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Distributed Filtering for Discrete Time Varying System Using Maximum Error First Protocol

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Abstract:- The problem of distributed filtering is designed for discrete time varying system considering sensor network with certain topological structure. Generally, the topology of sensor network is sparse in nature and the information are quite difficult to collect. In this paper, the sensor network is considered as discrete time varying system and the distributed recursive filtering problem is addressed. Stochastic nonlinearities are introduced into the system with gaussian inputs. Communication burden can be reduced by introducing Maximum Error First protocol (MEF) and also it saves the communication resource. The optimal distributed filter is designed with minimum variance for the considered discrete time varying stochastic nonlinear system. An upper bound of the error covariance matrix is arrived in terms of solving the Riccati type difference equation. The filter gain is derived in virtue of minimizing the upper bound of filtering error covariance. To deal with the sparsity of the sensor network a new matrix simplification technique is used. Results are derived by considering some of the sample values of time varying matrixes and nonlinear functions are simulated for the proposal and outputs are plotted in graph.

Keywords*:* Discrete Time varying Systems, Stochastic nonlinearities, Maximum Error Protocol and Gaussian Noise

1. Introduction

Recent advancement in the field of engineering and technology have made it easy to address the problem in developing and deploying complex communication network. Wired and wireless communication network collects and process large quantities and wide range of physical variable. The sensor and monitoring network have been incorporated in many filed such as, smart home; robotics; industrial monitoring; environmental and earth sensing [1-4]. Rapid development insensors plays a vital role in advancement of communication technology. They are used to measure/detect the physical variables such as pressure, temperature, and motion etc., also sensors are having the capability to communicate. Also, they are the important component in the field of Internet of Things (IoT) for developing future world products. Numerous sensors can be connected in a topology to form a sensor network [2-6]. Sensor networks can be wired or wireless;In common, wired sensor networks are using ethernet cables or fiber optic cable (OFC) for communication. Whereas wireless sensor network (WSN) uses communication technologies like Bluetooth, Radio Frequency, Infrared, General Pocket Radio Service (GPRS), Near Frequency Communication (NFC) and Wi-Fi to connect the sensors.

2. Problem Formulation

For addressing the problem let consider the discrete time-varying system as follows:

$$x_{n+1} = A_n x_n + \gamma_n f(x_n, n) + B_n \omega_n + h(x_{n-\tau}) \quad (1)$$

$$y_{i,n} = C_{i,n} x_n + V_{i,n} \quad (2)$$

Let,

$G = (V, E, H)$, a fixed discrete graph

$V = \{1,2, \dots, n\}$ a set of nodes

$E \in V \times V$, the set of edges

$H = [h_{ij}]$ the weighted adjacency matrix and each weighted element satisfies the property $h_{ij} > 0, \forall_{ij} \in E$

$N_i \triangleq \{j \in V/(i,j) \in E\}$ represents the neighbors of the node I plus the node itself

$x_n \in R^{N_x}$ represents the state vector

$y_{i,n} \in R^{N_y}$ represents the measurement output

$\omega_n \in R^{N_\omega}$ represents the process noise

$V_{i,n} \in R^{N_y}$ represents the measurement noise

A_n, B_n *and* $C_{i,n}$ are known time varying matrices with appropriate dimensions

$f(x_n, n) \in R^{N_x}$ is the nonlinear function

$E[\omega_n] = 0, \quad E[V_{i,n}] = 0, \quad E[W_n W_j^T] = \delta_n \delta_{nj}$

$E[V_{i,n}, V_{i,j}^T] = Q_{in}\delta_{nj}, \quad E[\omega_n, V_{i,j}^T] = R_{in}\delta_{nj}$

Where Q_{in}, R_{in} and δ_n are known positive definite matrices with appropriate dimensions

γ_n is a random variable and satisfies Bernoulli distribution with

$P(\gamma_n = 1) = \overline{\gamma_n}, \quad P(\gamma_n = 0) = 1 - \overline{\gamma_n}$

Where $\overline{\gamma_n}$ is a known constant and $\overline{\gamma_n} = [0,1]$

3. Maximum Error First Protocol (MEF)

If the system allows more than one node information transmission, the system may happen collision inevitably. In this case, the MEF protocol is employed to choose next instants node.

