Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 4 (2024)

Steady State Probability Analysis of GI/M (a, b)/1/MWV Queueing Model with Breakdown

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Abstract: This paper analyses a Non –Markovian Queuing model of General Input Single Server pattern with batch service where server undergoes multiple working vacation with server breakdown. The customers are served by a single server with general bulk service rule in which the server requires atleast “a” number of customers to function and maximum number of the customers are “b”. The server experiencing breakdown in regular busy period is considered in this model. Customer arriving follows General distribution and the service follows Exponential distribution. The breakdown that happens during regular busy periods is referred as β_D.The steady state probability distribution and mean queue length are calculated. Further performance measures of the considered model are discussed.

Keywords: *General Arrival, Batch service, Regular busy period, multiple vacation, Breakdown.*

1. Review of Literature:

There are few chances in which customers arrival occur in groups and also the service is provided in groups. Passengers travelling by share auto, service in rope car, packing the products in a manufacturing company are some the real time examples of batch service. The concepts of batch queue was first introduced through the research of Bailey, (1954) [2]. The general bulk service rule was pioneered by Neuts(1967)[13] with Poisson arrival and general service time distribution. Chaudhry M.I and Templeton J.G.C (1983) [3] discussed a first course in bulk queuing.Recent Developments in bulk and queuing models of GI/M (a,b)/ 1 queuing system was assessed by Medhi (1984)[12]. Choi and Han (1994) [4] analyzed the GI/ M (a,b) / 1 queue with multiple vacations. Servi. L.D and Finn S.G (2002) [15] have assessed M/M/1 queues with working vacations (M/M/1/WV). GI/M/1 working vacation model was examined by Baba (2005) [1]. Tian, Li and Zhang (2007) [10] provided a survey results of working vacation queues. Zhang and Xu, (2008) [18] have examined the M/M/1 queue with multiple working vacation. Gross, D. and Harris, C.M (2008) [8] have proposed fundamentals of queuing theory. Julia Rose Mary and Afthab Begum,(2009), [9] have analyzed the Markovian M/M(a,b)/1 queuing model under multiple working vacation and derived the steady state probability distribution and mean queue length for the model. Ghimere R.P and Ghimire S. (2011) [6] analyzed heterogeneous arrival and departure M/M/1 queue with vacation and service breakdown. Sushil Ghimire, R.P. Ghimire, Gyan Bahadur Thapa [2014] [16] examined the Mathematical models of the M^b/M/1 bulk arrival queuing system. “Steady state analysis of M/M (a,b)/MWv/BR queuing model” was studied by Rajalakshmi, Pavithra, Julia Rose Mary (2016)[17]. Easton G.D and Chaudhry M L (2018) [5] discussed about the Queueing system E_k/ M (a,b)/1 and its numerical analysis. Gopinath Panda, Abhijit Datta Banik and Dibyajyoti Guha (2018) [7] assessed “Stationary Analysis and optimal control under multiple working vacation policy in a GI/M(a,b)/1 queue”. Extended analysis and computationally efficient results for the GI/ $M^{a,b}$ /1 queuing system was analyzed by Samanta S.K Bank.B (2020)[14]. Mohan Chaudhry and jing Gai(2022)[11] proposed analytic and computational analysis of GI/M(a,b)/c queuing system. With the assistance of existing sources this paper analyses GI/M (a.b)/1/MWV/Br queuing model with server breakdown in regular busy period.

3. Model Description:

In this model the server serves the customers in batches according to the general bulk service rule (GBSR) introduced by Neuts (1967). This rule states that server starts service only when there are

atleast “a” customers in the system and maximum service capacity is “b”. After completing each service if the server finds at most “b” customers in the system then the server takes all “b” customers in a single batch to serve. If he finds more than “b” then he takes first “b” customers as first batch for service. Thus the service time of batches of size “k” (a ≤ k≤ b) is assumed to be independent identically distributed. If there are less than “a” customers in the system then the server takes vacation. During vacation if the server finds the system with more than “a” customers then the server starts the service with lower service rate $\mu_{WV.}$ The server repeats another working vacation if he finds less than “a” customers in the system which is known as multiple vacation process. In this model the arrival process is assumed as Poisson distribution with the parameter λ. Regular service follows exponential distribution and it is denoted by the parameterμ_R. Service rate during vacation is assumed to be μ_{WV} follows exponential distribution. The working vacation time is that the server completes a service and finds if the customers are less than ‘a’ server takes a vacation which is exponentially distributed and is assumed by η. The server’s breakdown during regular busy period follows Poisson distribution with a parameter β_D.

