Neighbour Vertex Distinguishing Total Coloring of Comb Product of Some Graphs

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Abstract

In this paper, we determine the neighbour vertex distinguishing total chromatic number of comb product of some graph path by path, path by cycle and cycle by path. Also determine chromatic number of the skew product and converse skew product of P_m & P_n in detail.

Keywords: simple graph, Neighbour vertex distinguishing total coloring, comb product, skew product, converse skew product, path, cycle.

1 Introduction

All the graphs considered here are undirected graph, simple and connected graph $G = \big(V(G), E(G)\big)$. For every vertex $x, y \in V(G)$, $xy \in E(G)$ denotes the edge connecting two vertices. For all other standard concepts of graph theory, we see [1], [2], [4].

[5], [6] The neighbour vertex distinguishing total coloring (abbreviated as $NVDTC$) of the graph G is a mapping $\phi : V(G) \cup E(G) \longrightarrow \{1,2,\cdots,\mathrm{t}\}, \mathrm{t} \in \mathbb{Z}^+$ such that any two adjacent or incident elements in $V(G) \cup E(G)$ have different colors. Then $C_\phi(x) \neq C_\phi(y)$ whenever $xy \in E(G)$, where $C_\phi(y)$ is the color class of the vertex y (with respect to ϕ).

$$\begin{aligned} C_\phi(x) &= \{\{\phi(x)\} \cup \{\phi(xy) \mid xy \in E(G)\}\} \\ \overline{C}_\phi(x) &= \{1,2,\cdots,t\} \setminus C_\phi(x) \end{aligned}$$

The minimum number of colors required to give an $NVDTC$ to the graph G is denoted by $\chi_{nvt}(G)$. AVDTC of tensor product of graphs are discussed in literature [5] and Quadrilateral snake in [6]. $\chi_{nvt}(G) \leq \Delta(G) + 2$, if G is a bipartite graph. For all other basic terminology of graph theory, we see [1], [2], [4].

2 NVDTC of $G_1 \rhd G_2$

[3] Let G_1 and G_2 be two connected graphs. Let v be a vertex of G_2. The comb product denoted by $G_1 \rhd G_2$ between G_1 and G_2, is a graph obtained by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and grafting the i-th copy of G_2 at the vertex v to the i-th vertex of G_1. By the definition of comb product, we can say that $V(G_1 \rhd G_2) = \{\phi(a;x)|\ a \in V(G_1); x \in V(G_2)\}$ and $(a,x)(b,y) \in E(G_1 \rhd G_2)$ whenever $a = b\ and\ xy \in E(G_2)$, or $ab \in E(G_1)$ and $x = y = v$. In this paper, we will show the results related to comb product of path and cycle graph namely $Pm \rhd P_n$, $Pm \rhd C_n$ and $Cm \rhd P_n, m, n \ \in N$.

The $NVDTC$ of comb product of two graphs G_1 and G_2 are discussed in this section. Let $\{v_1, v_2, \cdots, v_m\}$ be the vertices of G_1 and $\{w_1, w_2, \cdots, w_n\}$ be the vertices of G_2. By the definition of comb product, we obtain a graph with vertex set $\{u_{r,s}\ for\ r = 1,2,\cdots,m\ and\ s = 1,2,\cdots,n\}$.

Theorem 2.1.

$$\chi_{nvt}(P_m \rhd P_n) = 5,\ m \geq 4$$

Proof. Let $\{v_1, v_2, \cdots, v_m\}$ is the vertex set of P_m and $\{w_1, w_2, \cdots, w_n\}$ is the vertex set of P_n. Define $\phi: V(P_m \rhd P_n) \cup E(P_m \rhd P_n) \to \{1,2,\cdots,t\}, \mathrm{t}\ \in \mathbb{Z}^+$. The vertex and edge set of $P_m \rhd P_n$ is given by

$$\begin{aligned} V(P_m \rhd P_n) &= \{u_{r,s} \mid r = 1,2,3,\cdots,m,\ \ s = 1,2,3,\cdots,n\} \\ E(P_m \rhd P_n) &= \left\{\left(\bigcup_{r=1}^{m-1} u_{r,1}u_{r+1,1}\right) \cup \left(\bigcup_{r=1}^{m} u_{r,s}u_{r,s+1}\right),\quad s = 1,2,3,\cdots,(n-1)\right\} \end{aligned}$$

Clearly, $P_m \rhd P_n$ has mn vertices and $mn - 1$ edges.

