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INTUITIONISTIC FUZZY SOFT MODULES IN BCK −ALGEBRA

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Abstract: Since Iseki defined the notion of BCK-algebra in 1966, many authors conducted wide research on BCK-algebra in ordinary and fuzzy cases. In this article, the concept of intuitionistic fuzzy soft BCK-modules has been investigated and some basic properties and related results are studied. Also, some operations of intuitionistic fuzzy soft BCK- submodules and (β,γ) level set are presented.

Key Words: Intuitionistic Fuzzy soft set; BCK-algebra; Intuitionistic fuzzy soft BCK- algebra, Intuitionistic fuzzy soft BCK-modules.

1. Introduction

The concept of fuzzy set was introduced by L. A. Zadeh in 1965 [1]. The relation between fuzzy sets and group theory developed by A. Rosenfeld and regulated the notion of fuzzy subgroups [2]. Since then these ideas have been applied to other algebraic structure such as Fuzzy ring [3, 5], Fuzzy Subfields [5, 6], Fuzzy ideals [7, 8] Fuzzy linear Spaces [9] and Fuzzy modules [10]. The notion of Intuitionistic fuzzy sets (IFS) established by Atanassov in 1986 that involved basic and fundamental as the generalizations of fuzzy sets [11, 12]. In fact, the theory of IFS has been more benefited to solve incomplete and vague information. This concept is wide useful as an intuitionistic fuzzy sets, related to the degree of nonmembership and membership in a unit closed interval [0,1], while a fuzzy set is related to the degree of membership of an element in a specified set. Numerous ideas have been developed via IFS theory to intuitionistic fuzzy subgroup of fuzzy group by Biswas [13]. Further more many mathematicians worked in this area such as Intuitionistic Fuzzy ring [14], Intuitionistic fuzzy ideal [15], Intuitionistic fuzzy modules [16] etc.

In 1966, Kiyoshi Iseki [19] defined and studied BCI-algebra and it's characterization was discussed by Yoshinari Arai et al. [21] . Jun et al. [17] proposed fuzzy soft set theory applied to BCI/BCK-algebra in 2010. Cigdem and Sadi Bayramov [16] established intuitionistic fuzzy soft modules in 2011 and Mahmood Bakshi [22] applied fuzzy set theory to BCK-modules. In this article, we discuss intuitionistic fuzzy soft modules in BCK-algebra and various related results.

2. Preliminaries

In this section, we recollect some relevant basic definitions and results of this article [22].

Definition 2.1 An algebra $(X,*,0)$ of type $(2,0)$ is called BCK-algebra if it satisfies the following axioms:

1. $((x*y)*(x*z))*(y*z))=0$
2. $(x*(x*y))*y=0$
3. $x*x=0$
4. $0*x=0$
5. $x*y=0$ and $y*x=0 \Rightarrow x=y$ for all $x,y,z\in X$.

Definition 2.2 A partial ordering "$\leq$" is defined on X by $x\leq y \Leftrightarrow x*y=0$.

A BCK − algebra X is said to be

1. bounded if there is an element $1 \in X$ s.t. $x \leq 1$, for all $x \in X$.
2. commutative if it is satisfies the identity $x \wedge y = y \wedge x$, where $x \wedge y = x * (y * x)$, for all $x, y \in X$.
3. implicative if $x * (y * x) = x$, for all $x, y \in X$.

Example 2.3 Let A be a non-empty set and $X = \mathcal{P}(A)$, the power set of A, then $(X, -, \phi)$ is a $BCK-$ algebra.

Definition 2.4 [1] A mapping $\mu: X \longrightarrow [0,1]$ is a fuzzy set of a non-empty set X. Then the complement of μ is denoted by μ^c or $\overline{\mu}$, is the fuzzy set in X given by $\overline{\mu(x)} = 1 - \mu(x)$ for all $x \in X$.

