility and effectiveness of renal DTI and tractography in the context of fMRU among children. A total of nine children (six boys and three girls) were prospectively enrolled, with a mean age of 4.3 years (ranging from six months to 14.8 years). All participants underwent MRI at a 3.0 tesla magnetic field strength. DTI was conducted using an echo-planar sequence (TR/TE = 2,300/69 ms, b = 300 s/mm^2), incorporating twelve non-collinear directions and three signal averages. The functional MRU findings were employed to classify the renal moieties as normal or abnormal. DTI parameters were measured in designated regions of interest within the medulla and cortex, and the tractography-derived parenchymal volumes were compared to those obtained from fMRU.

Using Finsler information geometry, authors present [3] a technique for ship detection in synthetic aperture radar (SAR) amplitude images in this work. They concentrate on metric space in Finsler geometry. This offers profound, cohesive viewpoints on the use of Finsler geometry. Three steps make up the suggested approach: first, the statistical information of intensity SAR images is represented using the Weibull manifold model; next, the Finsler metric is built to realize the distance measurement between probability distributions in Weibull manifold space; and last, saliency representation and ship detection are accomplished using Finsler metric space. Using typical real SAR images, theoretical analysis and extensive experimental findings show the resilience and efficacy of the suggested technique.

A non-invasive imaging method called diffusion tensor magnetic resonance imaging (DT-MRI) makes it possible to calculate the molecular self-diffusion tensors of water in the surrounding tissue. In a post-processing step, reconstructed tensor pictures typically need some kind of regularization because magnetic resonance images have a low signal-to-noise ratio. In terms of the reconstruction or regularization process, earlier methods are either less than ideal. In order to overcome the drawbacks of other methods, this work proposes a Bayesian strategy for the simultaneous reconstruction and regularization of DT-MR images. In order to achieve this, tensor valued pictures that are regarded as Riemannian manifolds are generalized to estimation theory notions. By doing this, authors [4] can obtain a maximum a posteriori estimate of the tensor image that takes into account the nonlinear structure of tensor valued pictures as well as the statistical properties of the Rician noise present in MR images. The benefit of taking into account both the statistical and geometrical aspects of DT-MRI is confirmed by experiments conducted on both synthetic and actual DT-MRI data.

Authors [5] investigate three-dimensional volumes of higher-order tensors through the lens of Finsler geometry. The focus of their application is medical image analysis, particularly in the context of High Angular Resolution Diffusion Imaging (HARDI) [1] of the brain. They identify effective methods to elucidate the architecture of neural fibers within the white matter of the brain. In Diffusion Tensor Imaging (DTI), the diffusion of water is represented by a symmetric positive definite second-order tensor, predicated on the assumption that a single dominant direction of fibers constrains the thermal motion of water molecules, which naturally leads to a Riemannian framework. HARDI has the potential to address the limitations of DTI by accommodating multiple relevant directions; however, this invalidates the Riemannian approach. Finsler geometry, on the other hand, offers a suitable geometric generalization for multi-fiber analysis. In this paper, the authors present a precise criterion for determining whether a field of spherical functions possesses a Finsler structure. They also introduce a fiber tracking method within the Finsler framework. Their model includes a scale parameter, which proves advantageous given the inherently noisy nature of the data. The authors validate their methods using both analytic and real HARDI datasets

Authors [6] investigate a prominent scalar quantity in Riemannian geometry, known as the Ricci scalar, within the framework of diffusion tensor imaging (DTI), a novel non-invasive medical imaging technique. They provide a physical interpretation of the Ricci scalar and examine its experimental relevance in DTI. Furthermore, they broaden the definition of the Ricci scalar to accommodate high angular resolution diffusion imaging (HARDI) through the application of Finsler geometry. It is noteworthy that the Ricci scalar is applicable not only to tensor-valued image analysis but can also be calculated for any mapping.

An inner product on the tangent space determines a distance function in Riemannian geometry. A norm in Riemann-Finsler geometry can be used to determine this distance function. This allows for greater flexibility in the form of the set of unit vectors, also known as the so-called indicatrix. This has some intriguing uses, such as in the processing of medical images, particularly in diffusion weighted imaging (DWI). By measuring the restricted diffusion of water within the tissue, DWI is used to infer the local architecture of the tissue, which is usually made up of thin, elongated structures like muscle fibers or axons. The diffusion orientation distribution function (dODF), which can be represented as a spherical polynomial and shows the relative diffusivity in all directions, can be estimated using high angular resolution diffusion imaging (HARDI) data. The (second order) diffusion tensor technique is directly generalized by expressing this dODF as an equivalent spherical monomial (higher order tensor). Authors [7] present a straightforward and effective algorithm to invert even order spherical monomials, which extends the well-known inversion of diffusion tensors, i.e., symmetric matrices, to facilitate the efficient computation of Riemann–Finslerian quantities on diffusion weighted (DW)-images, such as the metric/norm tensor.

Magnetic Resonance Imaging (MRI) is recognized [8] as a distinctive radiological imaging technique due to its capability to conduct tomographic imaging of the body without employing any harmful ionizing radiation. Radiologists utilize MRI to obtain detailed insights into the anatomy of various organs, including the brain, while biomedical researchers leverage this modality to enhance their understanding of brain structure and function. Nevertheless, conventional MRI is limited by its resolution and contrast, which restricts its ability to depict the microstructure of the brain. Diffusion Tensor Imaging (DTI) builds upon the principles of conventional MRI to analyze the diffusion behavior of water molecules within tissues, thereby indirectly constructing a three-dimensional representation of brain anatomy. DTI facilitates the visualization of brain tissue microstructure, proving invaluable for the comprehension of numerous neuropathologies and neurodegenerative conditions. This review will provide an overview of the foundational concepts and operational principles of DTI, followed by an examination of contemporary trends in its applications for biomedical and clinical research concerning various brain diseases and disorders.

Authors [9] express a JC papovavirus infection causes progressive multifocal leukoencephalopathy, a severe demyelinating illness of the central nervous system. 90% of patients have a dismal prognosis and die within a year. Their case study demonstrates that diffusion tensor imaging can provide early and precise information regarding the state and severity of a congenital HIV infection in a 15-year-old girl with progressive multifocal leukoencephalopathy.

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