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Subclass of Negative Coefficient Univalent Functions Involving Raducanu-Orhan Differential Operator Connected with Pascal Distribution Series

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Abstract: - Recent years have shown us how fascinating the univalent function is; many new publications have been written in this field. Currently, operators of normalized analytic functions, differential and integral operators are highly sought after. Numerous researchers have examined and debated a great deal of material for the operators. This work introduces a new subclass $PQ_{q,\delta,\mu}^{n,r}(\theta)$ of the function class for univalent functions defined by the Raducanu-Orhan differential operator connected with Pascal distribution series. Our goal in this work is to further our understanding and make inferences regarding the functions that are a part of these new subclass. Furthermore, the convexity of the subclass, growth and distortion, radius of starlike, extreme points, and integral means of inequalities are obtained. All this research was performed inside an open unit disc.

Keywords: Analytic function, univalent functions, differential operator, subordination, coefficient inequality, starlike and convexity.

1. Introduction

Consider that the class A of univalent functions has the following form:

$$f(\xi) = \xi + \sum_{\upsilon=2}^{\infty} a_\upsilon \xi^\upsilon, \;\; \xi \in U := \{\xi \in C : |\xi < 1|\}. \qquad (1)$$

Which is analytic in the unit disk U, and

$$g(\xi) = \xi + \sum_{\upsilon=2}^{\infty} a_\upsilon \xi^\upsilon, \xi \in U. \qquad (2)$$

Then the convolution of (1) and (2) is represented by

$$(f * g)(\xi) = \xi + \sum_{\upsilon=2}^{\infty} a_\upsilon b_\upsilon \xi^\upsilon, \xi \in U. \qquad (3)$$

The Schwarz function in U, $w(\xi)$ exists if and only if $f(\xi)$ and $g(\xi)$ are analytical. It is our claim that $f(\xi)$ is subordinate to $g(\xi)$; that is, $f(\xi) \prec g(\xi)$. In this case, $w(0) = 0$ and $|w| < 1$ such that $f(\xi) = g(w(\xi))$. As proven, $f(\xi) \prec g(\xi)$ and $f(U) \subset g(U)$ implied by $f(0) = g(0)$.

Ma and Minda [14] used the idea of subordination to create various subclasses of radii of convexity and starlikeness. To achieve this goal, a univalent function $\phi(\xi)$ is taken into A consideration. This function is analytic and defined on U with a positive real portion, such that $\phi'(0) > 0$ and $\phi(0) = 1$.

Let $f(\xi) \in A$, then $f(\xi)$ is convex in U, if $Re\left(1+\left(\frac{\xi f''(\xi)}{f'(\xi)}\right)\right) > 0, \xi \in U$.

Let $f(\xi) \in A$, then $f(\xi)$ is starlike in U, if $Re\left(\left(\frac{\xi f'(\xi)}{f(\xi)}\right)\right) > 0, \xi \in U$.

Starlike and convex mapping are closely related in the definition that follow.

For $f(\xi) \in A$, Raducanu-Orhan [4] introduced the differential operator

$$Q_{\delta,\mu}^{n} f(\xi) = \xi + \sum_{\upsilon=2}^{\infty} [1 + (\upsilon - 1)(\delta - \mu + \upsilon\delta\mu)]^{n} a_{\upsilon} \xi^{\upsilon}, \xi \in U, \quad (4)$$

where $n \in N0 = N \cup 0, N = \{1,2,\dots,\}, \mu, \delta \geq 0, \xi \in U$.

Remark: $Q_{\delta,0}^{n} = D_{\delta}^{n}$ yields the Al-Oboudi differential operator [5] and $Q_{1,0}^{n} = D^{n}$ provide the Salagean differential operator [7].

