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An Analytical Solution to Vibration Analysis of cylindrical Shells

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Abstract

More simplified beam models can anticipate the dynamic behavior of a cylindrical shell in a variety of applications. The goal of this work is to identify the design parameters that allow a cylindrical shell to function like a beam. The governing equations for long cylinders with simply-supported boundary conditions at both ends are produced using the Hamilton's principle, along with the analytical solution for both the Flugge and Donnell-Mushtari shell theories. Next, the shell-to-beam transition conditions for both theories are determined by equalizing the vibration frequencies of the shell and the beam. The finite element approach is used to determine the ideal transition conditions with the fewest approximations possible, taking into consideration the effects of shear distortion and the shell's rotatory inertia. Lastly, the frequency response and the transition parameters are examined in relation to boundary conditions. The requirements that were established just state whether or not the shell may be taken to be a beam under particular geometrical and material circumstances.

Keywords: Vibration; shell mode, beam mode, transition condition, design requirement

1.Introduction

One of the most functional constructions in a variety of technical applications, such as storage tanks, pipelines and ducts, and spacecraft, is a cylindrical shell. The cylindrical shells are susceptible to different loading situations during operation, which changes the intricate issue of the shell structure's dynamic response i.e., natural frequency and mode forms which becomes a difficult challenge.

A typical cylinder's mode shapes may be classified into three types based on the circumferential deformations: axisymmetric (breathing), beam, and self-balancing modes as seen in Fig. 1. The circumferential wave number is represented by the quantity "n". A continuous radial shift in circumferential direction is described by the axisymmetric mode (n=0), which implies that the circular cross-sections maintain their circular origin.

In the case when the cylindrical shell is taken to have a circular cross-section beam, its behavior is described by the beam mode (n=1). In another way, only the circle's origin varies while the circular cross-section stays constant in the beam mode.

As seen in Fig. 1, the self-balancing modes occur when the circular cross section's form varies. The beam-like modes are of interest in many practical instances, including low-frequency vibration of pipes in industrial facilities. Additionally, for a variety of applications, more straight forward beam models are able to correctly anticipate the dynamic behavior of the cylindrical shell. These estimates come in particularly handy when dealing with the difficult issue of how a spacecraft responds to dynamic loads.

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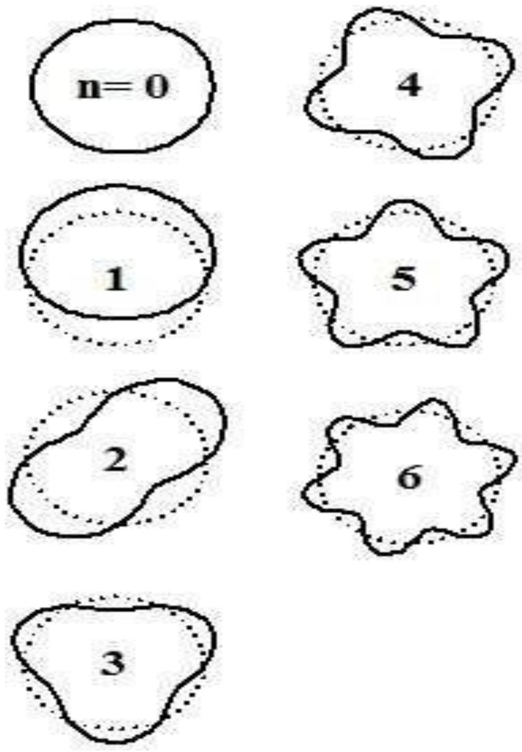

Fig. 1. Shell modes ($n \neq 1$) and beam mode (n=1) for cylindrical shells

Much work has been done in the last few years to accurately anticipate the dynamic behavior of cylindrical shells using several shell theories [1-2]. The complicated (often eighth-order) equations are handled for every shell problem as the shell equations should be permitted for scenarios with many circumferential waves. The minimal natural frequency, however, often corresponds to the axisymmetric and beam-type modes for long and moderately thick cylinders [1], which may be successfully predicted for a broad range of instances by using a simple beam model.