Now, the general graph $P_m \rhd P_n$ for $m, n \geq 4$.

$$For\ 1 \leq r \leq m,\ \ \phi(u_{r,1}) = \begin{cases} 1, & \text{for } r \equiv 1\ (mod\ 2) \\ 2, & \text{for } r \equiv 0\ (mod\ 2) \end{cases}$$

$$For\ 1 \leq r \leq m-1,\ \ \phi(u_{r,1}u_{r+1,1}) = \begin{cases} 3, & \text{for } r \equiv 1\ (mod\ 2) \\ 4, & for\ r \equiv 0\ (mod\ 2) \end{cases}\ and\ \phi(u_{r,1,}u_{r,2}) = \{5\}$$

$$For\ r \equiv 1\ (mod\ 2),\ \ \phi(u_{r,s}) = \begin{cases} 1, & \text{for } s \equiv 1\ (mod\ 2) \\ 2, & \text{for } s \equiv 0\ (mod\ 2) \end{cases}$$

$$For\ s \equiv 0\ (mod\ 2),\ \ \phi(u_{r,s}) = \begin{cases} 1, & \text{for } s \equiv 0\ (mod\ 2) \\ 2, & \text{for } s \equiv 1\ (mod\ 2) \end{cases}$$

$$For\ 2 \le s \le n-1\ \ and\ \ 1 \le r \le m,\ \ \phi(u_{r,s}u_{r,s+1}) = \begin{cases} 3, & \text{for } s \equiv 0\ (mod\ 2) \\ 4, & \text{for } s \equiv 1\ (mod\ 2) \end{cases}$$

The color classes are

$$For\ 2 \le r \le m-1,\ \ \overline{C}_\phi(u_{r,1}) = \begin{cases} 1, & \text{for } r \equiv 0\ (mod\ 2) \\ 2, & \text{for } r \equiv 1\ (mod\ 2) \end{cases}$$

$$C_\phi(u_{1,1}) = \{1,3,5\},$$

$$C_\phi(u_{m,1}) = \begin{cases} \{2,3,5\}, & \text{if } m \equiv 0\ (mod\ 2) \\ \{1,4,5\}, & \text{if } m \equiv 1\ (mod\ 2) \end{cases}$$

$$C_\phi(u_{r,2}) = \begin{cases} \{2,3,5\}, & \text{for } r \equiv 0\ (mod\ 2) \\ \{1,3,5\}, & \text{for } r \equiv 1\ (mod\ 2) \end{cases}\ \ and\ \ C_\phi(u_{r,3}) = \begin{cases} \{1,3,4\}, & for\ r\ \equiv 0\ (mod\ 2) \\ \{2,3,4\}, & for\ r\ \equiv 1\ (mod\ 2) \end{cases}$$

$$For\ 3 \le r \le m\ and\ s \equiv 1\ (mod\ 2)\ \ C_\phi(u_{r,s}) = \begin{cases} \{1,3,4\}, & \text{for } r \equiv 1\ (mod\ 2) \\ \{2,3,4\}, & \text{for } r \equiv 0\ (mod\ 2) \end{cases}$$

$$For\ 3 \le r \le m\ and\ s \equiv 0\ (mod\ 2)\ \ C_\phi(u_{r,s}) = \begin{cases} \{2,3,4\}, & \text{for } r \equiv 1\ (mod\ 2) \\ \{1,3,4\}, & \text{for } r \equiv 0\ (mod\ 2) \end{cases}$$

∴ The color classes of any two neighbour vertices are different.

$$\chi_{nvt}(P_m \rhd P_n) = 5,\ \text{for } m \ge 4$$

Hence the theorem. □

Theorem 2.2. *The comb product $P_m \rhd C_n$ admits AVDTC and*

$$\chi_{nvt}(P_m \rhd C_n) = 6,\ m \ge 3,\ n \ge\ 4$$

Proof. Let $V(P_m) = \{v_1, v_2, \cdots, v_m\}$ and $V(C_n) = \{w_1, w_2, \cdots, w_n\}$.