Recent studies have focused on a subclass of univalent functions associated with distribution series. These include the Borel, Pascal, Binomial, Poisson, Mittag-Lefer-type, geometric, exponential, and generalized distributions as well as a generalized discrete probability distribution. In recent years, various subclass of univalent functions related to Pascal distribution series have been studied by the following authors, B.A. Frasin et al. [2], S. Porwal [9], Anitha Lakshminarayanan et al. [1], G. Murugusundramoorthy [6], R. M. El-Ashwah, W. Y. Kota [10], T. Bulboaca and G. Murugusundaramoorthy [13], and B.A. Frasin et al. [3]. By examining the subclasses, researchers hope to gain a deeper understanding of the structure and behaviour of analytic functions, hence advancing their knowledge of complex analysis and its applications, which provides an extensive investigation of this area of study.

2. The Subclass $TQ_{q,\delta,\mu}^{n,r}(\theta)$

The probabilities $(1-q)r$, $q2r(r+1)(1-q)r\ 2!$, $qr(1-q)r\ 1!$, $q3r(r+1)(r+2)(1-q)r\ 3!$, … correspond to a variable x with values of 0, 1, 2, and 3, respectively, where q, and r are called the parameters, and thus

$$P(X = x) = \binom{x+r-1}{r-1} q^{x}(1-q)^{r}, x \in \{0,1,2,3,\dots\}. \quad (5)$$

According to S. M. El-Deep et al. [12], the power series of equation (6) is examined, with its coefficients representing probabilities of the Pascal distribution, that is

$$P_{q}^{r}(\xi) = \xi + \sum_{\upsilon=2}^{\infty} \binom{\upsilon+r-2}{r-1} q^{\upsilon-1}(1-q)^{r}\xi^{\upsilon}, \qquad \xi \in U, r \geq 1, 0 \leq q \leq 1. \quad (6)$$

Further let

$$T := \left\{ f(\xi) \in A; f(\xi) = \xi - \sum_{\upsilon=2}^{\infty} a_{\upsilon}\xi^{\upsilon} \right\}$$

be the subclass of A introduced by Silverman [11] in 1975, and the linear operator $D_{q}^{r}: A \to A$ is defined by

$$D_q^r(f(\xi)) = P_q^r * f(\xi) = \xi - \sum_{\upsilon=2}^{\infty} \binom{\upsilon + r - 2}{r - 1} q^{\upsilon-1}(1-q)^r a_\upsilon \xi^\upsilon, \ \xi \in U. \qquad (7)$$

By using the convolution (Hadamard product) of two equations (4) and (7), the linear operator $PD_{q,\delta,\mu}^{n,r}: A \to A$ is defined by

$$PD_{q,\delta,\mu}^{n,r} f(\xi) = \xi - \sum_{\upsilon=2}^{\infty} (C_\upsilon)\, a_\upsilon \xi^\upsilon, \qquad (8)$$

Where $C_\upsilon = [1 + (\upsilon - 1)(\delta - \mu + \upsilon\delta\mu)]^n \binom{\upsilon + r - 2}{r - 1} q^{\upsilon-1}(1-q)^r$

Now, the new subclass is defined in the following definition:

Definition 2.1

Let $TQ_{q,\delta,\mu}^{n,r}$ represents a class of $f(\xi) \in A$. Then

$$Re\left(1 + \frac{1}{b}\left(\frac{\xi\left(PD_{q,\delta,\mu}^{n,r} f(\xi)\right)'}{PD_{q,\delta,\mu}^{n,r} f(\xi)} - 1\right)\right) > \theta, \qquad (9)$$

where $r \geq 1,\ 0 \leq q \leq 1, \mu, \delta \geq 0,\ n \in N_0, 0 \leq \theta < 1, b \in C - \{0\}, \xi \in U$.