At each time instant n, the absolute error of sensor node I can be defined as follows

$$e_{i,n} = y_{i,n} - y_{i,n}^*, \quad i = 1,2, \dots n$$

Where $y_{i,n}^*$represents the latest transmitted signal. Until the next update instant $y_{i,n}^*$ will be held.

At time n, we choose the sensor node i at the next time by relying on the following formulae

$$l_n = min\left\{\arg \max_{1 \le i \le N} |e_{i,n}|\right\}$$

Where $l_n \in \{1,2, \dots N\}$ it means that the node i with the greatest absolute error chosen.

Let $y_n = [y_{1,n}^T, \dots y_{N,k}^T]^T$represent the received signal by remote filter after the MEF protocol. Clearly $\bar{y}_n = [\bar{y}_{1,n}, \dots \bar{y}_{N,n}]^T$, where $y_n \neq \bar{y}_n$ at each sampling instant n, the relationship can be expressed as

$$y_{i,n} = \begin{cases} \bar{y}_{1,n}, & i = l_n \\ \bar{y}_{i,n-1}, & otherwise \end{cases}$$

The update function cab be defined as

$\wedge l_n = diag\{\delta(1 - l_n), \dots, \delta(N - l_n)\}$ (3)

We define $y_{i,n}$

$y_{i,n} = \delta(i - l_n)\bar{y}_{1,n} + [1 - \delta(1 - l_n)]y_{i,n-1}$ (4)

Assumption1:

The nonlinear function $f(x_n, n)$ is continuous and satisfies sector bounded conditions in the following

$[f(x_n, n) - \Gamma_1\bar{x}_n]^T[f(x_n, n) - \Gamma_2\bar{x}_n] \le 0$ (5)

Where Γ_1 *and* $\Gamma_2$are real-valued matrices with appropriate dimensions and $\Gamma_1 - \Gamma_2 > 0$

Define

$f(\bar{x}_n, n) = \begin{bmatrix} \bar{f}(\bar{x}_n, n) \\ 0 \end{bmatrix}$, $B_{i,n} = \begin{bmatrix} \bar{B}_n & 0 \\ 0 & \delta(i - l_n)I \end{bmatrix}$

$$A_{i,n} = \begin{bmatrix} \bar{A}_n & 0 \\ \delta(i - l_n)\bar{C}_{i,n} & 1 - \delta(i - l_n)I \end{bmatrix}$$

$X_{i,n} = \begin{bmatrix} x_n \\ y_{i,n-1} \end{bmatrix}$, $\tilde{\gamma}_n = \begin{bmatrix} \gamma_n I & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{\omega}_{i,n} = \begin{bmatrix} \bar{\omega}_n \\ \bar{V}_{i,n} \end{bmatrix}$

$$C_{i,n} = \left[\delta(i - l_n)\bar{C}_{i,n}\left(1 - \delta(i - l_n)\right)\mathrm{I}\right]$$

Then we have the new forms of systems with MEF protocol

$X_{i,n+1} = A_{i,n}X_{i,n} + \tilde{\gamma}_n f(\bar{x}_n, n) + B_{i,n}\tilde{\omega}_{i,n}$ (6)

$\bar{y}_{i,n} = C_{i,n}X_{i,n} + \delta(i - l_n) + \bar{V}_{i,n}$ (7)

We design the distributed filter with MEF protocol

$\hat{x}_{i,n+1/n} = A_{i,n}\hat{x}_{i,n/n} + \tilde{\gamma}_n f\left(\hat{x}_{i,n/n}, n\right)$ (8)

$\hat{x}_{i,n+1/n+1} = \hat{x}_{i,n+1/n} + \sum_{j\in N_i} h_{ij,n+1}\, G_{ij,n+1} \times \left(y_{j,n+1} - C_{j,n+1}\hat{x}_{j,n+1/n}\right)$ (9)

Where $\hat{x}_{i,n+1/n}$ denotes the one-step prediction, $\hat{x}_{i,n+1/n+1}$ denotes the filter, and $G_{ij,n+1}$ denotes the filter gain.

