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# Solution of the Forward Problem of ECG using Radial basis Function Network

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**Abstract**

ECG inverse problem is a mathematical formulation of cardiac electrical activity on the heart surface that can be solved to determine the electric potential on the heart surface. In this paper, a forward problem to ECG will be formulated using a mathematical formulation similar to the ECG forward problem. The forward problem will be solved using radial basis collocation techniques by taking different discretization points of N=9 and N=25 points to predict the solution u(x,t) to the forward problem.

**Keywords:** radial basis function, inverse problem, forward problem, electric potential, body surface.

## 1. Introduction

Universally Cardiovascular diseases (CVDs) remain the basic cause of death among people [11]. In 2019, around 18 million people died due to CVDs [4]. Proper treatment at the early stage is the only way of preventing deaths due to CVDs [12]. ECG (Electrocardiogram) who's another name is EKG is used mostly to measure the electrical activity on the heart surface to detect heart disease. From the measured electric potential physicians identify anomalies on the heart surface such as coronary artery disease, ischemia, arrhythmia, atrial fibrillation etc. ECG lacks in sensitivity and specificity and its resolution is limited to a great extent.

Lots of work has been done to recreate the electric potential from the body surface potential on the heart surface using different methods. This process of obtaining the electric potential is called forward/inverse problem in ECG. The forward/inverse problem is a mathematical model which has the form of Laplace equation with Neumann boundary conditions. To obtain the electric potential on the heart surface the forward problem is first solved and then the inverse problem.

Numerical methods are used to address the forward/inverse problem. Boundary Element Methods (BEM) [8], finite element methods (FEM) [8], and finite volume methods (FVM) [3] are some of the most commonly used numerical approaches. Numerical approaches necessitate domain meshing, which is difficult in irregular geometry and results in weakly solving DE's (Differential Equations) over discretization. Due to the uneven geometry of the heart, typical methods for solving the forward problem of the ECG do not yield promising results. The generated solutions are discrete or have limited differentiability, which is a shortcoming of this method. In [11] the authors solved the inverse problem using the domain decomposition method. It is put to the test using fictitious data. The authors find that even with a high amount of noise, the wave front is successfully captured. In [5] the authors looked at the use of the generalised minimal residual (GMRes) method, which has been shown to be effective in tackling ill-posed issues in image processing. The performance of GMRes in imaging normal and aberrant electrical activity in calves is evaluated by the authors.

As the traditional method requires mesh generation which is complex for irregular domain geometry, many researchers are using meshless method to find the solution to the PDE. Radial basis function (RBF) is a meshless method which is used by many researchers for solving PDE, control problems etc. and it gives promising result as compared to the traditional method [7,9 9, 11, 13, 14, 15].

In this paper, a forward problem to ECG will be formulated by a similar mathematical formulation. RBF method will be used to evaluate the solution to the forward problem. In the first phase, collocation techniques will be utilized that converts the equation and boundary conditions into a nonlinear system of equations. By taking different values of discretization points equations will be solved using Matlab.

## 2. Radial Basis Function Approximation

RBF is a feed forward neural network model with an input, hidden and output layer. The network collects the user data through the input nodes and then pass on for processing to the inner layer. The network model is presented in Figure 1, with $x_1, x_2, \ldots .. x_n$ representing input nodes and $c_1, c_2, \ldots \ldots c_n$ representing hidden nodes respectively. The input information is collected and gives an output $\emptyset_j(x_i) = \emptyset(\|x_i - c_j\|)$, where $\emptyset_j(x_i)$ is the result of the activation function used.

**Table 1**. Radial Basis Function.

| | Basis Function |
|---|---|
| Gaussian | $\emptyset(r) = e^{-\pi^2}$ |
| Multiquadric | $\emptyset(r) = \sqrt{s^2 + r^2}$ |
| Inverse multiquadric | $\emptyset(r) = 1/\sqrt{s^2 + r^2}$ |
| Thin Plate Splines | $\emptyset(r) = r^2 \log(r)$ |

$Y = f(x_i) = \sum_{j=1}^{k} w_j \emptyset_j(x_i)$ is the output of the RBF. The weight associated with the $j^{th}$ hidden node and the output node is $w_j, j = 1,2, \ldots . k$. The shape parameter is denoted by s, while the distance between $x_i$ and the radial centre $c_j$ is denoted by $r = \|x_i - c_j\|$. With weights related between the hidden and output layers, the RBF network's layers are fully connected. Hence the output can be written in the matrix form as,

$$A^T W = f \qquad (1)$$

Where, $A = \begin{bmatrix} \emptyset_1(x_1) & \emptyset_1(x_2) & \ldots & \emptyset_1(x_n) \\ \emptyset_2(x_1) & \emptyset_2(x_2) & \ldots & \emptyset_2(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \emptyset_n(x_1) & \emptyset_n(x_2) & \ldots & \emptyset_n(x_n) \end{bmatrix}, W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}, f = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$

Equation (1) is a nonlinear system of equation which will be solve by using MATLAB. Equation (1) is ill posed where the condition number can be determined by using $C(A) = \|A\|\|A^{-1}\|$.