Definition 2.5 [11, 12] An intuitionistic fuzzy set (IFS) A of a nonvoid set X is described by the formation $A = \langle x, \mu_A(x), \nu_A(x) | x \in X \rangle$, where $\mu_A: X \to [0,1]$ is the degree of membership and $\nu_A: X \to [0,1]$ is the degree of non membership of the element $x \in X$, and we have $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \langle x, \mu_A(x), \nu_A(x) | x \in X \rangle$.

Definition 2.6 [11, 12] Consider μ^c, the complement of μ which is determined by $\mu_A^c(x) = 1 - \mu_A(x)$ and the complement of ν determined by $\nu_A^c(x) = 1 - \nu_A(x)$.

Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two IFS of X. Then the following statements have introduced earlier for all $x \in X$, as follows;

1. $A \subseteq B$ iff $\mu_A(x) \leq \nu_B(x)$ & $\mu_A(x) \geq \nu_B(x)$
2. $A = B$ iff $A \subseteq B$ & $B \subseteq A$
3. $A^c = \langle \nu_A(x), \mu_A(x) \rangle$
4. $A \cup B = \langle \mu_A(x) \vee \nu_B(x), \mu_A(x) \wedge \nu_B(x) \rangle$
5. $A \cap B = \langle \mu_A(x) \wedge \nu_B(x), \mu_A(x) \vee \nu_B(x) \rangle$
6. $\Box A = \langle \mu_A(x), \mu_A^c(x) \rangle$
7. $\Diamond A = \langle \nu_A^c(x), \nu_A(x) \rangle$

Proposition 2.7 [11, 12] Let X be non empty set. Then for every IFS A;

1. $\Box A = \overline{\Diamond \overline{A}}$;
2. $\Diamond A = \overline{\Box \overline{A}}$;
3. $\Box A \subset A \subset \Diamond A$;
4. $\Box\Box A = \Box A$;
5. $\Box \Diamond A = \Diamond A$;
6. $\Diamond \Box A = \Box A$;
7. $\Diamond\Diamond A = \Diamond A$.

3. Intuitionistic Fuzzy Soft $BCK-$ algebra

Here, we give the definition of the soft set defined by Molodstov [18]. Let U be initial universe set and E be set of parameters. Let $\mathcal{P}(U)$ denotes the power set of U and $A \subset E$.

Definition 3.1 [18] A pair (F, A) is called a soft set over U if and only if F is mapping from A into the set of all subset of the set U. i.e. $F: A \to \mathcal{P}(U)$.

In other words, a soft set is parameterized family of subsets of the universe U for $\alpha \in A$, $F(\alpha)$ may be considered as the set of $\alpha-$ approximate element of the soft set (F, A). In this manner, a soft set (F, A) is given as $(F, A) = \{F(\alpha): \alpha \in A\}$. Clearly every set is a soft set, but a soft set is a not a set.

Definition 3.2 [16] Let U be an initial universe set and E be set of parameters. Let $\mathcal{F}(U)$ denote the set of all intuitionistic fuzzy sets in U. Then $(\tilde{F}, A)$ is called an intuitionistic fuzzy soft set over U, where $A \subseteq E$ and $\tilde{F}$ is a mapping $\tilde{F}: A \to \mathcal{F}(U)$.

In general, for every $\alpha \in U$, $\tilde{F}(\alpha)$ is an intuitionistic fuzzy set in U and it is called an intuitionistic fuzzy value set of parameter α.

Clearly, $\tilde{F}[\alpha]$ can be written as an intuitionistic fuzzy set such that

$$\tilde{F}[\alpha] = \{\langle X, \mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x): x \in U\rangle\},$$

where $\mu_{\tilde{F}[\alpha]}(x)$ and $\nu_{\tilde{F}[\alpha]}(x)$ denotes the degree of membership and non-membership functions, respectively. If for every $x \in U$, $\mu_{\tilde{F}[\alpha]} = 1 - \nu_{\tilde{F}[\alpha]}$ and $\nu_{\tilde{F}[\alpha]} = 1 - \mu_{\tilde{F}[\alpha]}$, then $\tilde{F}[\alpha]$ will be generated to be a standard fuzzy set and then $(\tilde{F}, A)$ will be generated to be traditional fuzzy soft sets.