3. Main Results

Theorem 3.1 (Coefficient Inequality)

Let $f(\xi) \in TQ_{q,\delta,\mu}^{n,r}$. Then

$$\sum_{\upsilon=2}^{\infty} (\phi_\upsilon) C_\upsilon |a_\upsilon| \leq (1-\theta)|b|, \qquad (10)$$

where $\phi_\upsilon = |b + \upsilon - 1 - \theta b|$, $C_\upsilon = [1 + (\upsilon - 1)(\delta - \mu + \upsilon\delta\mu)]^n \binom{\upsilon + r - 2}{r - 1} q^{\upsilon-1}(1-q)^r$.

Proof. Let

$$F(\xi) = 1 + \frac{1}{b}\left(\frac{\xi\left(PD_{q,\delta,\mu}^{n,r} f(\xi)\right)'}{PD_{q,\delta,\mu}^{n,r} f(\xi)} - 1\right) - \theta$$

$$= 1 + \left(\frac{\xi\left(PD_{q,\delta,\mu}^{n,r} f(\xi)\right)' - bPD_{q,\delta,\mu}^{n,r} f(\xi) - \theta bPD_{q,\delta,\mu}^{n,r} f(\xi)}{bPD_{q,\delta,\mu}^{n,r} f(\xi)}\right)$$

By the condition of the class, $F(\xi) \prec \frac{1+\xi}{1-\xi}$.

A Schwarz function $w(\xi)$ exists, and $w(0) = 0$ as $F(\xi) \prec \frac{1+w(\xi)}{1-w(\xi)}$, where $|w| < 1$.

Therefore $w(\xi) \prec \left|\frac{F(\xi)-1}{F(\xi)+1}\right|$.

Then

$$\left|\frac{F(\xi)-1}{F(\xi)+1}\right| = \left|\frac{\xi\left(PD_{q,\delta,\mu}^{n,r} f(\xi)\right)' - (1+\theta b)PD_{q,\delta,\mu}^{n,r} f(\xi)}{\xi\left(PD_{q,\delta,\mu}^{n,r} f(\xi)\right)' - (1+\theta b - 2b)PD_{q,\delta,\mu}^{n,r} f(\xi)}\right|$$

$$= \left|\frac{\xi - \sum_{\upsilon=2}^{\infty} \upsilon C_\upsilon a_\upsilon \xi^\upsilon - (1+b\theta)\xi + \sum_{\upsilon=2}^{\infty}(1+b\theta)C_\upsilon a_\upsilon \xi^\upsilon}{\xi - \sum_{\upsilon=2}^{\infty} \upsilon C_\upsilon a_\upsilon \xi^\upsilon - (1+b\theta-2b)\xi + \sum_{\upsilon=2}^{\infty}(1+b\theta-2b)C_\upsilon a_\upsilon \xi^\upsilon}\right|$$

$$\leq \frac{\theta|b| + \sum_{\upsilon=2}^{\infty}|(1+b\theta-\upsilon)||C_\upsilon||a_\upsilon||\xi^{\upsilon-1}|}{(2-\theta)|b| - \sum_{\upsilon=2}^{\infty}|(1+b\theta-\upsilon-2b\upsilon)||C_\upsilon||a_\upsilon||\xi^{\upsilon-1}|}$$

Which is bounded by 1, if

$$\theta|b| + \sum_{\upsilon=2}^{\infty}|(1+b\theta-\upsilon)||C_\upsilon||a_\upsilon||\xi^{\upsilon-1}| \leq (2-\theta)|b| - \sum_{\upsilon=2}^{\infty}|(1+b\theta-\upsilon-2b)||C_\upsilon||a_\upsilon||\xi^{\upsilon-1}|$$

Which implies that $\sum_{\upsilon=2}^{\infty}|(b+\upsilon-1-\theta b)|C_\upsilon|a_\upsilon| \leq (1-\theta)|b|$.

Hence equation (10) holds.