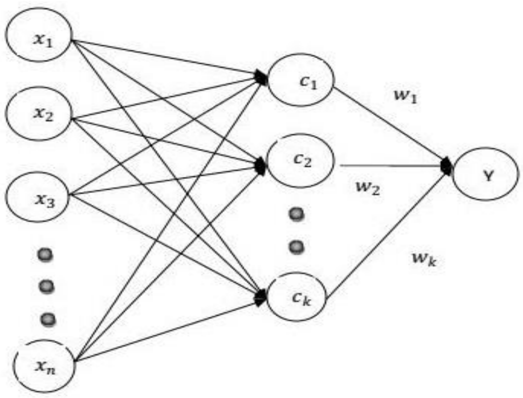


**Figure -1 Radial Basis Function Network.**

## 3. Statement of the Problem

The forward/inverse problem of ECG is constructed as a Laplace equation with neumann boundary conditions, where the forward equation is,

$\nabla.(\alpha\nabla u) = 0$ in $B$ (3)

$\vec{n}.(\alpha\nabla u) = 0$ on $\partial B$ (4)

$u = g$ on $\partial H$ (5)

where, B, $\partial H$, $\partial B$ and $g$ represents body, heart surface and body surface boundary and electric potential, $\vec{n}$ indicate normal on the body surface, which is shown in figure 2. Equation (3)-(5), satisfies $g$ on $\partial H$. Solving the above equation is finding $u(x, y)$ on $B$. The above equation is well-posed [15].

The inverse problem is,

$\nabla.(\alpha\nabla u) = 0$ in $B$ (6)

$\vec{n}.(\alpha\nabla u) = 0$ on $\partial B$ (7)

$u = d$ on $\partial B$ (8)

Equation (6)-(8) satisfies the electric potential $d$ measured on the body surface. Here $\vec{n}$ is the outward normal on the surface of the body. Solution to this equation is ill-posed [15].

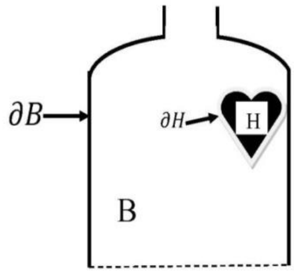


**Figure -2 Human torso.**

To solve the forward problem, we consider a simple mathematical model similar to the ECG forward model, consider a unit square $\Omega = (0,1)X(0,1)$,

$$\sigma_1 := \{(x, 0): 0 \leq x \leq 1\},$$

$$\sigma_2 := \{(1, y): 0 \leq y \leq 1\},$$

$$\sigma_3 := \{(x, 1): 0 \leq x \leq 1\},$$

$$\sigma_4 := \{(0, y): 0 \leq y \leq 1\},$$

With $\partial\Omega = \sigma_1 \cup \sigma_2 \cup \sigma_3 \cup \sigma_4$ as shown in figure 3.

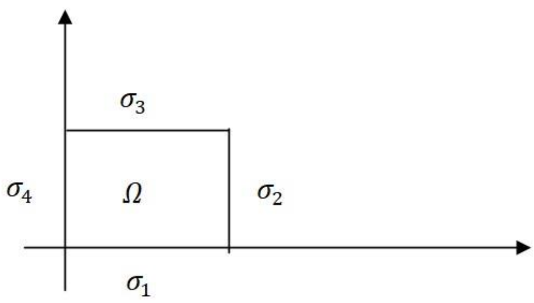


**Figure -3 Boundary of Forward Problem**

The forward problem is formulated as,

$\Delta u = 0$ for $(x, y)\epsilon\Omega$ (9)

$\nabla u.n = 0$ for $(x, y)\epsilon\sigma_1 \cup \sigma_2 \cup \sigma_4$ (10)

$u = g$ for $(x, y) \in\cup \sigma_3$ (11)

Equation (9)-(11) is a direct problem and the solution is finding $u(g)(x, y)$ on $\sigma_1$. The inverse problem is

$\Delta u = 0$ for $(x, y)\epsilon\Omega$ (12)

$\nabla u.n = 0$ for $(x, y)\epsilon\sigma_3$ (13)

$u = d$ for $(x, y) \in\cup \sigma_1$ (14)

Solving equation (12)-(14) is finding the solution on $\sigma_3$.

**4. Numerical Example**

Here in this paper, we will concentrate on solving the mathematical model of the forward problem. To evaluate the performance of the forward problem given the potential on heart surface $\sigma_3$, we consider the forward problem.