Definition 3.3 [17] Let (F, A) be a soft set over BCK-algebra X, where A is the subset of E. We say that (F, A) is a soft BCK-algebra over a BCK-algebra X if $F[\alpha]$ is a BCK-sub algebra of X for all $\alpha \in A$.

Definition 3.4 [16] Let $(\tilde{F}, A)$ be an intuitionistic soft set over BCK-algebra X, where A is the subset of E. We say that $(\tilde{F}, A)$ is an intuitionistic fuzzy soft BCK-algebra over a BCK-algebra X if $\tilde{F}[\alpha]$ is an intuitionistic fuzzy BCK-sub algebra in a BCK-algebra X for all $\alpha \in A$.

Definition 3.5 [20] The extended intersection of two intuitionistic fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over a common universe U is an intuitionistic fuzzy soft set $(\tilde{H}, C)$, where $C = A \cup B$ and for every $\alpha \in C$

$$\tilde{H}[\alpha] = \begin{cases} \tilde{F}[\alpha], \text{if} \quad \alpha \in A - B \\ \tilde{G}[\alpha], \text{if} \quad \alpha \in B - A \\ \tilde{F}[\alpha] \cap \tilde{G}[\alpha], \text{if} \quad \alpha \in A \cap B \end{cases}$$

In this case, we write $(\tilde{F}, A) \cap_e (\tilde{G}, B) = (\tilde{H}, C)$.

Proposition 3.6 [20] Extended intersection of two intuitionistic fuzzy soft BCK-algebra over a BCK-algebra X is also an intuitionistic fuzzy soft BCK-algebra over BCK-algebra X.

Definition 3.7 [20] Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U such that $A \cap B \neq \phi$. Then the restricted intersection of $(\tilde{M}, A)$ and $(\tilde{N}, B)$ is denoted by $(\tilde{F}, A) \cap_r (\tilde{G}, B)$ and is defined as $(\tilde{F}, A) \cap_r (\tilde{G}, B) = (\tilde{H}, C)$, where $C = A \cap B$ and for all $\beta \in C$, $\tilde{H}[\beta] = \tilde{F}[\beta] \cap \tilde{G}[\beta]$.

Proposition 3.8 [20] Restricted intersection of two intuitionistic fuzzy soft BCK-algebras over BCK-algebra X, is also an intuitionistic fuzzy soft BCK-algebra over BCK-algebra X.

Definition 3.9 [20] Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U. The union of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ is defined by $(\tilde{H}, C)$ satisfying the following conditions:

1. $C = A \cup B$
2. for all $\alpha \in C$

$$\tilde{H}[\alpha] = \begin{cases} \tilde{F}[\alpha], \text{if} \quad \alpha \in A - B \\ \tilde{G}[\alpha], \text{if} \quad \alpha \in B - A \\ \tilde{F}[\alpha] \cup \tilde{G}[\alpha], \text{if} \quad \alpha \in A \cap B \end{cases}$$

In this case, we write $(\tilde{F}, A) \cup (\tilde{G}, B) = (\tilde{H}, C)$.

Proposition 3.10 [20] The union of two intuitionistic fuzzy soft BCK-algebra over a BCK - algebra X is also an intuitionistic fuzzy soft BCK-algebra over a BCK-algebra X.

4. Fuzzy BCK- Modules

Definition 4.1 [22] Let X be a BCK-algebra. Then by a left X-module (abbreviated X-module) we mean an abelian group M with an operation $X \times M \longrightarrow M$ with $(x, m) \mapsto xm$ satisfies the following conditions for all $x, y \in X$ and $m, n \in M$:

1. $(x \wedge y)m = x(ym)$

2. $x(m+n) = xm + xn$
3. $0m = 0$

Moreover, if X is bounded and M satisfies $1m = m$, for all $m \in M$, then M is said to be unitary.

Example 4.2 [22] If A is a non-empty set, then $X = \mathcal{P}(A)$, the power set of A, is an X-module with $xm = x \wedge m$, for any $x, m \in X$.