4. Discrete –Time Markov Chain pre arrival Queue length.

Let $t_{n,}$ n= 1, 2, 3, 4… be the arrival epoch of the n th customer. If there is no customer arrives then

t_0 = 0. The inter arrival times { t_n ,n=1,2,3,4,….} are in-depended and uniformly distributed with a general distribution function G(t) with mean 1/λ and the Laplace S T of G(t) is denoted by G*(φ).

For the model GI/ M(a,b)/1/MWV/Br, Let $N_q(t)$ be the number of customers in the queue at time “t” and S_n (t) = 0 , 1, 2, 3 where server is in idle vacation , busy on working vacation, regular busy state, breakdown state respectively. In this model the process {($N_q(t_n$ - 0), S_n) ; n ≥ 1} is an embedded Markov chain with state space $\overline{C}$={ (n≥0) ; s=1,2,3}U{0≤n≤a-1;s=0}.

Further the system size probabilities at a time t are as given below

$A_n(t)$ = Pr {Nq (t) =n, S_n =1}, n≥0

$B_n(t)$ = Pr {Nq (t) =n, S_n =2}, n≥0

$C_n(t)$ = Pr {Nq (t) =n, S_n =0}, 0≤n≤a-1

$D_n(t)$ = Pr {Nq (t) =n, S_n =3}, n≥0

Assuming the steady state probabilities when the server is, in regular busy period, in working vacation period, in idle vacation period, in breakdown period respectively are as follows

$A_n = \lim_{t\to\infty} A_n(t);\ B_n = \lim_{t\to\infty} B_n(t);\ C_n = \lim_{t\to\infty} C_n(t);\ D_n = \lim_{t\to\infty} D_n(t)$

During working vacation and regular busy period there will be n number customers in the queue and the system has k (a ≤ k ≤ b) customers in service, whereas in idle vacation period the number customers in the queue and in the system are same.

Let r_k denote the probability that k batches are served at regular service rate μ in an inter arrival time. Then

$$r_k = \int_0^\infty e^{-\mu_R t}\ \frac{(\mu_R t)^k}{k!}\ d\,G(t) \qquad k \geq 0$$

Let w_k denote the probability that working vacation time is greater than an interarrival time and the service completion of k – batches occur at a rate μ_{WV} in an inter-arrival time. Then

$$w_k = \int_0^\infty e^{-\eta t} \frac{e^{-\mu_{WV} t}}{k!} \mu_{WV}\, t)^k dG(t) \qquad k \geq 0$$

Let t_k denote the probability that the server returns from vacation in an inter arrival time and k service completions occur in an inter arrival time. Then

$$t_k = \int_0^\infty \sum_{i=0}^{k} \left\{ \int_0^t \eta e^{-\eta x} \frac{(\mu_v\, x)^t}{i!}\, e^{-\mu_v\, x} \frac{(\mu(t-x))^{k-i}}{(k-i)!}\, e^{-\mu(t-x)}\, dx \right\} dG(t) \qquad k \geq 0$$

Let b_k denote the probability that the server gets breakdown at the rate of β_D .Then

$$b_k = \int_0^\infty e^{-\beta_D t} \frac{(\beta_D t)^k}{k!}\, dG(t), \qquad k \geq 0$$

The steady state equations at pre arrival time are calculated by observing the displacement between the state of Markov chain and are given as follows

$$A_n(t) = \sum_{k=0}^{\infty} A_{kb+n-1}(r_k + b_k) + \sum_{k=0}^{\infty} B_{kb+n-1}\, t_k \qquad n \geq 1 \quad \ldots\ldots (1)$$

$$A_0(t) = \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} A_{(k-1)b+j}\, (r_k + b_k) + \sum_{k=0}^{\infty} \sum_{j-a-1}^{b-1} B_{(k-1)b+j}\, t_k + C_{a-1} t_0 \ldots\ldots (2)$$

$$B_n(t) = \sum_{k=0}^{\infty} B_{kb+n-1}\, w_k \qquad n \geq 1 \ldots\ldots (3)$$

$$B_0(t) = \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} B_{(k-1)b+j}\, r_k + C_{a-1} w_0 \ldots\ldots (4)$$

$$C_n(t) = C_{n-1} \sum_{k=0}^{\infty} B_{kb+n-1}(1 - \sum_{i=0}^{k}(w_i + t_i)) + \sum_{k=0}^{\infty} A_{kb+n-1}\ (1 - \sum_{i=0}^{k}(r_i + b_i)) \quad 1 \leq n \leq a-1$$

$$\ldots\ldots (5)$$

$$C_0(t) = \sum_{k=1}^{\infty} \sum_{j-a-1}^{b-1} B_{kb+n-1}\, (1 - \sum_{i=0}^{k}(w_i + t_i)) + \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} A_{kb+n-1}\, (1 - \sum_{i=0}^{k}(r_i + b_i)) + C_{a-1}\ (1 - (t_0 + w_0)$$

$$\ldots\ldots (6)$$

Using forward displacement operator Equations (1) & (3) become

$$E - \sum_{k=0}^{\infty} E^{kb}(r_k + b_k)\, A_n = \sum_{k=0}^{\infty} B_{kb+n}\, t_k \qquad n \geq 0 \quad \ldots\ldots (7)$$

$$(E - \sum_{k=0}^{\infty} E^{kb}\, w_k)B_n = 0 \quad , n \geq 0 \quad \ldots\ldots (8)$$