In the following, let the one-step prediction error and the filtering error be

$$e_{i,n+1/n} = A_{i,n}e_{i,n/n} + \bar{\tilde{\gamma}}_n\left(f(\bar{x}_n, n) - f\left(\hat{x}_{i,n/n}, n\right)\right) + (\tilde{\gamma}_n - \bar{\tilde{\gamma}}_n)f(\bar{x}_n, n) + B_{i,n}\tilde{\omega}_{i,n}$$

$$e_{i,n+1/n+1} = e_{i,n+1/n} - \sum_{j\in N_i} h_{ij,n+1}\, G_{ij,n+1}\{C_{j,n+1} \times e_{j,n+1/n} + \delta(j - l_{n+1})\bar{V}_{j,n+1}\}$$

We introduce the following notations

$$E_i \triangleq diag\left\{\underbrace{0, \dots, 0}_{i-1}, \mathrm{I}, \underbrace{0, \dots, 0}_{N-i}\right\}$$

$\bar{\gamma}_n \triangleq diag_N\{\bar{\tilde{\gamma}}_n\}$, $X_{n+1} \triangleq Col_N\{X_{i,n+1}\}$,$\gamma_n \triangleq diag_N\{\tilde{\gamma}_n\}$

$A_n \triangleq diag_N\{A_{i,n}\}$,$B_n \triangleq diag_N\{B_{i,n}\}$,$C_n \triangleq diag_N\{C_{i,n}\}$

$f_n \triangleq Col_N\{f(\bar{x}_n, n)\}$,$e_{n+1/n+1} \triangleq Col_N\{e_{i,n+1/n+1}\}$

$\hat{f}_n \triangleq Col_N\{f\left(\hat{x}_{i,n/n}, n\right)\}$,$e_{n+1/n} \triangleq Col_N\{e_{i,n+1/n}\}$

$H_i \triangleq diag\{h_i, \mathrm{I}, \dots,\ h_{iN}, \mathrm{I}\}$,$G_n \triangleq \{G_{ij,n}\}_{N\times N}$

$\omega_n \triangleq Col_N\{\tilde{\omega}_{i,n}\}$,$V_n \triangleq \{\bar{V}_{i,n}\}$,$\bar{y}_n \triangleq Col_N\{\bar{y}_{i,n}\}$

From the above notations, the one-step prediction error and the filtering error can be rewritten

$e_{n+1/n} = A_n e_{n/n} + \bar{\gamma}_n\left(f_n - \hat{f}_n\right) + (\gamma_n - \bar{\gamma}_n)f_n + B_n\omega_n + h(x_n - \tau)$ (10)

$$e_{n+1/n+1} = (I - \sum_{i=1}^{N} E_i G_{n+1} H_i C_{n+1}) e_{n+1/n} - \sum_{i=1}^{n} E_i G_{n+1} H_i [\Omega(l_{n+1}) \otimes \mathrm{I}]\, \gamma_{n+1} \qquad (11)$$

Lemma 1

Given vector $F, V_1, V_2 \in R^n$ if $(F - V_1)^T (F - V_2) \leq 0$

Holds, there exists a constant $\ominus \leq (0,1)$ such that $\|F\|^2 \leq \frac{\ominus - \ominus^{-1}}{2(1-\ominus)} \|V_1\|^2 + \frac{\ominus^{-1}}{(1-\ominus)} \|V_2\|^2$ (12)