Example 4.3 [22] Let X be a bounded implicative BCK-algebra. Then $(X, +, 0)$ is an X-module, where "+" is defined as $x + y = (x * y) \vee (y * x)$ and $xy = x \wedge y$.

Example 4.4 [22] A subset A of BCK-module M is a BCK-sub module of M if and only if $a - b, xa \in A$, for every $a, b \in A$ and $x \in X$.

Definition 4.5 [22] A fuzzy subset μ of M is said to be a fuzzy BCK-sub module if for all $m, m_1, m_2 \in M$ and $x \in X$, the following axioms hold;

1. $\mu(m_1 + m_2) \geqslant \min\{\mu(m_1), \mu(m_2)\}$
2. $\mu(-m) = \mu(m)$
3. $\mu(xm) \geqslant \mu(m)$.

Example 4.6 [22] Let X = {0,a,b,c} and consider the following table;

*	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

Then $(X,*)$ is a bounded implicative BCK-algebra and so is a BCK-module over itself. let $t_0, t_1, t_2 \in [0,1]$ be such that $t_0 > t_1 > t_2$. Define $\mu: X \to [0,1]$ by $\mu(0) = t_0, \mu(a) = t_1, \mu(b) = \mu(c) = t_2$. Then μ is a fuzzy BCK- sub module of X.

5. Intuitionistic Fuzzy Soft BCK − Module

Here, we propose the following definition of intuitionistic fuzzy soft BCK-submodule and related theorems.

Definition 5.1 Let X be a BCK-algebra and M is a BCK -module, then an intuitionistic fuzzy soft subset $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$ in M is said to be an intuitionistic fuzzy soft BCK-submodule of M if for all $m, m_1, m_2 \in M$ and $x \in X$, the following conditions satisfy;

1. $\mu_{\tilde{F}[\alpha]}(m_1 + m_2) \geqslant \min\{\mu_{\tilde{F}[\alpha]}(m_1), \mu_{\tilde{F}(\alpha)}(m_2)\}$
2. $\mu_{\tilde{F}[\alpha]}(-m) = \mu_{\tilde{F}[\alpha]}(m)$
3. $\mu_{\tilde{F}[\alpha]}(xm) \geqslant \mu_{\tilde{F}[\alpha]}(m)$
4. $\nu_{\tilde{F}[\alpha]}(m_1 + m_2) \leq \max\{\nu_{\tilde{F}[\alpha]}(m_1), \nu_{\tilde{F}(\alpha)}(m_2)\}$
5. $\nu_{\tilde{F}[\alpha]}(-m) = \nu_{\tilde{F}[\alpha]}(m)$
6. $\nu_{\tilde{F}[\alpha]}(xm) \leq \nu_{\tilde{F}[\alpha]}(m)$

Theorem 5.2 An intuitionistic fuzzy soft subset $\tilde{F}[\alpha]$ of M is an intuitionistic fuzzy soft BCK-sub module of M if and only if

1. $\mu_{\tilde{F}[\alpha]}(m_1 - m_2) \geqslant \min\{\mu_{\tilde{F}[\alpha]}(m_1), \mu_{\tilde{F}(\alpha)}(m_2)\}$; $\nu_{\tilde{F}[\alpha]}(m_1 - m_2) \leq \max\{\nu_{\tilde{F}[\alpha]}(m_1), \nu_{\tilde{F}(\alpha)}(m_2)\}$
2. $\mu_{\tilde{F}[\alpha]}(xm) \geqslant \mu_{\tilde{F}[\alpha]}(m)$; $\nu_{\tilde{F}[\alpha]}(xm) \leq \nu_{\tilde{F}[\alpha]}(m)$.