A few studies are trying to use beam approximations to examine the complicated shell equations. Using beam approximations, Forsberg [3] investigated the breathing and beam-type vibrations of cylindrical shells and contrasted the findings with explicit solutions from Flugge's shell equations. The beam functions were utilized by Soedel 4] to approximate the vibration of cylindrical shells, while Farshidianfar et al [5] extracted the modal amplitudes of a long cylindrical shell using the same methodology. Using beam approximation functions, Olizadeh et al [6, 7] investigated the free vibration of simply supported cylindrical shells with four distinct shell theories and reported the impact of various parameters on mode shapes and natural frequencies. The natural frequencies of thin cylindrical shells with radial stresses were studied by Kumar et al. [8].In their study of the beam mode vibration of an infinitely filled cylindrical shell, Sakar and Sonti [9] discovered that at low frequencies, the cylindrical shell in a Timoshenko beam is formed by the beam mode. Vinson [10] investigated the beam-type vibration of cylindrical shells using Green's functions and the Rayleigh-Ritz approach. The stability and beam-mode dynamic properties of cantilevered functionally graded fluid-conveying shells were investigated by Shen et al. [11]. The vibrations of a spinning cylindrical shell with a noncircular cross-section in beam mode were studied by Pavlov and Kuptsov [12]. The axisymmetric (n=0) vibration of an orthotropic cylindrical shell with end disks was investigated by Lopatin and Morozov [13] using clamped-clamped beam approximation functions for axial displacement and deflection. The dynamic behavior of multiwall carbon nanotubes in beam and axisymmetric modes was investigated by Wang et al [14]. In order to discover shell-like modes in the free vibration of thin-walled isotropic and composite structures, Fazzolari [15] devised a Ritz formulation. Using the beam approach, Winfield et al. [16] investigated the vibration of a long, thick laminated conical tube. While the simplified beam models are often employed to examine the shell structures, little is known about the circumstances under which the beam-type modes predominate. Put differently, there is a need to place greater emphasis on the criteria that govern the shift from shell modes (that is, axisymmetric and self-balancing modes) to beam modes. The majority of earlier studies take into account a few particular circumstances and use beam approximations to solve the shell equations. For instance, Blaauwendraad and Hoefakker [17] established the similar process for cylindrical shell static analysis and specified the circumstances under which the shell functions as a beam. A long, thin shell (L/R>10 and R/h>10) was taken into consideration by Forsberg [3], who also discussed the drawbacks of these approximations. To the best of the authors' knowledge, no documented condition exists that specifies the changeover condition between axisymmetric and shell modes to beam modes.

This work proposes a design criterion wherein the circular cylindrical shell exhibits beam-like behavior. Both the linear shell and beam equations are used to get the natural frequencies for this purpose. Both Flugge and Donnell-Mushtari shell theories are used to derive the differential equations, and for long cylinders with simply-supported boundary conditions at both ends, an analytical solution is found. We then derive the basic shell-to-beam transition criteria for both theories. The finite element approach is utilized to discover the optimal transition conditions with the fewest approximation assumptions because the effects of shear distortion and the shell's rotatory inertia have been overlooked in thin shell theories. Lastly, the frequency response and the transition parameters are examined in relation to boundary conditions. The requirements that were established just state whether or not the shell may be taken to be a beam under particular geometrical and material circumstances.

2 An analytical solution to the shell to beam mode transition

2.1 Displacements and strains

Figure 1 depicts a typical isotropic cylindrical shell with coordinates of (x, θ, z), where x, θ, and z stand for longitudinal, circumferential, and radial coordinates, respectively. Consequently, the shell's radius, length, and thickness are represented by the parameters R, L, and h. The symbols u, v, and w represent the deflection of the center surface of the shell along the x, θ, and z axes, respectively. The displacement field (u, v, w) is caused by the Kirchhoff theory to be

$$u(x,\theta,z,t) = u(x,\theta,t) - z\frac{\partial w_o\,(x,\theta,t)}{\partial x}$$

$$v(x,\theta,z,t) = v(x,\theta,t) - \frac{z}{R}\frac{\partial w_o\,(x,\theta,t)}{\partial \theta}$$

$$w(x,\theta,z,t) = w_o(x,\theta,t) \qquad (1)$$

where (u_0, v_0, w_0) represent the mid-plane displacements along x and θ directions, respectively.