Corollary 3.2

Let $f(\xi) \in TQ_{q,\delta,\mu}^{n,r}$. Then we have

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$$a_\upsilon \le \frac{(1-\theta)|b|}{(\phi_\upsilon)C_\upsilon},$$
and
$$f(\xi) = \xi + \frac{(1-\theta)|b|}{(\phi_\upsilon)C_\upsilon}\xi^\upsilon, \upsilon = 2,3,4, \ldots \quad (11)$$
equals itself.

The function $f(\xi)$ is defined as the subclass of $\overline{TQ_{q,\delta,\mu}^{n,r}} \subset TQ_{q,\delta,\mu}^{n,r}$, then the extreme point of the subclass $\overline{TQ_{q,\delta,\mu}^{n,r}}$ has been determined as follows.

Theorem 3.3 (Extreme points)
Let
$$f_1(\xi) = \xi, f_\upsilon(\xi) = \xi + \sum_{\upsilon=2}^{\infty}\eta_\upsilon \frac{(1-\theta)|b|}{(\phi_\upsilon)C_\upsilon}\xi^\upsilon, \upsilon \ge 2.$$
Then $f \in \overline{TQ_{q,\delta,\mu}^{n,r}}$ strictly if $f(\xi) = \sum_{\upsilon=1}^{\infty}\eta_\upsilon f_\upsilon(\xi)$, where $\eta_\upsilon > 0$ and $\sum_{\upsilon=2}^{\infty}\eta_\upsilon = 1$.
Proof. Let
$$f(\xi) = \sum_{\upsilon=1}^{\infty}\eta_\upsilon f_\upsilon(\xi)$$
$$= \xi + \sum_{\upsilon=2}^{\infty}\eta_\upsilon \frac{(1-\theta)|b|}{(\phi_\upsilon)C_\upsilon}\xi^\upsilon$$
$$= \sum_{\upsilon=2}^{\infty}\eta_\upsilon \frac{(1-\theta)|b|}{(\phi_\upsilon)C_\upsilon}(\phi_\upsilon)C_\upsilon$$
$$= (1-\theta)|b|\sum_{\upsilon=2}^{\infty}\eta_\upsilon$$
$$= (1-\theta)|b|(1-\eta_1)$$
$$< (1-\theta)|b|$$
Which shows that, $f \in \overline{TQ_{q,\delta,\mu}^{n,r}}$.
Conversely, suppose that $f \in \overline{TQ_{q,\delta,\mu}^{n,r}}$. Since $a_\upsilon \le \frac{(1-\theta)|b|}{(\phi_\upsilon)C_\upsilon}, \upsilon = 2,3, \ldots$
Let
$$\eta_\upsilon \le \frac{(\phi_\upsilon)C_\upsilon}{(1-\theta)|b|}, \eta_1 = 1 - \sum_{\upsilon=2}^{\infty}\eta_\upsilon.$$
Then we obtain
$$f(\xi) = \sum_{\upsilon=1}^{\infty}\eta_\upsilon f_\upsilon(\xi).$$

Definition 3.4 (Little wood subordination theorem [8].)
Considering that f and g in U are analytic, and that $f(\xi) \prec g(\xi)$, then
$$\int_0^{2\pi}|f(\xi)|^\mu d\theta \le \int_0^{2\pi}|g(\xi)|^\mu d\theta, \mu > 0, \text{ and } \xi = re^{i\theta}, 0 < r < 1.$$
Theorem 3.4 (Integral means of inequality)
Let $f(\xi) \in TQ_{q,\delta,\mu}^{n,r}$ and suppose that $g(\xi) = \xi + \sum_{\upsilon=2}^{\infty}\frac{(1-\theta)|b|\epsilon_\upsilon}{(\phi_\upsilon)C_\upsilon}\xi^\upsilon, \quad \upsilon = 2,3, \ldots,$ $|\epsilon_\upsilon| = 1$. If $w(\xi)$ is real, it is given by $(w(\xi))^{\upsilon-1} = \frac{(\phi_\upsilon)C_\upsilon}{(1-\theta)|b|\epsilon_\upsilon}\sum_{\upsilon=2}^{\infty}a_\upsilon\xi^{\upsilon-1}$, then $\int_0^{2\pi}|f(\xi)|^\mu d\theta \le \int_0^{2\pi}|g(\xi)|^\mu d\theta$, for $\mu > 0$, and $\xi = re^{i\theta}, 0 < r < 1$.
Proof. We need to demonstrate that to finish the theorem
$$\int_0^{2\pi}\left|1 + \sum_{\upsilon=2}^{\infty}a_\upsilon\xi^{\upsilon-1}\right|^\mu d\theta \le \int_0^{2\pi}\left|1 + \sum_{\upsilon=2}^{\infty}\frac{(1-\theta)|b|\epsilon_\upsilon}{(\phi_\upsilon)C_\upsilon}\xi^{\upsilon-1}\right|^\mu d\theta$$