Proof: Let $\tilde{F}[\alpha]$ be an intuitionistic fuzzy soft BCK- sub module of M, then

1. $\mu_{\tilde{F}[\alpha]}(m_1 - m_2) = \mu_{\tilde{F}[\alpha]}\{(m_1 + (-m_2)\}$

$\geq \min\{\mu_{\tilde{F}[\alpha]}(m_1), \mu_{\tilde{F}[\alpha]}(-m_2)\}$

$= \min\{\mu_{\tilde{F}[\alpha]}(m_1), \mu_{\tilde{F}[\alpha]}(m_2)\}$

and

$$\nu_{\tilde{F}[\alpha]}(m_1 - m_2) = \nu_{\tilde{F}[\alpha]}\{(m_1 + (-m_2)\}$$

$\leq \max\{\nu_{\tilde{F}[\alpha]}(m_1), \nu_{\tilde{F}[\alpha]}(-m_2)\}$

$= \max\{\nu_{\tilde{F}[\alpha]}(m_1), \nu_{\tilde{F}[\alpha]}(m_2)\}$

2. $\mu_{\tilde{F}[\alpha]}(xm) \geqslant \mu_{\tilde{F}[\alpha]}(m)$ and $\nu_{\tilde{F}[\alpha]}(xm) \leq \nu_{\tilde{F}[\alpha]}(m)$.

Conversely, suppose $\tilde{F}[\alpha]$ holds conditions 1 and 2, then by definition we have

$\mu_{\tilde{F}[\alpha]}(-m) \geqslant \mu_{\tilde{F}[\alpha]}(m)$ and $\mu_{\tilde{F}[\alpha]}(m) = \mu_{\tilde{F}[\alpha]}\{-(-m)\} \geq \mu_{\tilde{F}[\alpha]}(-m)$. Thus $\mu_{\tilde{F}[\alpha]}(m) = \mu_{\tilde{F}[\alpha]}(-m)$. Similarly, $\nu_{\tilde{F}[\alpha]}(m) = \nu_{\tilde{F}[\alpha]}(-m)$

Also we have

$\mu_{\tilde{F}[\alpha]}(m_1 + m_2) = \mu_{\tilde{F}[\alpha]}\{m_1 - (-m_2)\}$

$\geq \min\{\mu_{\tilde{F}[\alpha]}(m_1), \mu_{\tilde{F}[\alpha]}(-m_2)\}$

$= \min\{\mu_{\tilde{F}[\alpha]}(m_1), \mu_{\tilde{F}[\alpha]}(m_2)\}$.

similarly, $\nu_{\tilde{F}[\alpha]}(m_1 + m_2) \leq \max\{\nu_{\tilde{F}[\alpha]}(m_1), \nu_{\tilde{F}(\alpha)}(m_2)\}$

Thus $\tilde{F}[\alpha]$ is an intuitionistic fuzzy soft BCK-sub module of M.

Theorem 5.3 Let $\tilde{F}[\alpha] \in \mathcal{F}(M)$. Then $\tilde{F}[\alpha]$ is an intuitionistic fuzzy soft BCK-sub module of M if and only if

1. $\mu_{\tilde{F}[\alpha]}(0) \geqslant \mu_{\tilde{F}[\alpha]}(m)$ and $\nu_{\tilde{F}[\alpha]}(0) \leq \nu_{\tilde{F}[\alpha]}(m)$
2. $\mu_{\tilde{F}[\alpha]}(xm_1 - ym_2) \geqslant \min\{\mu_{\tilde{F}[\alpha]}(m_1), \mu_{\tilde{F}[\alpha]}(m_2)\}$ and

$\nu_{\tilde{F}[\alpha]}(xm_1 - ym_2) \leq \max\{\nu_{\tilde{F}[\alpha]}(m_1), \mu_{\tilde{F}[\alpha]}(m_2)\}$

Proof. ($\Rightarrow$) it follows from theorem 5.2 and that $0m = 0$, for all $m \in M$.

($\Leftarrow$) we have

$\mu_{\tilde{F}}(xm) = \mu_{\tilde{F}}(xm - y0)$

$\geq \min\{\mu_{\tilde{F}}(xm), \mu_{\tilde{F}}(0)\}$

$= \mu_{\tilde{F}}(m)$

and

$\mu_{\tilde{F}}(m - n) = \mu_{\tilde{F}}(1.m - 1.n)$

$\geq \min\{\mu_{\tilde{F}}(m), \mu_{\tilde{F}}(n)\}$

Similarly $\nu_{\tilde{F}[\alpha]}(xm) \leq \nu_{\tilde{F}[\alpha]}(m)$ and $\nu_{\tilde{F}}(m - n) \leq \max\{\nu_{\tilde{F}}(m), \nu_{\tilde{F}}(n)\}$.