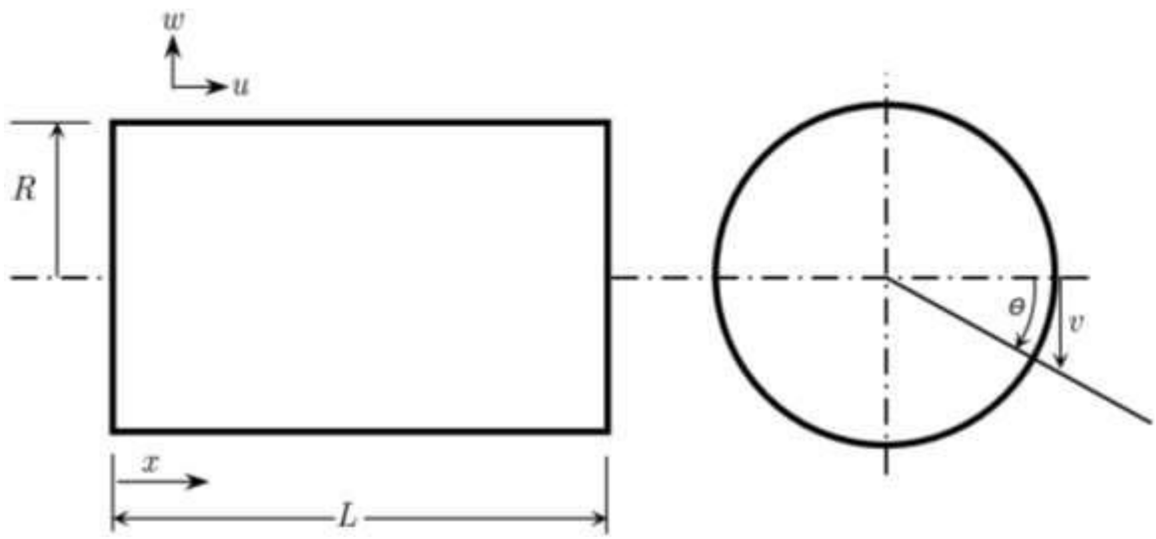

Fig.2.Geometry of a typical cylindrical shell.

Using the thin shell theory of Donnell-Mushtari, the strains can be written as [1]

$$\begin{Bmatrix} e_x \\ e_\theta \\ e_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \epsilon_x \\ \epsilon_\theta \\ \gamma_{x\theta} \end{Bmatrix} + z \begin{Bmatrix} \mathrm{K}_x \\ \mathrm{K}_\theta \\ \mathrm{K}_{x\theta} \end{Bmatrix} \quad (2)$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_\theta \\ \gamma_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right) \\ \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} \end{Bmatrix} \quad , \begin{Bmatrix} \mathrm{K}_x \\ \mathrm{K}_\theta \\ \mathrm{K}_{x\theta} \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2} \\ -\frac{2}{R}\frac{\partial^2 w}{\partial x \partial \theta} \end{Bmatrix} \quad (3)$$

The parameters $(\epsilon_x, \epsilon_{\theta,}\gamma_{x\theta})$ are membrane strains and $(\mathrm{K}_x, \mathrm{K}_{\theta,}\mathrm{K}_{x\theta})$ are the curvatures.

2.2 Constitutive relations

For isotropic materials, the constitutive relations can be shown as

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} e_x \\ e_\theta \\ e_{x\theta} \end{Bmatrix} \quad (4)$$

Where $Q_{11} = Q_{22} = \frac{E}{1-\upsilon^2}, Q_{12} = \frac{\upsilon E}{1-\upsilon^2}, Q_{66} = \frac{E}{2(1+\upsilon)}$ (5)

where E, G and v are Young's modulus, shear modulus and Poisson's ratio, respectively.

2.3 Governing equations for vibration of cylindrical shells

The governing equations of isotropic cylindrical shells are derived by the use of Hamilton'sprinciple as

$$\int_0^T (\delta U + \delta V - \delta K)dt = 0 \quad (6)$$

where δU, δV and δK are virtual strain energy, virtual potential energy and virtual kinetic energy respectively and defined as

$$\delta K = \int \rho \left(\dot{u}_1 \delta \dot{u}_1 + \dot{v}_1 \delta \dot{v}_1 + \dot{w}_1 \delta \dot{w}_1\right) dV$$