Define $\phi: V(P_m \rhd C_n) \cup E(P_m \rhd C_n) \to \{1,2,\cdots,t\}, t \in \mathbb{Z}^+$.

$$V(P_m \rhd C_n) = \{u_{r,s} \mid r = 1,2,3,\cdots,m,\ \ s = 1,2,3,\cdots,n\}$$
$$E(P_m \rhd C_n) = \left\{\left(\bigcup_{r=1}^{m-1} u_{r,1}u_{r+1,1}\right) \cup \left(\bigcup_{r=1}^{m} u_{r,n}u_{r,1}\right)\left(\bigcup_{r=1}^{m} u_{r,s}u_{r,s+1}\right),\quad s = 1,2,3,\cdots,(n-1)\right\}$$

Clearly, $P_m \rhd C_n$ has mn vertices and $m + mn - 1$ edges.

Case I. When $n\ is\ even$

Now, we consider the general graph $P_m \rhd C_n$ for $m \ge 3$ and $n \ge 4$.

$$For\ 1 \le r \le m,\ \ \phi(u_{r,1}) = \begin{cases} 1, & \text{for } r \equiv 1\ (mod\ 2) \\ 2, & \text{for } r \equiv 0\ (mod\ 2) \end{cases}$$

$$For\ 1 \le r \le m-1,\ \ \phi(u_{r,1}u_{r+1,1}) = \begin{cases} 5, & \text{for } r \equiv 1\ (mod\ 2) \\ 6, & for\ r \equiv 0\ (mod\ 2) \end{cases}$$

$$For\ 1 \le s \le n-1\ and\ for\ 1 \le r \le m,\ \ \phi(u_{r,s}u_{r,s+1}) = \begin{cases} 3, & \text{for } s \equiv 1\ (mod\ 2) \\ 4, & \text{for } s \equiv 0\ (mod\ 2) \end{cases}$$

$$For\ r \equiv 1\ (mod\ 2),\ \ and\ for\ \ 1 \le s \le n\ \ \phi(u_{r,s}) = \begin{cases} 1, & \text{for } s \equiv 1\ (mod\ 2) \\ 2, & \text{for } s \equiv 0\ (mod\ 2) \end{cases}$$

$$For\ r \equiv 0\ (mod\ 2),\ \ and\ for\ \ 1 \le s \le n\ \ \phi(u_{r,s}) = \begin{cases} 1, & \text{for } s \equiv 0\ (mod\ 2) \\ 2, & \text{for } s \equiv 1\ (mod\ 2) \end{cases}$$

$$For\ 1 \le r \le m,\ \ \phi(u_{r,n}u_{r,1}) = 4$$

∴ The color classes of any two neighbour vertices are different.

$$\chi_{nvt}(P_m \rhd C_n) = 6,\ \text{for } m \ge 3\ and\ n\ is\ even.$$

Case (II), When $n\ is\ odd$

Now, we consider the general graph $P_m \rhd C_n$.

$$For\ 1 \le r \le m,\ \ \phi(u_{r,1}) = \begin{cases} 1, & \text{for } r \equiv 1\ (mod\ 2) \\ 2, & \text{for } r \equiv 0\ (mod\ 2) \end{cases}$$

$$For\ 1 \le r \le m-1,\ \ \phi(u_{r,1}u_{r+1,1}) = \begin{cases} 5, & \text{for } r \equiv 1\ (mod\ 2) \\ 6, & for\ r \equiv 0\ (mod\ 2) \end{cases}$$

For $1 \le s \le n-1$

$$If\ r\ is\ odd,\ \ \phi(u_{r,s}) = \begin{cases} 1, & \text{for } s \equiv 1\ (mod\ 2) \\ 2, & for\ s \equiv 0\ (mod\ 2) \end{cases}\ and\ \ If\ r\ is\ even,\ \ \phi(u_{r,s}) = \begin{cases} 1, & \text{for } s \equiv 0\ (mod\ 2) \\ 2, & for\ s \equiv 1\ (mod\ 2) \end{cases}$$