Lemma 2

Let $A = [\alpha_{ij}]_{m\times m}$ be real-valued matrix,

$B = \{\beta_1, \beta_2, \dots, \beta_m\}$ be a diagonal random matrix, thus we have

$$E\{BAB^T\} = \begin{bmatrix} E\{\beta_1^2\} & E\{\beta_1\beta_2\} & \dots E\{\beta_1\beta_m\} \\ E\{\beta_2\beta_1\} & E\{\beta_2^2\} & \dots E\{\beta_2\beta_m\} \\ \vdots & \vdots & \vdots \quad \vdots \\ E\{\beta_m\beta_1\} & E\{\beta_m\beta_2\} & \cdots\ E\{\beta_1\beta_2\} \end{bmatrix} o \qquad (13)$$

Where 'o' represents the Hadamard product

Lemma 3

Let U, V, W *and* X be compatible dimensions matrices thus the following equations hold

$$\frac{\partial tr(VX^T)}{\partial X} = U\,, \qquad \frac{\partial tr(XU)}{\partial X} = U^T$$

$$\frac{\partial tr(UXV)}{\partial X} = U^T V^T\,, \qquad \frac{\partial tr(UX^T V)}{\partial X} = VU$$

$$\frac{\partial tr(UXVX^T W)}{\partial X} = U^T W^T X V^T + WUXV$$

Furthermore, for any symmetric matrix P, the following equation is true

$$\frac{\partial tr\big((UXV)P(UXV)^T\big)}{\partial X} = 2U^T U\, X\, VPV^T$$

We introduce some notations as follows

$$\psi_n \triangleq \{(\gamma_n - \bar{\gamma}_n)(\gamma_n - \bar{\gamma}_n)^T\} \qquad (14)$$

$$= \ diag_N \left\{ diag\{\bar{\bar{\gamma}}_n(1 - \bar{\bar{\gamma}}_n)\mathrm{I}, 0, \bar{\bar{\gamma}}_n(1 - \bar{\bar{\gamma}}_n)\mathrm{I}, 0\} \right\}$$

$$\widetilde{W}_{i,n} \triangleq E\{\widetilde{\omega}_{i,n} \widetilde{\omega}_{i,n}{}^T\} = \begin{bmatrix} S_n & R_{i,n} \\ R_{i,n}{}^T & Q_{i,n} \end{bmatrix} \qquad (15)$$

$$W_n \triangleq E\{\omega_n \omega_n{}^T\} = \ (\widetilde{W}_{i,n})_{N\times N} \qquad (16)$$

$$V_n \triangleq E\{V_n V_n{}^T\} = \ diag_N\{Q_{i,n}\} \qquad (17)$$

Main Result

In this section, our aim is to design the optimal filter gain $G_{ij,n+1}$. Firstly, the upper bound of the stat covariance is shown below

Lemma 4

The stat covariance matrix $X_{n+1} = E[x_{n+1} x_{nn}{}^T]$ satisfies the following inequality

$$X_{n+1} \leq \phi_{n+1} \qquad (18)$$

Where,

$$\rho_1 = \frac{\ominus + \ominus^{-1}}{2(1 - \ominus_1)}, \qquad \rho_2 = \frac{\ominus^{-1}}{(1 - \ominus_1)}$$

$$\phi_{n+1} = (1+\gamma_1)A_n\phi_n A_n{}^T + B_n S_n B_n{}^T + (1+\gamma_1{}^{-1})\bar{\gamma}\rho_1 \times tr\{\Gamma_1\phi_n\Gamma_1{}^T\}\mathrm{I} + (1+\gamma_1{}^{-1})\bar{\gamma}\rho_2 tr\{\Gamma_2\phi_n\Gamma_2{}^T\}\mathrm{I} \quad (19)$$

Now we can further obtain the upper bound of one-step prediction error covariance and filtering error covariance.