That proves F is an intuitionistic fuzzy soft BCK-sub module of M.

Theorem 5.4: Let $\tilde{F}[\alpha] \in \mathcal{F}(M)$. Then F is an intuitionistic fuzzy soft BCK - sub module if and only if for all $t \in [0,1]$, $\tilde{F}[\alpha]_t \neq \phi$ is an BCK - sub module of M.

Proof: Here $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$ ($\Rightarrow$) Let $\tilde{F}[\alpha]_t \neq \phi$, $t \in [0,1]$ and $m, n \in \tilde{F}[\alpha]_t$; then $\mu_{\tilde{F}[\alpha]}(m), \mu_{\tilde{F}[\alpha]}(n) \geqslant t$ and $\nu_{\tilde{F}[\alpha]}(m), \nu_{\tilde{F}[\alpha]}(n) \leq t$, since $\tilde{F}[\alpha]$ is an intuitionistic fuzzy soft BCK-sub module $\tilde{F}[\alpha] \geq \min\{\mu_{\tilde{F}[\alpha]}(m), \mu_{\tilde{F}[\alpha]}(n)\} \geq t$ and $\tilde{F}[\alpha] \leq \max\{\nu_{\tilde{F}[\alpha]}(m), \nu_{\tilde{F}[\alpha]}(n)\} \leq t$. So $m - n \in \mu_{\tilde{F}[\alpha]_t}$ and $m - n \in \nu_{\tilde{F}[\alpha]_t}$. This shows that $\tilde{F}[\alpha]_t$ is a subgroup of M. Now let $m \in \tilde{F}[\alpha]_t$ and $x \in X$, then $\tilde{F}[\alpha](xm) \geq \mu_{\tilde{F}[\alpha]}(m) \geq t$

and $\tilde{F}[\alpha](xm) \leq \nu_{\tilde{F}[\alpha]}(m) \leq t$, i.e. $xm \in \tilde{F}[\alpha]_t$. Therefore $\tilde{F}[\alpha]_t$ is BCK-sub module of M.

(⇐) Let $t \in (\tilde{F}[\alpha](m), \tilde{F}[\alpha](n))$, for $m, n \in M$. then $m, n \in \tilde{F}[\alpha]_t$ and so $m - n \in \tilde{F}[\alpha]_t$ which means that $\tilde{F}[\alpha](m-n) \geq t = \min\{\mu_{\tilde{F}[\alpha]_t}(m), \mu_{\tilde{F}[\alpha]_t}(n)\}$ and $\tilde{F}[\alpha](m-n) \leq t = \max\{\mu_{\tilde{F}[\alpha]_t}(m), \mu_{\tilde{F}[\alpha]_t}(n)\}$. Now let $s = \tilde{F}[\alpha](m)$. Then $m \in \mu_{\tilde{F}[\alpha]_s}$ and so $xm \in \mu_{\tilde{F}[\alpha]_s}$ similarly $xm \in \nu_{\tilde{F}[\alpha]_s}$ which means that $\mu_{\tilde{F}[\alpha]}(xm) \geq s = \tilde{F}(m)$.

Therefore F is an intuitionistic fuzzy soft BCK-sub module of M.

Definition 5.5 Let $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$ be an intuitionistic fuzzy soft set in BCK-sub module M and let $\beta, \gamma \in [0,1]$ such that $0 \leq \beta + \gamma \leq 1$. then the set $\tilde{F}[\alpha]_{(\beta,\gamma)} = \{m \in M | \mu_{\tilde{F}[\alpha]} \geq \beta, \nu_{\tilde{F}[\alpha]} \leq \gamma\}$ is called an (β, γ) level set of $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$.