$$= \int_A \int_{-h/2}^{h/2} \rho \left[\left(\dot{u} - z\frac{\partial \dot{W}}{\partial x}\right)\left(\delta \dot{u} - z\frac{\partial \delta \dot{w}}{\partial x}\right) + \left[\dot{v} + \frac{1}{R} z \left(\dot{v} cos\alpha - \frac{\partial \dot{w}}{\partial \theta}\right)\right]\left[\delta \dot{v} + \frac{1}{R} z \left(\delta \dot{v} cos\alpha - \frac{\partial \delta \dot{w}}{\partial \theta}\right)\right] + \dot{w}\delta \dot{w} \right] R dx d\theta dz$$

$$= \int_A \left\{ I_o(\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w}) + \frac{1}{2} I_2 \delta \left[\left(\frac{\partial \dot{w}}{\partial x}\right)^2 + \frac{1}{R^2}\left(\dot{v}\, cos\alpha - \frac{\partial \dot{w}}{\partial \theta}\right)^2 \right] \right\} R dx d\theta \quad (7)$$

$$\delta U = \int_V \sigma_{ij} \delta \varepsilon_{ij} dV = \int_A \int_{-h/2}^{h/2} \sigma_{ij} \delta \varepsilon_{ij}\ R ds d\theta\ dz$$

$$= \int_A [N_x \delta \varepsilon_x + M_x \delta K_x + N_\theta \delta \varepsilon_\theta + M_\theta \delta K_\theta + N_{x\theta} \delta \varepsilon_{x\theta} + M_{x\theta} \delta K_{x\theta}]\ R ds d\theta \quad (8)$$

$$\delta V = \int_\Gamma \left[\widehat{N}_x \delta u + \widehat{T}_x \delta v + \widehat{S}_x \delta w + \widehat{M}_x \delta \left(\frac{\partial w}{\partial x}\right) \right] R d\theta \quad (9)$$

where ρ is the density and I_i s are the mass inertias defined as

$$I_i = \int_{-h/2}^{h/2} \rho z^i dz \qquad (i = 0,2) \tag{10}$$

In addition , parameters $\widehat{N}_x, \widehat{T}_x, \widehat{S}_x, \widehat{M}_x$ are stress resultants due to applied axial load, and (N, M) are stress resultants measured per unit length and defined as

$$\begin{bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \\ z\,\sigma_x \\ z\,\sigma_\theta \\ z\,\sigma_{x\theta} \end{bmatrix} dz \tag{11}$$

Substituting Eqs. (3), (4) and (11) into Eqs. (7), (8) and (9), neglecting I_2 due to thin shell assumptions and then imposing all into Eq. (6), we have [1]

$$\delta u: \frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial \theta} = I_0 \frac{\partial^2 u}{\partial t^2}$$

$$\delta v: \frac{1}{R}\frac{\partial N_\theta}{\partial \theta} + \frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R^2}\frac{\partial M_\theta}{\partial \theta} = I_0 \frac{\partial^2 v}{\partial t^2}$$

$$\delta w: -\frac{1}{R} N_\theta + \frac{\partial^2 M_x}{\partial x^2} + \frac{1}{R}\frac{\partial^2 M_{x\theta}}{\partial x \partial \theta} + \frac{1}{R}\frac{\partial^2 M_{x\theta}}{\partial x \partial \theta}\frac{1}{+R^2}\frac{\partial^2 M_\theta}{\partial \theta^2} = I_0 \frac{\partial^2 w}{\partial t^2} \tag{12}$$

2.4 Vibration analysis of long cylindrical shell

In the case that the longitudinal wavelength is long, the solution functions can be consideredas [1]

$$u(x, \theta, t) = U\, cosn\theta e^{i\omega t}$$

$$v(x, \theta, t) = V\, sinn\theta e^{i\omega t}$$

$$w(x, \theta, t) = W\, cosn\theta e^{i\omega t} \tag{13}$$

where (U, V, W) are the amplitudes, n is the circumferential wave number and ω is the frequency of the cylindrical shell. Applying Eq. (13) in governing equations yields a set of algebraic equations as [1]

$$\begin{bmatrix} \frac{1-\upsilon}{2}n^2 - \frac{\rho(1-\upsilon^2)R^2\omega_S{}^2}{E} & 0 & 0 \\ 0 & n^2 - \frac{\rho(1-\upsilon^2)R^2\omega_S{}^2}{E} & n \\ 0 & n & (1+kn^4) - \frac{\rho(1-\upsilon^2)R^2\omega_S{}^2}{E} \end{bmatrix} \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{14}$$