The Little wood subordination theorem can be used to demonstrate that

$$1+\sum_{\upsilon=2}^{\infty} a_\upsilon \xi^{\upsilon-1} \prec 1+\sum_{\upsilon=2}^{\infty} \frac{(1-\theta)|b|\epsilon_\upsilon}{(\phi_\upsilon)C_\upsilon} \xi^{\upsilon-1}.$$

Let

$$1+\sum_{\upsilon=2}^{\infty} a_\upsilon \xi^{\upsilon-1} \prec 1+\sum_{\upsilon=2}^{\infty} \frac{(1-\theta)|b|\epsilon_\upsilon}{(\phi_\upsilon)C_\upsilon} (w(\xi))^{\upsilon-1}.$$

Therefore

$$(w(\xi))^{\upsilon-1}=\frac{(\phi_\upsilon)C_\upsilon}{(1-\theta)|b|\epsilon_\upsilon}\sum_{\upsilon=2}^{\infty} a_\upsilon \xi^{\upsilon-1}.$$

Hence $w(0)=0$.

Furthermore, if $f(\xi) \in A$ satisfy $(\phi_\upsilon)C_\upsilon \le (1-\theta)|b|$.

$$|(w(\xi))|^{\upsilon-1}=\left|\frac{(\phi_\upsilon)C_\upsilon}{(1-\theta)|b|\epsilon_\upsilon}\right|\sum_{\upsilon=2}^{\infty}|a_\upsilon||\xi^{\upsilon-1}| \le |\xi| \le 1.$$

Theorem 3.5 (Convex of order σ)

Let $f(\xi) \in TQ_{q,\delta,\mu}^{n,r}$. Then f is convex of order σ in $|\xi|<R_3$, since

$$R_3 := inf\left(\frac{(1-\sigma)(\phi_\upsilon)C_\upsilon}{\upsilon(\upsilon-\sigma)(1-\theta)|b|}\right)^{\frac{1}{\upsilon-1}}, (\upsilon \ge 2). \tag{12}$$

Proof. Assuming that $|\xi|<R_3$ and the inequality (12) is valid, it is demonstrated that

$$\left|\frac{\xi f''(\xi)}{f'(\xi)}\right| \le 1-\sigma. \tag{13}$$

It is adequate to show that

$$|\xi| \le \left(\frac{(1-\sigma)(\phi_\upsilon)C_\upsilon}{\upsilon(\upsilon-\sigma)(1-\theta)|b|}\right)^{\frac{1}{\upsilon-1}}, (\upsilon \ge 2).$$

From (13), we obtain

$$\left|\frac{\sum_{\upsilon=2}^{\infty} \upsilon(\upsilon-1)a_\upsilon \xi^{\upsilon-1}}{1-\sum_{\upsilon=2}^{\infty} \upsilon a_\upsilon \xi^{\upsilon-1}}\right| \le 1-\sigma.$$

Hence,

$$|\xi| \le \left(\frac{(1-\sigma)(\phi_\upsilon)C_\upsilon}{\upsilon(\upsilon-\sigma)(1-\theta)|b|}\right)^{\frac{1}{\upsilon-1}}, (\upsilon \ge 2).$$

