Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 4 (2024)

Methods of Development Spatial Representation of Students by Teaching Solving Geometric Problems

Gulnora Goyibnazarova

Associate Professor, Department of Mathematics and Physics at Alfraganus University

Abstract: This article is devoted to the issues of improving the geometric training of prospective teachers of mathematics. The article provides examples of using the method of geometric transformations in solving problems on construction, with the aim of developing graphic skills. Specially selected exercises are offered to develop students' graphic skills. Analysis and synthesis are used together when solving problems. It is substantiated that the formation of the ability to predict, to foresee the results that each individual step in the process of finding a solution to a problem will lead to, is a component of the development of students' geometric training.

Key words: *Geometric training, planimetry, stereometry, problem, graphic skills, geometric transformation.*

1. Introduction

In didactics, a teaching method is understood as an ordered way of interconnected activity of a teacher and students, aimed at achieving educational and upbringing tasks. Traditional teaching methods were developed by experienced teachers, formed as a result of long-term teaching practice. By combining different methods: both traditional and "new", a teacher can achieve substantial success in his work. And for this, he needs to clearly understand the advantages and disadvantages of each method, and the conditions of its applicability.

When choosing one or another method, we proceed from the set goal - the development of geometric training. And since the most important component of geometric training is graphic skills, we will show the development of graphic skills when solving problems in the course of geometry.

In stereometry lessons, a drawing for a problem or proof of a theorem is important, so we consider it necessary to show the development of graphic skills when solving problems on construction, using the method of geometric transformations.

Solving problems on construction consists of four parts [1]: analysis, construction, synthesis or proof, and research of the solution to the problem.

2. Objectives

The textbook contains many problems on construction using the method of parallel transfer, symmetry, the method of similarity, the use of rotation in solving problems, the method of geometric places, and each of these solutions is divided into four parts - analysis, construction, synthesis, research, which is clear and convenient for students.

It is not enough to show ready-made images in a textbook or on the screen; students must see the process of construction itself. By observing where the teacher begins to draw, in what sequence and how he draws lines, when and how he uses drawing tools, students receive the most important information about the art of drawing.

In order to develop graphic skills, students themselves should draw, primarily in notebooks. In stereometry lessons, it should be explained that the first drawing of a figure may be unsuccessful, so you can suggest that students draw on a code film, and then, after discussing and choosing the best location for the figure, correcting the mistakes, offer their own options.

3. Methods

Work on developing students' graphic skills should not be postponed until the time when the study of polyhedrons begins. This work has to be constant. Already in the first lessons of stereometry, students should be advised that a straight line lying on a plane is better depicted on the entire limited part of this plane (Fig. 1), as a straight line (a) on a plane (α), and the depiction of a straight line (*b)* is unsuccessful.

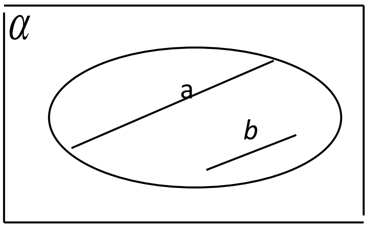
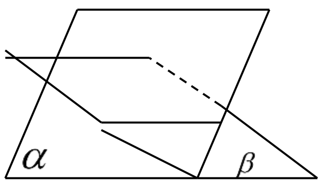


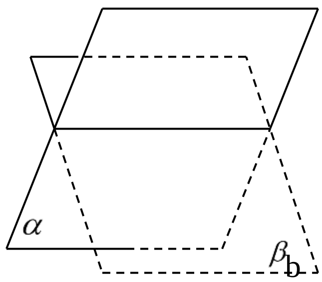
Fig. 1

Of great importance is also the careful writing of letters in the drawing. So, letters denoting a straight line should be written on one side of it, so that they do not intersect other lines of the drawing. Letters that denote planes are best written on the side, so that they do not interfere with subsequent constructions. When depicting the line of intersection of two planes, it is necessary to connect the points of intersection of the boundaries of parts of the planes with a segment, as in Fig. 2: a - unsuccessful, b - successful.



a

Fig. 2



To develop basic graphic skills that will be used in the future, you can offer students, in addition to the tasks in the textbook, the following exercises:

1. Draw a plane and a line a that:

a) intersects the plane at point A;

b) lies in the plane;

c) has no common points with the plane;

2. Draw planes α and β if they are:

a) parallel; b) intersect along a straight line c

3. Draw intersecting straight lines a and b.

4. Draw perpendicular AB and inclined AC to plane α.

5. Draw:

a) two mutually perpendicular planes;