For $1 \le r \le m$

$$\phi(u_{r,n}) = 4$$
$$\phi(u_{r,n-1}u_{r,n}) = 1$$
$$\phi(u_{r,n}u_{r,1}) = 2$$

$$For\ 1 \le s \le n-2,\ \ \phi(u_{1,s}u_{1,s+1}) = \begin{cases} 3, & \text{for } r \equiv 1\ (mod\ 2) \\ 4, & for\ r \equiv 0\ (mod\ 2) \end{cases}$$

It is clear that the color classes of any two neighbour vertices are distinct.

$$\chi_{nvt}(P_m \rhd C_n) = 6,\ \text{for } m \ge 3$$

Hence the theorem. □

Theorem 2.3. *The comb product $C_m \rhd P_n$ admits NVDTC and*

$$\chi_{nvt}(C_m \rhd P_n) = 5,\ m \geq 4$$

Proof. Define a function $\phi: V(C_m \rhd P_n) \cup E(C_m \rhd P_n) \to \{1,2,\cdots,t\}, t \in \mathbb{Z}^+$. The vertex and edge set of $C_m \rhd P_n$ are

$$V(C_m \rhd P_n) \;=\; \{u_{r,s} \mid r = 1,2,3,\cdots,m,\ \ s = 1,2,3,\cdots,n\}$$
$$E(C_m \rhd P_n) \;=\; \left\{\left(\bigcup_{r=1}^{m-1} u_{r,1}u_{r+1,1}\right) \cup (u_{m,1}u_{1,1}) \cup \left(\bigcup_{r=1}^{m} u_{r,s}u_{r,s+1}\right),\quad s = 1,2,3,\cdots,(n-1)\right\}$$

Clearly, $P_m \rhd C_n$ has mn vertices and $mn + m - 1$ edges.

Now, the general graph $C_m \rhd P_n$ for $m, n \geq 4$.

Case I. When $m\ is\ even$

For $1 \leq r \leq m,\ \ 1 \leq s \leq n$

$$If\ r\ is\ odd,\ \ \phi(u_{r,s}) = \begin{cases} 1, & \text{for } s \equiv 1\ (mod\ 2) \\ 2, & for\ s \equiv 0\ (mod\ 2) \end{cases}\ and\ \ If\ r\ is\ even,\ \ \phi(u_{r.s}) = \begin{cases} 1, & \text{for } s \equiv 0\ (mod\ 2) \\ 2, & for\ s \equiv 1\ (mod\ 2) \end{cases}$$

$$For\ 1 \leq r \leq m,\ \phi(u_{r,1}u_{r,2}) = 5,\quad \phi(u_{m,1}u_{1,1}) = 4$$

For $1 \leq r \leq m - 1$

$$\phi(u_{r,1}u_{r+1,1}) = \begin{cases} 3, & \text{for } r \equiv 1\ (mod\ 2) \\ 4, & for\ r \equiv 0\ (mod\ 2) \end{cases}$$

For $1 \leq r \leq m$ and $2 \leq s \leq n - 1$

$$\phi(u_{r,s}u_{r,s+1}) = \begin{cases} 3, & \text{for } s \equiv 0\ (mod\ 2) \\ 4, & for\ s \equiv 1\ (mod\ 2) \end{cases}$$

Case II. When $m\ is\ odd$

For $1 \leq r \leq m - 1,\ \ 1 \leq s \leq n$

$$If\ r\ is\ odd,\ \ \phi(u_{r,s}) = \begin{cases} 1, & \text{for } s \equiv 1\ (mod\ 2) \\ 2, & for\ s \equiv 0\ (mod\ 2) \end{cases}\ and\ \ If\ r\ is\ even,\ \ \phi(u_{r,s}) = \begin{cases} 1, & \text{for } s \equiv 0\ (mod\ 2) \\ 2, & for\ s \equiv 1\ (mod\ 2) \end{cases}$$

$$For\ 1 \leq r \leq m - 1,\ \phi(u_{r,1}u_{r,2}) = 5,\quad \phi(u_{m,1}) = 4,$$

$\phi(u_{m-1,1}u_{m,1}) = 1,\quad \phi(u_{m,1}u_{1,1}) = 2.$

For $1 \leq r \leq m - 2$

$$\phi(u_{r,1}u_{r+1,1}) = \begin{cases} 3, & \text{for } r \equiv 1\ (mod\ 2) \\ 4, & for\ r \equiv 0\ (mod\ 2) \end{cases}$$