Theorem 3.6 (Starlike of order σ)

Let $f(\xi) \in TQ_{q,\delta,\mu}^{n,r}$. Then f is starlike of order σ in $|\xi|<R_2$, since

$$R_2 := inf\left(\frac{(1-\sigma)(\phi_\upsilon)C_\upsilon}{(\upsilon-\sigma)(1-\theta)|b|}\right)^{\frac{1}{\upsilon-1}}, (\upsilon \ge 2). \tag{14}$$

Proof. Assuming that $|\xi|<R_2$ and the inequality (14) is valid, it is demonstrated that

$$\left|\frac{\xi f'(\xi)}{f(\xi)} - 1\right| \leq 1 - \sigma. \quad (15)$$

It is adequate to show that

$$|\xi| \leq \left(\frac{(1-\sigma)(\phi_\upsilon)C_\upsilon}{(\upsilon-\sigma)(1-\theta)|b|}\right)^{\frac{1}{\upsilon-1}}, (\upsilon \geq 2).$$

From (15), we obtain

$$\left|\frac{\xi - \sum_{\upsilon=2}^{\infty} \upsilon a_\upsilon \xi^\upsilon}{\xi - \sum_{\upsilon=2}^{\infty} a_\upsilon \xi^\upsilon} - 1\right| \leq 1 - \sigma.$$

Hence,

$$|\xi| \leq \left(\frac{(1-\sigma)(\phi_\upsilon)C_\upsilon}{(\upsilon-\sigma)(1-\theta)|b|}\right)^{\frac{1}{\upsilon-1}}, (\upsilon \geq 2).$$

Theorem 3.7 (Close to Convex of order σ)

Let $f(\xi) \in TQ_{q,\delta,\mu}^{n,r}$. Then f is close to convex of order σ in $|\xi| < R_1$, since

$$R_1 := inf\left(\frac{(1-\sigma)(\phi_\upsilon)C_\upsilon}{\upsilon(1-\theta)|b|}\right)^{\frac{1}{\upsilon-1}}, (\upsilon \geq 2). \quad (16)$$

Proof. Assuming that $|\xi| < R_1$ and the inequality (16) is valid, it is demonstrated that

$$|f'(\xi) - 1| \leq 1 - \sigma. \quad (17)$$

It is adequate to show that

$$|\xi| \leq \left(\frac{(1-\sigma)(\phi_\upsilon)C_\upsilon}{\upsilon(1-\theta)|b|}\right)^{\frac{1}{\upsilon-1}}, (\upsilon \geq 2).$$

From (17), we obtain

$$\left|1 - \sum_{\upsilon=2}^{\infty} \upsilon a_\upsilon \xi^{\upsilon-1}\right| \leq 1 - \sigma.$$

Hence,

$$|\xi| \leq \left(\frac{(1-\sigma)(\phi_\upsilon)C_\upsilon}{\upsilon(1-\theta)|b|}\right)^{\frac{1}{\upsilon-1}}, (\upsilon \geq 2).$$

Theorem 3.8 (Growth theorem)

Let $f(\xi) \in TQ_{q,\delta,\mu}^{n,r}$. Then for $|\xi| = r^*$,

$$r^* - \frac{(1-\theta)|b|}{(1+b-\theta b)C_2} {r^*}^2 \leq |f(\xi)| \leq r^* + \frac{(1-\theta)|b|}{(1+b-\theta b)C_2} {r^*}^2 \quad (18)$$

Where $C_2 = \left(1 + (\delta - \mu + 2\delta\mu)\right)^n rq(1-q)^r$.