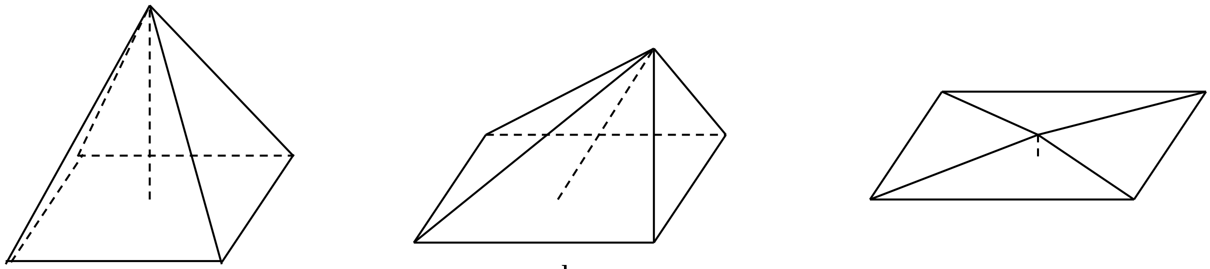
b) three mutually perpendicular planes;

6. Construct points A (2,4,-1), B (-2,0,3), C (0,0,5) in a rectangular coordinate system.

Most of the problems considered in stereometry are related to the depiction of polyhedra, solids of revolution and their combinations. Therefore, it is very important to develop students' skills in their competent depiction. First of all, it is advisable to give students some recommendations before starting work on the depiction of polyhedra and solids of revolution.

It is better to draw a pyramid starting from the base. A prism can be drawn starting from the upper base or the lower one. The base of a polyhedron is the most important part of the drawing. It is useful to think about how this polygon is depicted according to the design rules, which edges of the depicted base will be visible and which will not.

When it comes to a pyramid, the question of its visible and invisible edges is not always resolved unambiguously: it depends not only on the type of projection, but also on the ratio of the sizes of the polyhedron. For example, depending on the ratio of the height of a regular quadrangular pyramid to the edge of its base, either three of its edges must be depicted with dashed lines, or only one, or none (Fig. 3 a-c).



a b c

Fig. 3

When drawing polyhedrons in a notebook, it is advisable to first depict it with thin lines. Only after making sure that the drawing corresponds to the task, is clear and well-positioned, finally outline its invisible lines. If one drawing depicts the entire figure, and the other - some part of it, then it is necessary to ensure that both drawings have the same orientation and letter designations.

If it is necessary to depict a combination of some figures, then the inscribed figure is depicted with dashed lines, although other methods are possible.

In drawings for problems, it is necessary to observe metric relationships between the elements of the figures.

When drawing non-flat figures in class, students are guided by the properties of parallel projection, therefore it is necessary to conduct classes on frontal, dimetric and isometric projection.

Numerous facts show that one of the main reasons for the insufficient graphic culture is the low level of development of students' spatial representations. In order to teach students to represent spatial objects, to depict them correctly, to "read" drawings correctly, it is desirable to compare drawings of spatial figures with corresponding models - wireframe, glass, etc. Of course, models should not be overused in stereometry lessons. But in the first lessons on this subject or at the beginning of studying each section, models are necessary.

4. Results

To develop students' graphic skills, we use specially selected exercises. Here are some of them.

1. Draw an obtuse dihedral angle.

2. Draw a linear angle in an acute dihedral angle.

3. Draw a regular quadrangular prism whose height is half the base side.

4. Draw an inclined triangular prism.

5. Draw a regular hexagonal pyramid.

6. Draw an equilateral cylinder.

7. Draw a truncated cone.

8. Draw the figure obtained by rotating an equilateral trapezoid around a large base.

9. Construct a section of a regular quadrangular pyramid passing through the side of the base and the midline of the opposite lateral face.

10. Draw a section of a sphere by a plane passing through the middle of the radius perpendicular to it.

Using such exercises will allow focusing the students' attention on the image of figures. Experience shows that if students accompany a calculation or proof task with a drawing, then they pay primary attention to the calculations, identical transformations, and consider the drawing as something secondary. Therefore, in order to improve the students' graphic skills, special exercises aimed at achieving this goal are needed.

When solving these exercises, the skills acquired at school when solving problems will also be necessary. This includes searching and constructing a plan for solving a problem, taking into account the data and the unknown values. Construction tasks will be very useful for developing graphic and constructive skills and abilities. When searching for solutions to problems, the student first of all tries to find more and more new consequences from the data, without taking into account the unknown values. When the direction of the search changes, students try to find new statements by inertia, from which the unknown value follows, forgetting to refer to the data that has not yet been used.

The teacher recommends switching from the data to the unknown, and back, just at the moment when the student is convinced that the movement in one of the directions has not yet led to success. In the process of which the idea of solving the problem arises. Thus, when solving problems, analysis and synthesis are used together. Analysis can appear in two forms, when in reasoning they move from the unknown to the data of the problem or when the whole is divided into parts.