For $1 \leq r \leq m$ and $2 \leq s \leq n-1$

$$\phi(u_{r,s}u_{r,s+1}) = \begin{cases} 3, & \text{for } s \equiv 0 \ (mod\ 2) \\ 4, & for\ s \equiv 1 \ (mod\ 2) \end{cases}$$

Therefore, the color classes of any two neighbour vertices are different.

$$\chi_{nvt}(C_m \rhd C_n) = 5, \text{ for } m \geq 4$$

Hence the theorem. □

3 NVDTC of skew and converse skew product of graphs

In this section, the skew and converse skew product of path by path graph discussed. The skew product of two graphs G_1 and G_2 denoted by $G_1 \Delta G_2$ has the vertex set $V(G_1) \times V(G_2)$ and the edge set is

$$E(G_1 \Delta G_2) = \begin{cases} (x_1, y_1), (x_2, y_2) | \ x_1 = x_2 \ and \ y_1y_2 \in E(G_2) \\ \quad (or)x_1x_2 \ \in E(G_1) \ and \ y_1y_2 \ \in E(G_2) \end{cases}$$

The converse skew product of two graphs G_1 an G_2 denoted by $G_1 \nabla G_2$ has the same vertex set of $G_1 \Delta G_2$ and the edge set is

$$E(G_1 \nabla G_2) = \begin{cases} (x_1, y_1), (x_2, y_2) | \ y_1 = y_2 \ and \ x_1x_2 \in E(G_1) \\ \quad (or)x_1x_2 \ \in E(G_1) \ and \ y_1y_2 \ \in E(G_2) \end{cases}$$

In this section, we propose $P_m \Delta P_n$ and $P_m \nabla P_n$ admits neighbour vertex distinguishing total coloring conjecture.

Theorem 3.1.

$$\psi_{nvt}(P_m \Delta P_n) = 8, \qquad m, n \ \geq 4$$

Proof. Let $V(P_m) = \{x_1, x_2, \cdots, x_m\}$ and $V(P_n) = \{y_1, y_2, \cdots, y_n\}$ are the vertex set of the graph P_m and P_n respectively. Define $\phi : V\ (P_m \Delta P_n) \cup \ E(P_m \Delta P_n) \longrightarrow \ \{1,2,\cdots, t\}, t \ \in \mathbb{Z}^+$.

$$V(P_m \Delta P_n) = \{v_{r,s} \,|\, r = 1,2,3, \cdots m, \qquad s = 1,2,3, \cdots, n\}$$

$$E(P_m \Delta P_n) = \left\{\left(\bigcup_{r=1}^{m-1} (v_{r,s}v_{r+1,s+1}) \cup (v_{r+1,s}v_{r,s+1})\right) \cup \left(\bigcup_{r=1}^{m} v_{r,s}v_{r,s+1}\right), \right.$$
$$\left. s = 1,2,3, \cdots, (n-1) \right\}$$

Clearly, $P_m \Delta P_n$ has mn vertices and $(n\ -\ 1)(m\ -\ 2)$ edges.

For $1\ \leq\ r\ \leq\ m,\ 1\ \leq\ s\ \leq\ n,$

$$\phi(v_{r,s}) = \begin{cases} 1 & s \equiv 1(2) \\ 2 & s \equiv 0(2) \end{cases}$$

For $1 \leq r \leq m$,

$$\phi(v_{r,s}v_{r,s+1}) = \begin{cases} 3 & s \equiv 1(2) \\ 4 & s \equiv 0(2) \end{cases} \qquad for\ 1 \leq s \leq n-1$$

For $1 \leq r \leq m-1$ and $1 \leq s \leq n-1$

$$\phi(v_{r,s}v_{r+1,s+1}) = \begin{cases} 5 & r \equiv 1(2) \\ 6 & r \equiv 0(2) \end{cases} \quad and \quad \phi(v_{r+1,s}v_{r,s+1}) = \begin{cases} 7 & r \equiv 1(2) \\ 8 & r \equiv 0(2) \end{cases}$$

The color classes of any two adjacency vertices are different.