Theorem 1:

For systems (3) and (4), the one-step prediction error covariance $\rho_{n+1/n}$ and the filtering error covariance $\rho_{n+1/n+1}$ satisfy the following formulae

$$\rho_{n+1/n} \leq \Omega_{n+1/n}, \quad (20)$$

$$\rho_{n+1/n+1} \leq \Omega_{n+1/n+1} \quad (21)$$

Where

$$\Omega_{n+1/n} = (1+\epsilon_1+\epsilon_2)A_n\Omega_{n/n}A_n{}^T + (1+\epsilon_2{}^{-1}) \times B_n W_n B_n{}^T + N\tau_1\gamma_n\{\rho_1 tr(\Gamma_1\phi_n\Gamma_1{}^T)\mathrm{I} + \rho_2 tr(\Gamma_2\phi_n\Gamma_2{}^T)\mathrm{I}\}\gamma_n{}^T + \tau_2\gamma_n\hat{\mathrm{F}}_n\hat{\mathrm{F}}_n{}^T\gamma_n{}^T + \psi_n o\left\{N\{\rho_1 tr(\Gamma_1\psi_n\Gamma_1{}^T)\mathrm{I} + \rho_2 tr(\Gamma_2\psi_n\Gamma_2{}^T)\mathrm{I}\}\right\} \quad (23)$$

$$\Omega_{n+1/n+1} = \left(\mathrm{I} - \sum_{i=1}^{N} E_i\, G_{n+1}H_iC_{n+1}\right) \Omega_{n+1/n}\left(\mathrm{I} - \sum_{i=1}^{N} E_i\, G_{n+1}H_iC_{n+1}\right)^T + \left(\sum_{i=1}^{N} E_i\, G_{n+1}H_i\right)[\Omega(l_{n+1})\otimes I]V_{n+1} [\Omega(l_{n+1})\otimes I]^T(\textstyle\sum_{i=1}^{N} E_i\, G_{n+1}H_i)^T \quad (24)$$

With,

$$\rho_1 \triangleq \frac{\ominus_1+\ominus_1{}^{-1}}{2(1-\ominus_1)}, \qquad \rho_2 = \frac{\ominus_1{}^{-1}}{(1-\ominus_1)}, \ominus \in (0,1)$$

$$\tau_1 \triangleq (1+\epsilon_1{}^{-1})(1+\epsilon_3)$$

$$\tau_2 \triangleq (1+\epsilon_1{}^{-1})(1+\epsilon_3{}^{-1})$$

Where ϵ_1,ϵ_2 and ϵ_3 are arbitrary positive scalar

Proof: using $\mathrm{P}_{n+1/n} = \mathrm{E}[\mathrm{e}_{n+1/n}\mathrm{e}_{n+1/n}{}^T]$, we have

$$\mathrm{P}_{n+1/n} \leq (1+\epsilon_1+\epsilon_2)A_n\mathrm{P}_{n/n}A_n{}^T + (1+\epsilon_1{}^{-1})\gamma_n \mathrm{E}\left\{(\mathrm{F}_n - \hat{\mathrm{F}}_n)(\mathrm{F}_n - \hat{\mathrm{F}}_n)^T\right\}\gamma_n{}^T + \mathrm{E}\{(\gamma_n - \bar{\gamma}_n)\mathrm{F}_n\mathrm{F}_n{}^T \times (\gamma_n - \bar{\gamma}_n)^T\} + (1+\epsilon_2{}^{-1})B_n W_n B_n{}^T \quad (25)$$