The Probability Generating Function of r_k , w_k , b_k respectively becomes

$$R(z^b) = \sum_{k=0}^{\infty} r_k\, z^{kb} = G^*(\mu_R(1 - z^b)), \quad \ldots\ldots (9)$$

$$W(z^b) = \sum_{k=0}^{\infty} w_k\, z^{kb} = G^*(\eta + \mu_{wv}(1 - z^b)) \quad \ldots\ldots (10)$$

$$F(z^b) = \sum_{k=0}^{\infty} f_k\, z^{kb} = G^*(\beta(1 - z^b)) \quad \ldots\ldots (11)$$

The characteristic equation z= R (z^b) has a unique root r inside (0,1) if $\rho = (\frac{\lambda}{b\mu}) < 1$. Thus by the result of GI/M/1 model of Gross and Harris (1985), R(r^b)=r with 0< r< 1.

The characteristic equation z=W(z^b) has a unique root r_1 inside (0,1).Therefore by the result of Baba(2005) , W(r_1)= r_1 with 0 <r_1<1.

If $\rho = (\frac{\lambda}{b\mu})$ <1 then the equation $F(z^b) = z$ has unique root r_2 inside (0,1) then we have $F(r_2{}^b) = \beta$ where β tends to zero.

Hence the homogeneous equation has a solution

$B_n = r_1{}^n B_0 \qquad n \geq 0$ ………………………………………….. (12)

And the non-homogeneous difference equation has a solution

$A_n = (Lr^n + R_t r_1{}^n)\, B_0$ …………………………………………… (13)

Where $R_t = \frac{T(r_1{}^b)}{r_1 - R(r_1{}^b) - F(r_1{}^b)}$ and $Tr_1{}^b = \sum_{k=0}^{\infty} r_1{}^{kb} t_k = R_t\,(r_1 - R\,(r_1{}^b) - F(r_1{}^b))$ ……… (14)

Thus $A_n = \left[Lr^n + (\sum_{k=0}^{\infty} r_1{}^{kb}\, t_k)\frac{r_1{}^n}{r_1 - R(r_1{}^b) - F(r_1{}^b)}\right] B_0 \quad where\ r_1 \neq r$

To find the remaining probabilities the equation (5) & (6) are used.

Now

$$C_n = \left[L\left(\frac{1-r-\beta}{1-r}\right)\left(\frac{r^{a-1}-r^n}{1-r^b}\right) + (R_t+1)\left(\frac{r_1{}^{a-1} - 1 + (1-r_1{}^n)\left(1-\frac{\beta}{1-r_1}\right)}{1-r_1{}^b}\right) + \frac{r_1{}^{a-1}-r_1{}^n}{r_1{}^b\,(1-r_1)} + \frac{r_1{}^b - r_1{}^a}{w_0 r_1{}^b\,(1-r_1)}\right] B_0 \qquad 0 \leq n \leq a-1$$

$$C_n = \left[L\left(\frac{1-r-\beta}{1-r}\right)\left(\frac{r^{a-1}-r^n}{1-r^b}\right) + (R_t+1)\left(\frac{r_1{}^{a-1}-r_1{}^n}{1-r_1{}^b}\right) + \frac{r_1{}^{a-1}-r_1{}^n}{r_1{}^b\,(1-r_1)} + \frac{r_1{}^b-r_1{}^a}{w_0 r_1{}^b\,(1-r_1)}\right] B_0 \quad 0 \leq n \leq a-1$$

as β ⟶ 0

$$C_n = \left(L\left(\frac{1-r-\beta}{1-r}\right) h(r) + (R_t+1)\, h(r_1) + \frac{r_1{}^{a-1}-r_1{}^n}{r_1{}^b\,(1-r_1)} + \frac{r_1{}^b - r_1{}^a}{w_0 r_1{}^b\,(1-r_1)}\right) B_0$$

where $h(r) = \left(\frac{r^{a-1}-r^n}{1-r^b}\right)$ …………………….. (15)

Thus the steady state queue size probabilities are expressed in terms of B_0 and L.