The color classes are,

$$C(v_{1,1}) = \{1,3,5\}, \qquad C(v_{1,n}) = \begin{cases} (1,4,7), & n\ is\ odd \\ (2,3,7), & n\ is\ even \end{cases}$$

$$For\ 2 \leq s \leq n-1, \qquad C(v_{1,s}) = \begin{cases} (2,3,4,5,7), & s \equiv 0(2) \\ (1,3,4,5,7), & s \equiv 1(2) \end{cases}$$

$$For\ 2 \leq r \leq m-1, \quad and \quad 2 \leq s \leq n-1, \qquad \overline{C}(v_{r,s}) = \begin{cases} (2) & when\ s \equiv 1(2) \\ (1) & when\ s \equiv 0(2) \end{cases}$$

$For\ 2 \leq r \leq m-1,$

$$C(v_{r,1}) = \begin{cases} (1,3,6,7), & when\ r \equiv 0(2) \\ (1,3,5,8), & when\ r \equiv 1(2) \end{cases} \qquad and \qquad C(v_{m,1}) = \begin{cases} (1,3,7), & when\ m\ is\ even \\ (1,3,8), & when\ m\ is\ odd \end{cases}$$

$$If\ n\ is\ odd, then\ C(v_{r,n}) = \begin{cases} (1,4,5,8), & r \equiv 0(2) \\ (1,4,6,7), & r \equiv 1(2) \end{cases}$$

$For\ 2 \leq r \leq m-1, when\ n\ is\ even$

$$C(v_{r,n}) = \begin{cases} (2,3,5,8), & r \equiv 0(2) \\ (2,3,6,7), & r \equiv 1(2) \end{cases}$$

$$If\ n\ is\ odd, then\ C(v_{m,n}) = \begin{cases} (1,4,5), & m\ is\ even \\ (1,4,6), & m\ is\ odd \end{cases}$$

$$If\ n\ is\ even, then\ C(v_{m,n}) = \begin{cases} (2,3,5), & m\ is\ even \\ (2,3,6), & m\ is\ odd \end{cases} \quad For\ 2 \leq s \leq n-1$$

$$C(v_{r,n}) = \begin{cases} \begin{cases} (2,3,4,5,7), & s \equiv 0(2) \\ (1,3,4,5,7), & s \equiv 1(2) \end{cases} & when\ m\ is\ even \\ \begin{cases} (2,3,4,6,8), & s \equiv 0(2) \\ (1,3,4,6,8), & s \equiv 1(2) \end{cases} & when\ m\ is\ odd \end{cases}$$

It is clear that that the color classes of any to adjacent vertices are different. Hence

$$\psi_{nvt}(P_m \, \Delta \, P_n) = 8, \qquad m, n \geq 4$$

□

Theorem 3.2.

$$\psi_{nvt}(P_m \nabla P_n) = 8, \quad m,n \geq 4$$

Proof. Let $V(P_m) = \{x_1, x_2, \cdots, x_m\}$ and $V(P_n) = \{y_1, y_2, \cdots, y_n\}$ are the vertex set of the graph P_m and P_n respectively. Define $\phi: V(P_m \nabla P_n) \cup E(P_m \nabla P_n) \longrightarrow \{1,2,\cdots,t\}, t \in \mathbb{Z}^+$.

$$V(P_m \nabla P_n) = \{v_{r,s} \mid r = 1,2,3,\cdots m, \quad s = 1,2,3,\cdots,n\}$$

$$E(P_m \nabla P_n) = \left\{\left(\bigcup_{s=1}^{m-1}(v_{r,s}v_{r+1,s+1}) \cup (v_{r+1,s}v_{r,s+1})\right) \cup \left(\bigcup_{s=1}^{m} v_{r,s}v_{r+1,s}\right), \right.$$
$$\left. r = 1,2,3,\cdots,(m-1)\right\}$$