Proof. Since

$$a_\upsilon \leq \frac{(1-\theta)|b|}{(\phi_\upsilon)C_\upsilon}.$$

And

$$f(\xi) = \xi - \sum_{\upsilon=2}^{\infty} a_{\upsilon} \xi^{\upsilon}$$

Then

$$|f(\xi)| = r^{*} + \sum_{\upsilon=2}^{\infty} a_{\upsilon} (r^{*})^{\upsilon}$$

$$\Rightarrow |f(\xi)| = r^{*} + \sum_{\upsilon=2}^{\infty} \frac{(1-\theta)|b|}{(\phi_{\upsilon})C_{\upsilon}} (r^{*})^{\upsilon}$$

$$\Rightarrow |f(\xi)| \leq r^{*} + \frac{(1-\theta)|b|}{(1+b-\theta b)C_2} {r^{*}}^{2}.$$

Similarly

$$|f(\xi)| \geq r^{*} - \frac{(1-\theta)|b|}{(1+b-\theta b)C_2} {r^{*}}^{2}.$$

Theorem 3.9 (Distortion theorem)

Let $f(\xi) \in TQ_{q,\delta,\mu}^{n,r}$. Then for $|\xi| = r^{*}$,

$$1 - \frac{2(1-\theta)|b|}{(1+b-\theta b)C_2} r^{*} \leq |f'(\xi)| \leq 1 + \frac{2(1-\theta)|b|}{(1+b-\theta b)C_2} r^{*} \tag{19}$$

Where $C_2 = \left(1 + (\delta - \mu + 2\delta\mu)\right)^{n} rq(1-q)^{r}$.

Proof. Since

$$a_{\upsilon} \leq \frac{(1-\theta)|b|}{(\phi_{\upsilon})C_{\upsilon}}.$$

And

$$f(\xi) = \xi - \sum_{\upsilon=2}^{\infty} a_{\upsilon} \xi^{\upsilon}$$

Then

$$|f'(\xi)| = \left| 1 + \sum_{\upsilon=2}^{\infty} \upsilon a_{\upsilon} (\xi)^{\upsilon-1} \right|$$

$$\Rightarrow |f'(\xi)| \leq 1 + \frac{2(1-\theta)|b|}{(1+b-\theta b)C_2} r^{*}$$

Similarly

$$|f'(\xi)| \geq 1 - \frac{2(1-\theta)|b|}{(1+b-\theta b)C_2} r^{*}.$$

4. Application- stealth combat aircraft

Aerodynamics relies heavily on univalent functions, which are single-valued, conformal mappings in complex analysis. This is especially true when designing stealth combat aircraft. To reduce radar cross-section (RCS) and improve aerodynamic efficiency, univalent functions are useful in modelling and optimizing air foil forms.

Designers can study and enhance air ow properties by translating complicated shapes into simpler forms. These functions facilitate the study of ow patterns around an aircraft, allowing engineers to predict how changes in shape affect drag and lift, which is crucial for stealth performance. These features make it easier to analyse the patterns surrounding an aircraft, which enables engineers to forecast how shape changes will impact drag and lift an essential aspect of stealth performance [15]-[19]. The principles of univalent functions can be applied to develop surfaces that scatter radar waves in ways that reduce detectability. By controlling the geometry of the aircraft's surfaces, engineers can create shapes that detect radar signals away from the source. Develop surfaces that scatter radar waves in a way that decreases detectability using the concepts of univalent functions. Engineers can build shapes that detect radar signals away from the source of the signal by manipulating the aircraft's surface geometry.

5. Conclusion

We have examined the coefficient challenges related to the newly created subclass of univalent functions in U, as stated in Definition (2.1), in this work. Additionally, the radius of starlikeness, extreme points, development and distortion, convexity of the subclass, and integral means of inequalities are found. A greater comprehension of the composition and behaviour of analytic functions is offered by the examined subclass. Promising avenues for further research are indicated by the examination of Hankel determinants of orders between two and three on the before described subclass, as well as by the investigation and estimations associated with the Fekete-Szegö functional problem.

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