$\Delta u = 0$ for $(x, y)\epsilon\Omega$

$\nabla u.n = 0$ for $(x, y)\epsilon\sigma_1 \cup \sigma_2 \cup \sigma_4$

$u = g$ for $(x, y) \in\cup \sigma_3$

The analytic solution to the above problem is $u(g_i)(x, y) = \frac{1}{\cosh(i\pi)} \cos i\pi x \cosh i\pi y$, where $g_i = \cos i\pi x$. The above equation is then solved using RBF by taking different discretization points on the square boundary with different shape parameters and bias. The discretization points we have taken for our experiment is N=9 and N=25 and the performance are evaluated using root mean squared error (RMSE). By taking N=9 with different shapes parameter we observe that the solution converges around shape parameter 3.5111 giving RMSE convergence with 2 decimal places only as shown in table 2 and figure 3.

**Table 2. RMSE for U(x,t) and for N=9,**

| C(shape parameter) | RMS U(x,y) appox | bias |
|---|---|---|
| 3.5 | 0.0077 | 1 |
| 3.51 | 0.0025 | 1 |
| 3.511 | 0.002 | 1 |
| 3.5111 | 0.0019 | 1 |
| 3.51111 | 0.0019 | 1 |
| 3.49 | 0.0128 | 1 |
| 3.499 | 0.0082 | 1 |
| 3.4999 | 0.0078 | 1 |
| 3.49999 | 0.0077 | 1 |
| 3.52 | 0.0029 | 1 |
| 3.53 | 0.0083 | 1 |
| 3.54 | 0.0138 | 1 |
| 3.55 | 0.1959 | 1 |

| | | |
|---|---|---|
| 3.56 | 0.0254 | 1 |

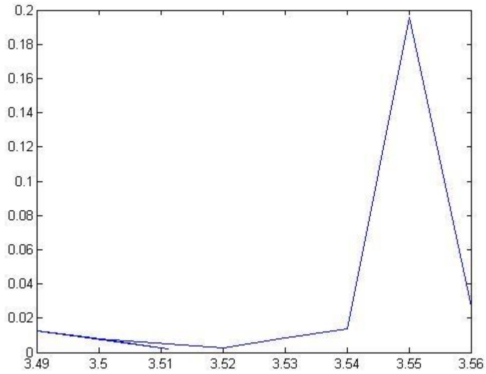


**Figure -4 RMSE for N=9**

For N=25 we observe as shown in Table 3 and Figure 4 that the solution converges around shape parameter 0.1. The RMSE gives the convergence with 1 decimal place only. Figure 5 shows the predicted solution of the forward problem for N=25.

**Table 3. RMSE for U(x,t) and for N=25,**

| C(shape parameter) | RMS U(x,y) appox | bias |
|---|---|---|
| 0.1 | 0.0936 | 1 |
| 0.09 | 0.1865 | 1 |
| 0.099 | 0.0828 | 1 |
| 0.0999 | 0.0800 | 1 |
| 0.09999 | 0.0780 | 1 |
| 0.11 | 0.0954 | 1 |
| 0.111 | 0.0963 | 1 |
| 0.1111 | 0.1321 | 1 |
| 0.1111 | 0.1045 | 1 |

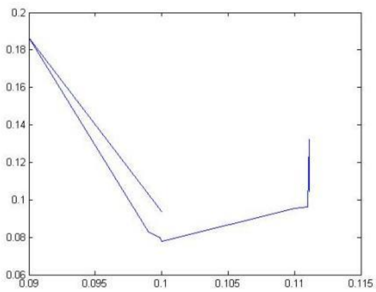


**Figure -5. RMSE for N=25**

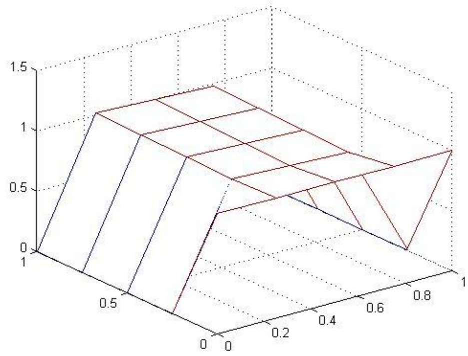


**Figure -6 RBF solution $u(x,t)$ for N=25**

## 6. Conclusion

RBF collocation method is used to solve a simple mathematical forward problem of ECG. The method is evaluated by taking N=9 and N=25 discretization. The performance is evaluated using RMSE, and from the predicted solution, it is observed that the method converges for N=9, around shape parameter 3.51, as can be seen in Fig 3, and for N=25, it converges around shape parameter 0.1. From the Table 2 and 3 it is observed that the solution converges only up to two decimal places which is not a very good prediction of the solution to the problem. In future we are using feed forward neural network method like and deep learning to find a better accurate solution to the mathematical formulation of the forward problem of ECG and later apply the method to the ECG inverse problem for accurate prediction of the electric potential on the heart surface.

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