Applying the inequality (5) and lemma 1, we have

$$\bar{\gamma}_n\mathrm{E}\left\{(\mathrm{F}_k - \hat{\mathrm{F}}_k)(\mathrm{F}_k - \hat{\mathrm{F}}_k)^T\right\}\bar{\gamma}_n{}^T \leq N(1+\epsilon_3{}^{-1})\bar{\gamma}_n\{\rho_1 tr(\Gamma_1 X_n\Gamma_1{}^T)\mathrm{I} + \rho_2 tr(\Gamma_2 X_n\Gamma_2{}^T)\mathrm{I}\}\bar{\gamma}_n{}^T + (1+\epsilon_3{}^{-1})\gamma_n\hat{\mathrm{F}}_n\mathrm{F}_n{}^T\gamma_n{}^T \quad (26)$$

According inequality (5), lemma 1 and lemma 2, we have

$$\mathrm{E}\{(\gamma_n - \bar{\gamma}_n)\mathrm{F}_n\mathrm{F}_n{}^T(\gamma_n - \bar{\gamma}_n)^T\} = \psi_n o\mathrm{E}[\mathrm{F}_n\mathrm{F}_n{}^T] < \psi_n o\{N\{\rho_1 tr(\Gamma_1 X_n\Gamma_1{}^T)\mathrm{I} + \rho_2 tr(\Gamma_2 X_n\Gamma_2{}^T)\}\mathrm{I}\} \quad (27)$$

Based on (25), (26) and (27) we obtain the following formula

$$\mathrm{P}_{n+1/n} \leq (1+\epsilon_1+\epsilon_2)A_n\mathrm{P}_{n/n}A_n{}^T + (1+\epsilon_2{}^{-1})B_nW_nB_n{}^T + N\tau_1\bar{\gamma}_n\{\rho_1 tr(\Gamma_1X_n\Gamma_1{}^T)\mathrm{I} + \rho_2 tr(\Gamma_2X_n\Gamma_2{}^T)\mathrm{I}\} \times \gamma_n{}^T + \tau_2\gamma_n\hat{\mathrm{F}}_k\hat{\mathrm{F}}_k{}^T\gamma_n{}^T + \psi_k o\left\{N\{\rho_1 tr(\Gamma_1X_n\Gamma_1{}^T)\mathrm{I} + \rho_2 tr(\Gamma_2X_n\Gamma_2{}^T)\mathrm{I}\}\right\}$$

$$\leq \Omega_{n+1/n}$$

Similarly, the upper bound of the filter error covariance $\Omega_{n+1/n+1}$ is obtained

We introduce some notations

$\mathrm{M}_{n+1} \triangleq \Omega_{n+1/n}\mathrm{C}_{n+1}{}^T, \mathrm{M}_{n+1} \triangleq \left[\mathrm{M}_{n+1}^{(i)}\right]_{\mathrm{N}\times 1}$

$\mathrm{G}_{n+1}^{(i)} \triangleq \left[G_{ij,n+1}\right]_{1\times\mathrm{N}}, \mathrm{G}_{n+1}^{(i)} \triangleq \left[G_{ij,n+1}\right]_{1\times\mathrm{N}}$

$G_{n+1} \triangleq \left[\mathrm{G}_{n+1}^{(i)}\right]_{\mathrm{N}\times 1}, \mathrm{O}_i \triangleq diag_N\left\{\sqrt{h_{ij}\mathrm{I}}\right\}$

$\mathrm{G}_{n+1}^{(i)} \triangleq \mathrm{M}_{n+1}^{(i)}\bar{\mathrm{O}}_i\left(\bar{\mathrm{O}}_i{}^T S_{n+1}\bar{\mathrm{O}}_i\right)^{-1}\bar{\mathrm{O}}_i{}^T$

$S_{n+1} \triangleq \mathrm{C}_{n+1}\Omega_{n+1/n}\mathrm{C}_{n+1}{}^T + [\Omega(l_{n+1})\otimes\mathrm{I}]V_{n+1} \times [\Omega(l_{n+1})\otimes\mathrm{I}]^T$ (28)

