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Matrix Transformations of some Generalized Sequence Spaces Into l_∞

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Abstract- In this paper, we shall find the necessary and sufficient conditions for an infinite matrix $A=(a_{nk});\, n,k=1,\ 2,\ldots$ to transform some generalized sequence spaces into the space l_∞.

Keywords: *Matrix transformations, Sequence spaces, sufficient condition.*

Introduction

In this paper, we shall study the included theorems on matrix transformations of some generalized sequence spaces and the convolution products of sum ability matrices characterized in connection with the generalized sequence spaces. Given below briefly deal with the required known properties of the sequence spaces and the other available results needed for our investigations.

One of the most important investigations in the theory of finding out the necessary and sufficient conditions on an infinite matrix in order thatit should transform one sequence space into the same space or another space. Problems of this type in the classical theory of matrix transformations are generally designated as 'Matrix Transformations of Sequence Spaces'. This problem remains unsolved for any two arbitrary sequence spaces in general, although solutions have been given for many particular cases. The basic results of such problems can be found in the books of R.G. Cooke [5], and I.J, Maddox [7].

Transform some generalized sequence spaces into the space l_∞. In we shall deal with the classes of matrices.

$(\ w(p), l_\infty)$ (Theorem 1)

and $(c(p), l_\infty)$ (Theorem 1)

Matrix Transformations of *w*(*p*) and *c*(*p*) Into l_∞ **:**

Theorem 1: When, $0 < p_k \le 1, A \in (w(p), l_\infty)$ if and only if (1.1) there exist an integer *M*> 1 such that

$$\sup_n \sum_{r=0}^{\infty} \max_r \left\{ \left(2^r{}_M{}^{-1}\right)^{1/p_k} \left(a_{nk}\right) \right\} < \infty \qquad (1)$$

Where $\max_r$ is the maximum over $2^r \le k < 2^{r+1}$.

Proof. For the sufficiency let (1) holds and $x = (x_k) \in w(p)$.

Then, $$2^{-r} \sum_r |x_k - L|^{p_k} \to 0 \; as \; r \to \infty \text{ for some } L$$

Where, $\sum_r$ stands for the summations over. $2^r \le k < 2^{r=1}$.

Hence there exists an integer $R > 0$, such that

$$2^{-r} \sum_r |x_k - L|^{p_k} < 1/2M$$

and $$2^{-r} \max(l, |L|) < 1/2M$$

for every $r>R$.

Therefore, $$2^{-r} M (x_k)^{p_k} < 1$$

So that, since $0 < p_k \le 1$, we have

$$2^{-r/p_k} M^{1/p_k} |x_k| < 2^{-r} M |x_k|^{p_k}.$$

Then for every $r>R$, we get

$$\sum_r |a_{nk} x_k| \le \max_r \left\{ \left(2^r M^{-1}\right)^{1/p_k} |a_{nk}| \right\} M.g(x)$$

Where $g(x) = \sup_r \left\{ 2^{-r} \sum_r |x_k|^{pk} \right\}$.

Hence for $r>R$, the sum $\sum_{k=2^R}^{\infty} a_{nk} x_k$ is bounded.

Now consider, $$\sum_{k=1}^{2^R - 1} a_{nk} x_k \qquad (2)$$

Since R is fixed and (1.1) implies (a_{nk}) is bounded for fixed n, it follows that the sum (1.2) is bounded.

Hence $(y_n) = \left(\sum_k a_{nk} x_k \right) \in l_\infty$ so that $A \in (w(p), l_\infty)$.

For the necessity, let $A \in (w(p), l_\infty)$. Since the matrix (a_{nk}) is applicable to each member of $w(p), (a_{nk})_{k\geq} \in w^\beta(p)$ for each $n \geq 1$ so that $\sum_k a_{nk} x_k$ converges for every $x = (x_k) \in w(p)$

.

Now the metric defined by. $g(x) = \sup_r \left\{2^{-r} \sum_r |x_k|^{p_k}\right\}$ where $\sum_r$ is the summation over. $2^r \leq k < 2^{r+1}$, Determines the topology on $w(p)$. It follows from the definition of. $g(x)$ That the coordinate functional are continuous, and

Hence, $A_{n,k}(x) = \sum_{k=1}^{k} a_{nk} x_k$

Is an element of $w^*(p)$. Taking n as fixed we are given that this tends to a limit $A_n(x)$ for every $x \in w(p)$. So by the uniform bounded principle there exists positive number δ_n, G_n (say) such that if $g(x) < \delta_n$, then $A_n(x) < G_n$. If $g(x) < \delta_n / r$ (r a positive integer), this can be made arbitrarily small by the choice of *r*:, thus we can make $A_n(x)$ absolutely small by making $g(x)$ sufficiently small.

Hence $A_n \in w^*(p)$.