Using Equation (2) L can be determined as follows

$$L\,u(r) + R_t\,u\,(r_1) = \frac{t_0}{w_0}\left(\frac{r_1{}^a - r_1{}^b}{r_1{}^b\,(1-r_1)}\right) \qquad \text{………………… (16)}$$

where $u(x) = \frac{x^a - x^b}{x^b(1-x)} + (r_0 + f_0)\left(\frac{x^b - x^{a-1}}{x^b\,(1-x)}\right) + \beta\left(\frac{x^{a-1}-x^b}{x^b\,(1-x)}\right)$

The value of B_0 is calculated by using normalizing conditions.

$$\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} B_n + \sum_{n=0}^{\infty} C_n = 1$$

$B_0{}^{-1} = L\,(h(r)) + (R_t+1)h\,(r_1) + \frac{1}{r_1{}^b(1-r_1)}\left(\frac{r_1{}^b - r_1{}^a}{w_0} + (r_1{}^{a-1} - r_1{}^b)\right)$ ………................ (17)

where $h(x) = \frac{1}{1-x^b}\left(\frac{x^a - x^b}{1-x} + x^{a-1}\right)$

Thus the steady state queue size probabilities for the general bulk service with break down are given by

$$B_n = r_1{}^n B_0 \qquad n \geq 0$$

$$A_n = (Lr^n + R_t r_1{}^n) B_0$$

where $R_t = \frac{T(r_1{}^b)}{r_1 - R(r_1{}^b) - F(r_1{}^b)}$ and $Tr_1{}^b = R_t\,(r_1 - R\,(r_1{}^b) - F(r_1{}^b))$

$$C_n = \left[L\left(\frac{1-r-\beta}{1-r}\right)\left(\frac{r^{a-1}-r^n}{1-r^b}\right) + (R_t+1)\left(\frac{r_1{}^{a-1} - 1 + (1-r_1{}^n)\left(1-\frac{\beta}{1-r_1}\right)}{1-r_1{}^b}\right) + \frac{r_1{}^{a-1} - r_1{}^n}{r_1{}^b\,(1-r_1)} + \frac{r_1{}^b - r_1{}^a}{w_0 r_1{}^b\,(1-r_1)} \right] B_0 \qquad 0 \le n \le a-1$$

B_0 is determined using equation (17)

5. Mean Queue Length

The mean queue length L_q of the model is as follows

$L_q = \sum_{n=0}^{\infty} nA_n + \sum_{n=0}^{\infty} nB_n + \sum_{n=0}^{\infty} nC_n$

After simplification it is found as

$$L_q = \left[L(\xi\,(\mathrm{r})) + (R_t+1)(\xi\,(r_1)) + \frac{a\,(a-1)}{2}\left(\frac{r_1{}^{a-1} - r_1{}^b}{r_1{}^b\,(1-r_1)} + \frac{r_1{}^b - r_1{}^a}{w_0 r_1{}^b\,(1-r_1)}\right)\right]$$

where $\xi\,(\mathrm{x}) = \frac{x}{(1-x)^2} + \frac{1-x-\beta}{(1-x)(1-x^b)}\left(\frac{a(a-1)x^{a-1}}{2} + \frac{ax^a(1-x) - x(1-x^a)}{(1-x)^2}\right)$

6. Performance Measures

Let M_v, M_I, M_{Busy} denote the server is in vacation , idle, regular busy respectively. Then

$M_v = \sum_{n=0}^{\infty} r_1{}^n B_0 = \frac{B_0}{1-r_1}$

$M_I = \sum_{n=0}^{a-1} C_n = \left[L\left(\frac{1-r-\beta}{1-r}\right)\frac{r^{a-1}}{1-r^b} + (R_t+1)\left(\frac{1-r_1-\beta}{1-r_1}\right)\left(\frac{r_1{}^{a-1}}{1-r_1{}^b}\right) - L\left(\frac{1-r-\beta}{1-r}\right)\left(\frac{1-r^a}{(1-r)(1-r^b)}\right) - (R_t+1)\left(\frac{1-r_1{}^a}{(1-r_1)\,(1-r_1{}^b)}\right) + \frac{r_1{}^{a-1} - r_1{}^b}{r_1{}^b\,(1-r_1)} + \frac{r_1{}^b - r_1{}^a}{w_0\, r_1{}^b\,(1-r_1)}\right] B_0$

$M_{Busy} = \sum_{n=0}^{\infty}(L\,r^n + R\,r_1{}^n)B_0 = \left(\frac{L}{1-r} + \frac{R}{1-r_1}\right) B_0$

7. Conclusion

The Non- Markovian queuing model of General Input single server with server break down

GI/M (a,b)/1/MWV/Br is discussed in this paper. Server experiencing breakdown during regular busy period is analyzed. The steady state solution, performance measures for the model are evaluated. In future the model can be extended to multiserver Non-Markovian queuing model with different cases of break down.

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