Theorem 5.6 Let $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$ be an intuitionistic fuzzy soft set in M such that $\tilde{F}[\alpha]_((\beta, \gamma)$ is a BCK- sub module of M, for all $(\beta, \gamma) \in [0,1]$ with $0 \leq \beta + \gamma \leq 1$. Then $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$ is an intuitionistic fuzzy soft BCK-sub module of M.

Proof: Let $m, m_1, m_2 \in M$ and $x \in X$ such that $\tilde{F}[\alpha](m_1) = (\beta_1, \gamma_1)$, $\tilde{F}[\alpha](m_2) = (\beta_2, \gamma_2)$ where $0 \leq \beta_i + \gamma_i \leq 1$ for $i = 1,2$. Then $m_1, m_2 \in \tilde{F}[\alpha]_{(\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))}$ and so $m_1 - m_2 \in \tilde{F}[\alpha]_{(\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))}$. Hence $\mu_{\tilde{F}[\alpha]}(m_1 - m_2) \geq \min(\beta_1, \beta_2)$ and $\mu_{\tilde{F}[\alpha]}(m_1 - m_2) \leq \max(\gamma_1, \gamma_2)$. Also if we put $s' = \tilde{F}[\alpha](m)$ and $t' = \tilde{F}[\alpha](m)$ where $0 \leq s' + t' \leq 1$, then $m \in \tilde{F}[\alpha]_{(s',t')}$. Since $\tilde{F}[\alpha]_{(s',t')}$ is a BCK- sub module of M, we have $xm \in \tilde{F}[\alpha]_{(s',t')}$. it follows that $\mu_{\tilde{F}[\alpha]}(xm) \geq s' = \mu_{\tilde{F}[\alpha]}(m)$ and $\nu_{\tilde{F}[\alpha]}(xm) \leq t' = \nu_{\tilde{F}[\alpha]}(m)$. Hence $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$ is an intuitionistic fuzzy soft BCK-module of M.

Theorem 5.7 If $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$ be an intuitionistic fuzzy soft BCK-sub module of M. Then $\Box\, \tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \bar{\mu}_{\tilde{F}[\alpha]}(x))$.

Theorem 5.8 If $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$ be an intuitionistic fuzzy soft BCK-sub module of M. Then $\Diamond\, \tilde{F}[\alpha] = (\bar{\nu}_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$.

Theorem 5.9 Let $\tilde{F}[\alpha] = (\mu_{\tilde{F}[\alpha]}(x), \nu_{\tilde{F}[\alpha]}(x))$ be an intuitionistic fuzzy soft BCK-sub module of M. Then $A = \{m | m \in M, \mu_{\tilde{F}[\alpha]}(m) = 1 \text{ and } \nu_{\tilde{F}[\alpha]}(m) = 0\}$ is a sub module of M.

Proof: Let $m, m_1, m_2 \in M$ and $x \in X$, then by definition 5.1

1. $\mu_{\tilde{F}[\alpha]}(m_1 - m_2) \geq \min\{\mu_{\tilde{F}[\alpha]}(m_1), \mu_{\tilde{F}[\alpha]}(m_2)\} = \min\{1,1\} = 1$, so $m_1 - m_2 \in A$.
2. $\nu_{\tilde{F}[\alpha]}(m_1 - m_2) \leq \max\{\nu_{\tilde{F}[\alpha]}(m_1), \nu_{\tilde{F}[\alpha]}(m_2)\} = \max\{0,0\} = 0$, so $m_1 - m_2 \in A$.
3. $\mu_{\tilde{F}[\alpha]}(xm) \geq \mu_{\tilde{F}[\alpha]}(m) = 1$, $xm \in A$.
4. $\nu_{\tilde{F}[\alpha]}(xm) \leq \nu_{\tilde{F}[\alpha]}(m) = 0$, $xm \in A$.

Hence A is a sub module of M.

6. Concluding Remarks

The present paper summarizes the basic concepts of intuitionistic fuzzy soft sets and intuitionistic fuzzy soft modules in BCK-algebra. We have discussed some algebraic properties of intuitionistic fuzzy soft modules in BCK-structure. This work may be helpful to study the homomorphism of intuitionistic fuzzy soft BCK- modules and its algebraic properties.

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