Since *w* (*p*) is a complete linear metric space under the metric $g(x)$ and since $\sup_n |A_n(x)| < \infty$ on *w* (*p*), there exists by the uniform bounded principle a number G independent of *n* and *x*, and a number $\delta \leq 1$ such that

$$|A_n(x)| \leq G \tag{3}$$

For every $x \in S[\theta, \delta]$ and every n where $S[\theta, \delta]$ denotes the closed sphere in *w* (*p*) with centre $\theta = (0,\ 0, \ldots)$ and radius δ.

Let M be an integer > 1 such that

$$M^{-1} < \delta \tag{4}$$

writing $A(r,k) = (2^r \delta)^{1/p_k} |a_{nk}|$ *for* $2^r \leq k < 2^{r=1}$, and suppose $k(r)$ is such that $\max_r A(r, k(r))$, taking n as fixed, define for any $s, x = (x_k) \in w(p)$ by

$$x_k = 0 \text{ for } k \geq 2^{s=1}, x_k(r) = (2^r \delta)^{1/p_k} \operatorname{sgn} a_{n,k}(r),$$

$$x_k = 0, (k \neq k(r)) \text{for } 0 \leq r < s.$$

Then $g(x) \leq \delta$ and $x = (x_k) \in S[\theta, \delta](w(p))$. hence (3) and (4) gives

$$\sum_{r=0}^{s} \max_r \left\{ \left(2^r M^{-1}\right)^{1/p_k} |a_{nk}| \right\} \le G.$$

Since this holds for any *s*, it follows that

$$\sum_{r=0}^{\infty} \max_r \left\{ \left(2^r M^{-1}\right)^{1/p_k} |a_{nk}| \right\} \le G.$$

This holds for any *n*. hence we have (1).

Theorem 2: Let $p = (p_k) \in l_\infty$. then $A \in (c(p), l_\infty)$ if and only if

(2.1) there exists an absolute constant *M*>1 such that

$$\sup_n \sum_k |a_{nk}| M^{-1/p_k} < \infty \tag{5}$$

and

$$\sup_n \sum_k |a_{nk}| < \infty \tag{6}$$

Proof. For the sufficiency, let (5) and (6) hold and $x(x_k) \in c(p)$. Then

$$|x_k - L|^{p_k} \to 0 \text{ as } k \to \infty \text{ for some L.}$$

Put $x_k - L = x_k^{'}. Then\ x' = (x_k^{'}) \in c_0(p)$ and

we have
$$|An(x)| = \left|\sum_k a_{nk} x_k\right| \le |A_n(x')| + L \sum_k |a_{nk}| \tag{7}$$

where $A_n(x') = \sum a_{nk} x_k^{'}$.

Since holds, by theorem of in this paper, we have $(A_n(x')) \in l_\infty$,

so that,
$$\sup_n |A_n(x')| < \infty \tag{8}$$

Now using (8) and (6), we have from (1.7)

$$(A_n(x')) \in l_\infty \text{ so that } A \in (c(p), l_\infty).$$

For the necessity, let $A \in (c(p), l_\infty). Now\ c_0(p) \subset c(p)$ and $c_0 \subset c(p)$ so that $(c(p), l_\infty) \subset (c_0(p), l_\infty)$ and $(c(p), l_\infty) \subset (c_0, l_\infty)$. Hence $A \in (c_o(p), l_\infty)$ and $A \in (c_o, l_\infty)$. Therefore the necessities of (5) and (6) follow from theorem of in this paper.

Corollary: $A \in (c_o, l_\infty)$ if and only if

$$\sup_n \sum_k |a_{nk}| < \infty$$

Proof. The proof follows from the theorem by taking $p_k = 1$ for all k.

Conclusion: The results in the above theorem generalize many of the results of *k*. ChandrasekharaRao [4] arid also give some more results regarding the closure properties of the convolution of the classes of matrices involved in the inclusion theorems of generalized sequence spaces.

References:

[1] Altay, B., Basar, F. and Mursaleen, M., On the Euler sequence spaces which include the spaces $\ell_p$and ℓ_∞, *Inform. Sci.*,176(10) (2006) 1450-1462.

[2] Akhmedov, A. M. and Basar, F., The fine spectra of the Cesáro operator C_1 over the sequence space $bv_p(1 \leq p < \infty)$, *Math. J. Okayama Univ.,* 50 (2008) 135-147.

[3] Aydın, C. and Basar, F., Some new sequence spaces which include the spaces $\ell_p$and ℓ_∞, *Demonstratio Math.*,38(3) (2005) 641-656.

[4] ChandrasekharaRao, K. Matrix transformations of some sequence spaces. *Pacific J.M*., 31 (1969), Pp. 171-174.

[5] Cooke, R.G. *Infinite matrices and sequence spaces*. Macmillan Co., London, 1949.

[6] Lascarides, C.G. and Maddox, I.J. Matrix transformations between some classes of sequences. *Proc. Camb. Phil. Soc.*, 68 (1970), Pp. 99-104.

[7] Maddox, I.J. and Willey, M.A.L. Continuous Operators on paranormed spaces and matrix transformations. *Pac. J. Math.*, **5**(1) (1974), Pp. 217-227.

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