The subscript “s” denotes the frequency formula obtained from shell equations and

$$k = \frac{h_e{}^4}{12} \tag{15}$$

in which h_e is the equivalent thickness, defined as

$$h_e = \sqrt{\frac{h}{R}} \tag{16}$$

It can be seen from Eq. (15) that the motion in axial (longitudinal) direction is completely uncoupled from the other two modes. Therefore, finding the roots of the second orderdeterminant for non-axial modes, arising from Eq. (15) yields the frequencies of the long cylindrical shells with Donnell-Mushtari shell theory as

$$\omega_s{}^2 = \frac{E}{2\rho(1-\upsilon^2)R^2}\left[(1+n^2+kn^4) \mp \sqrt{(1+n^2+kn^4)^2 - 4kn^6}\right] n \neq 0 \quad (17)$$

Following the same procedure, the frequency parameter for circumferential modes of long cylindrical shell according to Flugge shell theory can be extracted as [3]

$$\omega_s{}^2 = \frac{E}{2\rho(1-\upsilon^2)R^2}\left[(1+n^2+kn^4) \mp \sqrt{(1+n^2)^2 - 2kn^6}\right] n \neq 0 \quad (18)$$

2.5 Vibration of the simply-supported beam

The vibration characteristics of an Euler-Bernoulli beam with simply-supported boundary conditions at both ends can be easily found in any reference book (see for example Craig and Kurdila [19] as

$$\omega_b = \left(\frac{m\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}} \quad (19)$$

where m is the longitudinal half wave number and the subscript "*b*" denotes the frequency formula obtained from beam equation, I is the second moment of area and A is the cross-section area. For a thin tube, the Eq. (20) can be represented as

$$\omega_b = R\left(\frac{m\pi}{L}\right)^2 \sqrt{\frac{E}{2\rho}} \quad (20)$$

2.6 Condition of transition from beam modes to shell modes

The transition conditions in which, the vibration of the cylindrical shell converts from shell modes to the beam modes can be easily extracted when the frequencies from shell equations (i.e. either Eq. (18) or Eq. (19)) be equalized with the frequencies obtained from beam solution (Eq. (21)). Considering the beam mode (n=m=1), neglecting higher terms of k and performing some mathematical simplification, the transition conditions can be stated as

$$L_e h_e = C\sqrt[4]{(1-\upsilon^2)} \quad (21)$$

where Le is the equivalent length defined as

$$L_e = \frac{L}{R} \quad (22)$$

and C is the constant depends on the applied shell theory. For Donnel-Mushtari and Flugge shell theories the critical value of the C constant can be extracted as

C_{cr} □ 5.847 Donnell-Mushtari theory

C_{cr} □ 5.284 Flugge theory (23)

The above relation depicts that the shell acts as a beam when the geometrical and material conditions of the shells are in the way that C>C_{cr}, the shell behaves like a beam and vice versa.

3. Finite element analysis for the transition from shell to beam mode

The aim of this section is to find the transition conditions from shell-like modes to beam-like modes using finite element analysis. To this end, the finite element procedure is described first and then, the effects of material properties on the transition condition are studied. Finally, the value of the Ccr is obtained and the results are discussed in detail.

3.1 Finite element analysis

Since the shell theories are simplified with some assumptions, the finite element (FE) analysisis employed to find the best conditions (with less error) for transition from shell to beam modes. A cylindrical shell made from Aluminum (E=70 Gpa and ν=0.3) with different radii and thicknesses is considered. By changing the length, thickness and radius of the shell, the minimum value of the geometrical properties with beam-like mode (i.e. n=1) can be obtained. In the next step, the mechanical properties and boundary conditions are changed and the effects of these parameters on the shell-to-beam transition are studied.

The Finite element analysis is conducted by Ansys software with four nodes shell element 4-node Quadrilateral Shell Element (SHELL 181) that has six degrees of freedom at each node, three translational displacements in the nodaldirections and three rotational displacements about the nodal axes. SHELL181 is a linear element formulation, reduced-integration, and hourglass control. The eigenvalue problem is solved using Block Lanczos method.

Generally, the result of the FE analysis is affected by number of elements. Hence, theconvergence of the analysis results versus the number of total elements are studied. In order to select appropriate mesh size, a compromise between time and number of elements is needed. In other words, by increasing the number of elements, there is no specific change in the solution and indeed the cost of computation can overcome the changes of the solution. The final FE model of the structure is composed of 7560 elements and 7860 nodes.