Clearly, $P_m \nabla P_n$ has mn vertices and $(m-1)(n-2)$ edges. When $m,n \geq 3$, we have

For $1 \leq r \leq m,\ 1 \leq s \leq n$, then

$$\phi(v_{r,s}) = \begin{cases} 1 & r \equiv 1(2) \\ 2 & r \equiv 0(2) \end{cases}$$

For $1 \leq r \leq m-1, 1 \leq s \leq n$

$$\phi(v_{r,s}v_{r+1,s}) = \begin{cases} 3 & r \equiv 1(2) \\ 4 & r \equiv 0(2) \end{cases}$$

For $1 \leq r \leq m-1$ *and* $1 \leq s \leq n-1$

$$\phi(v_{r,s}v_{r+1,s+1}) = \begin{cases} 5 & s \equiv 1(2) \\ 6 & s \equiv 0(2) \end{cases} \quad and \quad \phi(v_{r+1,s}v_{r,s+1}) = \begin{cases} 7 & s \equiv 1(2) \\ 8 & s \equiv 0(2) \end{cases}$$

The color classes of any two neighbouring vertices are different.

The color classes are,

$$C(v_{1,1}) = \{1,3,5\}, \qquad for\ 2 \leq s \leq n-1, \quad C(v_{1,s}) = \begin{cases} (1,3,6,7), & s \equiv 1(2) \\ (1,3,5,8), & s \equiv 0(2) \end{cases}$$

$For\ 2 \leq r \leq m-1,$

$$C(v_{r,1}) = \begin{cases} (2,3,4,5,7), & when\ r \equiv 1(2) \\ (1,3,4,5,7), & when\ r \equiv 0(2) \end{cases} \quad and \quad C(v_{m,1}) = \begin{cases} (2,3,7), & when\ m\ is\ even \\ (1,4,7), & when\ m\ is\ odd \end{cases}$$

$$For\ 2 \leq r \leq m-1, \quad and \quad 2 \leq s \leq n-1, \qquad C(v_{r,s}) = \begin{cases} 2,3,4,5,6,7,8 & r \equiv 0(2) \\ 1,3,4,5,6,7,8 & r \equiv 1(2) \end{cases}$$

$For\ 2 \leq s \leq n-1,$

$$C(v_{m,s}) = \begin{cases} \begin{cases} (2,3,5,8), & s \equiv 0(2) \\ (2,3,6,7), & s \equiv 1(2) \end{cases} & when\ m\ is\ even \\ \begin{cases} (1,4,5,8), & s \equiv 0(2) \\ (1,4,6,7), & s \equiv 1(2) \end{cases} & when\ m\ is\ odd \end{cases}$$

$For\ 2 \leq r \leq m-1$

$$C(v_{r,n}) = \begin{cases} \begin{cases} (2,3,4,5,7), & r \equiv 0(2) \\ (1,3,4,5,7), & r \equiv 1(2) \end{cases} & when\ n\ is\ even \\ \begin{cases} (2,3,4,6,8), & r \equiv 0(2) \\ (1,3,4,6,8), & r \equiv 1(2) \end{cases} & when\ n\ is\ odd \end{cases}$$

$$When\ m\ is\ odd, \qquad C(v_{m,n}) = \begin{cases} (1,4,5), & n\ is\ even \\ (1,4,6), & n\ is\ odd \end{cases}$$

$$When\ m\ is\ even, \qquad C(v_{m,n}) = \begin{cases} (2,3,5), & n\ is\ even \\ (2,3,6), & n\ is\ odd \end{cases}$$

It is clear that all the neighbouring vertices have distinct color class. Hence

$$\psi_{nvt}(P_m \nabla P_n) = 8, \qquad m,n \geq 4$$

□

Conclusion.

In this paper, we have proved that the neighbouring vertex distinguishing total chromatic number of comb product of some graph path by path, path by cycle and cycle by path. Also, we proved that the skew and converse skew product of path by path graphs have the same neighbour vertex distinguishing total chromatic number.

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