Synthesis is reasoning from the data of the problem to the initial ones or when elements are combined into a whole. For example, when searching for solutions to the following problem, the analysis is in the form of reasoning from the unknown to the data.

Task 1. Prove that the intersection point of the extensions of the lateral sides of the trapezoid and the midpoints of its bases belong to the same line (Fig. 4).

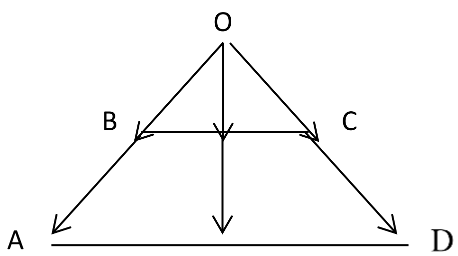


Fig. 4

Let ABCD be a trapezoid, O be the intersection point of its lateral sides, M and P be the midpoints of the bases. To prove that the points O, M, P lie on one straight line, it is sufficient to prove the collinearity of the vectors $\overrightarrow{OM}$ and $\overrightarrow{OP}$, i.e. $\overrightarrow{OP} = k\overrightarrow{OM}$.To prove the latter, we express $\overrightarrow{OP}$ and $\overrightarrow{OM}$ and through the same vectors. Given the data of the problem: $\Delta OBC \sim \Delta OAD \Rightarrow$ $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OB} + \overrightarrow{OC})$ $\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OD}) = k \cdot \frac{1}{2}(\overrightarrow{OB} + \overrightarrow{OC}) = k\overrightarrow{OM}$

which means points P, O, M lie on one straight line.

Analysis and synthesis are used together when solving problems in two forms:

1. Dissection (studying the conditions of the problem, identifying individual data, solving parts of the problem);

2. Reasoning from the initial to the data.

Analysis is also used in conjunction with other methods of scientific knowledge, such as inductive, generalization, specification, etc. Therefore, when searching for solutions to problems, all sorts of combinations of methods are used. When using the second form, two types of analysis are distinguished: ascending and descending. Ascending can be explained based on the solution to the problem. To prove some proposition A, it is enough to prove assertion B, from which A follows, and then C, from which B follows, etc. Descending - let's say it is necessary to prove assertion A. Let's assume that it is true, and we try to get the correct consequence from it.

An example of bottom-up analysis is the previous task. An example of top-down analysis is the following task:

Problem 2. "The lengths of the sides of a triangle a, b, c (a<b<c) form an arithmetic progression. Prove that $ac = 6Rr$ *, where R, r are the radii of the circumscribed and inscribed circles, respectively.*

Solution: Let the following equality be true $ac = 6Rr$, and because $R = \frac{abc}{4s}$, $r = \frac{s}{p}$, we get $ac = 6\frac{abc}{4s} \cdot \frac{s}{p}$; $a + b + c = 3b$, $\frac{a+c}{2} = b$, which is true, because a, b, c form an arithmetic progression.

All these transformations can be carried out in reverse order, therefore, the relationship being proven is true.

Analysis in the form of dismemberment is also a switch from data to source data and vice versa, when students are asked to find solutions to some part of the problem statement. It is especially often used when performing identical transformations.

Forecasting is of no small importance in finding solutions to problems. The ability to forecast, to anticipate events, the results obtained is one of the most important components of human mental activity. Formation of the ability to forecast, to foresee the results that each individual step in the process of finding a solution to a problem will lead to is an important component of the development of geometric training of students. For the purpose of such development, when discussing the idea of solving a problem, when one of the students suggests using a particular formula, theorem, definition, identical transformation, it is advisable to ensure that the student substantiates the reasonableness of his proposal and at least in general terms indicates what it will lead to. Thus, the analytical and synthetic course of reasoning of one of the students is revealed to the entire class, and the rest are accustomed to predicting the process of finding a solution to the problem.

Forecasting together with analysis, synthesis, generalization and a number of methodological recommendations helps students find a way to solve the problem. Forecasting is an important element in finding solutions and a powerful means of developing students' geometric training.

5. Discussion

The most important task of a teacher, the solutions to which are being developed by educators today, is the development of students' abilities and skills for independent work with educational literature. Students master thinking techniques in the process of studying the program material in a book. They perform specific tasks when reading books. This task directs students to use certain techniques of thinking activity. If the conditions of the pattern are met, then the students' activity is activated. Since the use of thinking techniques necessarily leads to a deeper understanding of the material, this material is well remembered. As a result, students simultaneously master the techniques of thinking activity and master the program material well.

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