Nothing that, $j \notin \mathbb{N}_i(h_{ij}=0)$ will cause corresponding column matrix of O_i to be zero matrix. Let $\bar{\mathrm{O}}_i$ represent the simplified matrix of O_i and $\bar{\mathrm{O}}_i$ can gotten by removing zero column of matrix O_i. $\mathrm{M}_{n+1}^{(i)}$ is the i^{th} row sub-matrix of M_{n+1} and $\mathrm{G}_{n+1}^{(i)}$ is the i^{th} row sub-matrix of G_{n+1}

Theorem 2:

Considering the system (6) and (7) and distributed filter (8) and (9), the upper bound of the filtering error covariance can be minimized by choosing filter gain as

$G_{ij,n+1} = \begin{cases} 0, & j \notin \mathbb{N}_i \\ \bar{G}_{ij,n+1}\bar{h}_{ij}, & j \notin \mathbb{N}_i \end{cases}$ (29)

Proof

According to (23) and taking the trace for the both sides, we obtain partial derivation $tr\{\Omega_{n+1/n+1}\}$ then taking the partial derivation of $tr\{\Omega_{n+1/n+1}\}$with respect to G_{n+1}, we have

$\circledast\{\Omega_{n+1/n+1}\} = -2\sum_{i=1}^{N} E_i\,\Omega_{n+1/n}C_{n+1}{}^T H_i + 2\sum_{i=1}^{n} E_i\,G_{n+1}H_iC_{n+1} \times \Omega_{n+1/n}C_{n+1}{}^T H_i + 2\sum_{i=1}^{n} E_i\,G_{n+1}H_i[\Omega(l_{n+1})\otimes\mathrm{I}] \times V_{n+1} \times [\Omega(l_{n+1})\otimes\mathrm{I}]^T H_i$ (30)

Formula is a sufficient condition to ensure that formula equals to zero

$\mathrm{G}_{n+1}^{(i)}H_iS_{n+1}H_i = \mathrm{M}_{n+1}^{(i)}H_i, i = 1,2,\ldots,N$ (31)

Nothing $H_i = \bar{\mathrm{O}}_i\bar{\mathrm{O}}_i{}^T (i = 1,2,\ldots,N)$, we have

$\mathrm{G}_{n+1}^{(i)}\bar{\mathrm{O}}_i\bar{\mathrm{O}}_i{}^T S_{n+1}\bar{\mathrm{O}}_i\bar{\mathrm{O}}_i{}^T = \mathrm{M}_{n+1}^{(i)}\bar{\mathrm{O}}_i\bar{\mathrm{O}}_i{}^T$ (32)

From (32), one has

$\mathrm{G}_{n+1}^{(i)}\bar{\mathrm{O}}_i\bar{\mathrm{O}}_i{}^T S_{n+1}\bar{\mathrm{O}}_i = \mathrm{M}_{n+1}^{(i)}\bar{\mathrm{O}}_i, i = 1,2,\ldots,N$ (33)

$\mathrm{G}_{n+1}^{(i)}\bar{\mathrm{O}}_i = \mathrm{M}_{n+1}^{(i)}\bar{\mathrm{O}}_i\left(\bar{\mathrm{O}}_i{}^T S_{n+1}\bar{\mathrm{O}}_i\right)^{-1}$ (34)

Using (28) and (32), we arrive at

$\bar{\mathrm{G}}_{n+1}^{(i)} = \mathrm{G}_{n+1}^{(i)}H_i$ (35)

Thus, the filter gain $G_{ij,n+1}$ can be obtained the proof of theorem 2 is finished.