3.2 Effects of elastic modulus

In this section, the effect of different values of Young moduli and Poisson ratios on the transition condition is investigated. It should be noted that from previous studied, we know thatthe elastic modulus has no effect on the transition conditions. However, to ensure from the correctness and accuracy of the results, the Young modulus is changed and the its effects on the results are studied.

To study the elastic modulus, a cylindrical shell with constant radius R=10mm and thickness h=1mm with ρ=2700Kg/m^3 and ν=0.3 is considered. The values of elastic modulus are varied from 3 to 300 GPa and the minimum value of the shell length, in which, the beam-like mode (n=1) occurs is obtained. The results are shown in Table 1. It can be seen that the elastic modulus has no effect on transition condition.

Table 1. Effects of Young's modulus on equivalent length and thickness.

***E* (GPa)**	3	70	100	150	180	210	240	300
he	0.3162	0.3162	0.3162	0.3162	0.3162	0.3162	0.3162	0.3162
Le	14	14	14	14	14	14	14	14

3.3 Effects of Poisson's ratio

From previous studies, it can be concluded that the shell-to-beam mode transition condition isproportional with the forth root of the (1-ν^2), as seen in Eq. (21). However, to ensure from the Poisson's ratio effects, the above-mentioned aluminum cylinder (i.e. R=10mm, h=1mm, ρ=2700Kg/m^3 and E=70GPa) is considered. The values of Poisson's ratio are varied from 0.18to 0.4 and the minimum length in which the beam mode occurs is obtained.

Table (2) Effects of Poisson's ratio on equivalent length and thickness of transition condition from shell to beam modes.

ν	0.18	0.22	0.26	0.3	0.35	0.4
he	0.3162	0.3162	0.3162	0.3162	0.3162	0.3162
Le	14	14	14	13	13	13

Table 2 shows the values of equivalent length and equivalent thickness for different values ofPoisson's ratio. It can be concluded that

$$L_e h_e \propto \sqrt[4]{(1-\nu)^2} \quad (24)$$

4.Shell-to-beam transition condition

The FE analysis was performed for a wide range of cylindrical shells with 0.001<R<1 m and 0.001<h/R<0.1. In each specific R and h, the values of the shell length are changed and the minimum value at which the beam-like mode is occurred is obtained. The results of FE analysis for all the geometries, along with Donnel-Mushtari and Flugge results (Eq. 22) are plotted in the Le-he space, as shown in Fig. 3. It can be seen that, due to the simplifications in Donnel- Mushtari and Flugge theories, the results of these theories are a bit different with FE results. Inother words, the shell-to-beam transition condition is a more than the present results.

To find the best values for transition condition, the best curve fitted to the FE results is obtained.The curve like the Eq. (23) is applied and the C_{cr} is obtained as

$$C_{cr} = 4.49 \quad (25)$$

Equation (25) denotes the transition condition from shell-like modes to beam-like modesobtained from FE analysis. In other words, if the geometrical and material properties of theshell are in the way that the value $\sqrt[4]{(1-\nu^2)}$ is more than C_{cr}, the shell can be assumed as a beam.

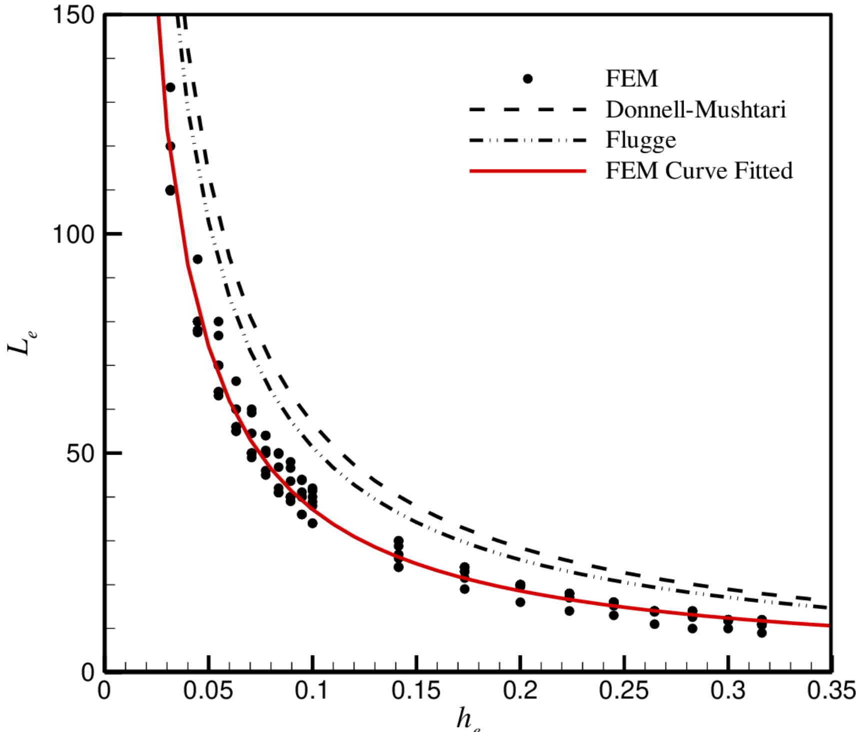

Fig. 3: transition condition from shell to beam modes, obtained from Donnel-Mushtari, Flugge and FEM.

5.Effect of boundary conditions

In this section, the effects of different boundary conditions on the transition conditions of cylindrical shell is studied. Three boundary conditions, including free-free (F-F), simply supported-simply supported (SS), and clamped-clamped (CC) are considered and the results are shown in Fig. 4. As can be seen, the results of different boundary conditions are very closeto each other. As a result, the effects of boundary conditions on the transition condition from shell-like modes to beam-like modes are negligible. It was not unexpected, because the beam

mode usually occurs for relatively long cylinders and the effects of boundary conditions on thedynamic behavior of the shells decrease with increase in the shell length.

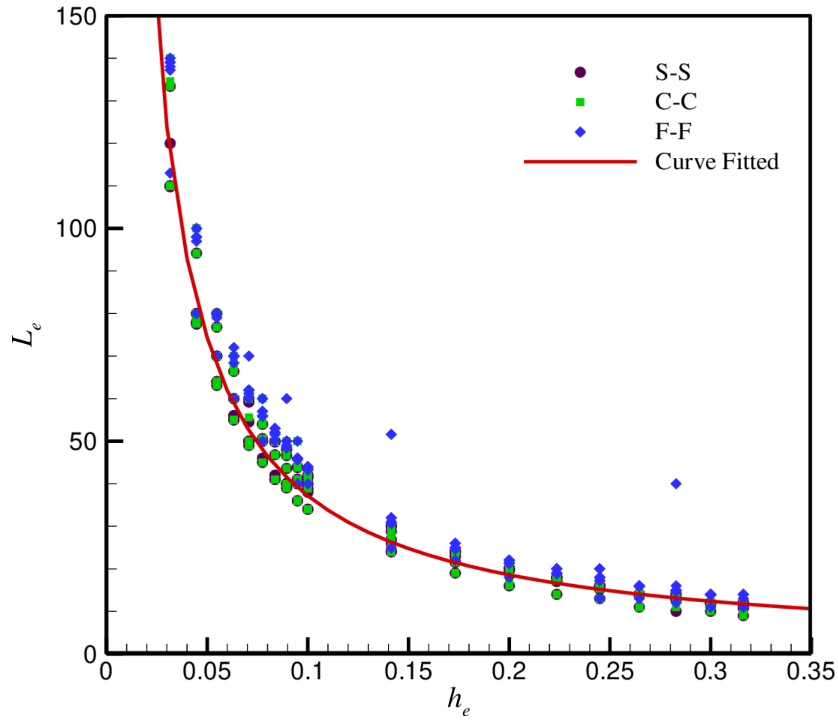

Fig. 4. Effects of boundary conditions on shell-to-beam transition condition.

6.Conclusion

This study presents a transition condition from shell-like modes (axisymmetric and self-balancing modes) to beam-like modes. If the cylindrical shell's geometrical and material qualities match this criteria, it acts like a beam and may be described using beam governing equations. This is a significant development since the dynamic behavior of the beam may be easily examined using simplified governing equations. The governing equations are derived using the Donnell-Mushtari and Flugge shell theories, and the transition condition is attained by equating the natural frequencies obtained from the shell and beam theories. The simple shell-to-beam transition conditions are then established, and the more correct connection is recovered using finite element analysis, taking into consideration the effects of shear distortion and rotatory inertias. n addition, the impact of boundary conditions on transition parameters is investigated. The obtained transition relation merely describes the circumstance under which the shell can be regarded to act as a beam.

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