4. Example

In this section, an example is considered for illustration to show the result of the proposed filter model. According to time varying system (1), considering the following system function:

$$A_k = \begin{bmatrix} -0.36 + 0.01\sin(k) & -0.21 + 0.02e^{-5k} \\ 0.02\cos(k) & -0.275 \end{bmatrix}$$

$B_k = \begin{bmatrix} 0.15 \\ 0.25 \end{bmatrix}$, $\quad D_k = 0.02$

The stochastic nonlinear function $f(x_k, \eta_k)$ can be selected as follows

$$f(x_k, \eta_k) = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}(0.3\,\mathrm{sign}(x_k^1)x_k^1\eta_k^1 + 0.4\,\mathrm{sign}(x_k^2)x_k^2\eta_k^2)$$

Which satisfies the condition (3) and (4) with $r = 1$ and

$$\Sigma_k^1 = \begin{bmatrix} 0.03 & 0.05 \\ 0.06 & 0.08 \end{bmatrix}, \qquad \Pi_k^1 = \begin{bmatrix} 0.08 & 0.00 \\ 0.00 & 0.15 \end{bmatrix}$$

Where $x_k = [x_k^1 \quad x_k^2]^T$

The deterministic nonlinear function $g(x_k)$ and $h(x_k)$ are chosen as

$$h(x_k) = 0.8\sin(x_k^2)$$
$$g(x_k) = 0.8\cos(x_k^1)$$

And satisfy the condition (2) with $a_1 = 0.8$ and $a_2 = 0$.

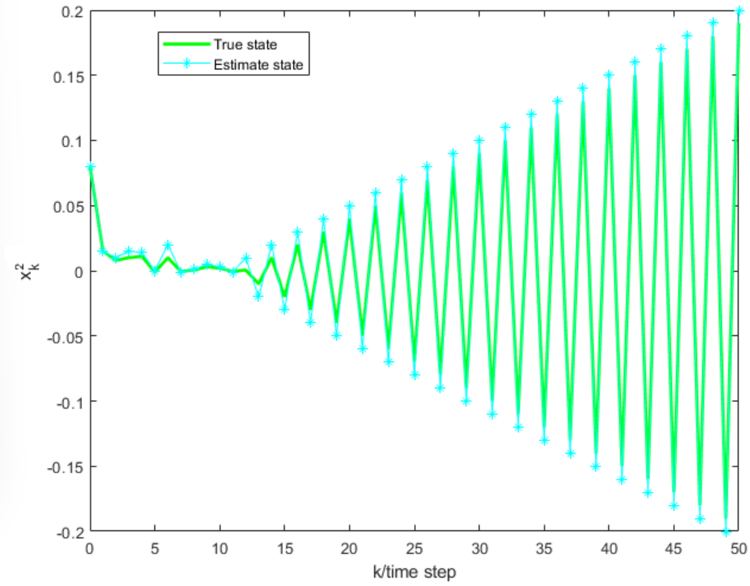

Fig. 1 Actual (True) state and estimated state

In the simulations, let the time delay step $\tau = 1$. The initial conditions are set as $x_0 = x_{-1} = [0.085 \quad 0.9]^T$, $\dot{x}_{0|0} = \dot{x}_{-1|-1} = [0.086 \quad 0.91]^T$ and $\dot{X}_{0|0} = X_{-T|-T} = \cdots = 0.01diag\{1,1\}$.

Other parameters are chosen as

$P_{1,k} = 0.1, P_{2,k} = 0.14,$

$$Q_{1,k} = 0.1, \qquad Q_{2,k} = 0.1,$$
$$\delta_1 = \delta_2 = 0.001, \epsilon_i = 0.35\ (i = 1, 2, \dots, 5.).$$

The attack signal and its upper bound are set as $\xi_k = 0.1\sin(k)$ and $\bar{\xi} = 0.11$. The expectation of the random variable ψ_k is set as $\bar{\psi} = 0.85$. Set the parameter of the measurement and process noises $M_k = N_k = 1$. Even though, let MSE_i denotes the mean square error for the estimation of ith (i=1,2) state with $\frac{1}{S}\sum_{k=1}^{S}\left(x_k^i - \dot{x}_{k|k}^i\right)^2$, where S represents the number of the samples.

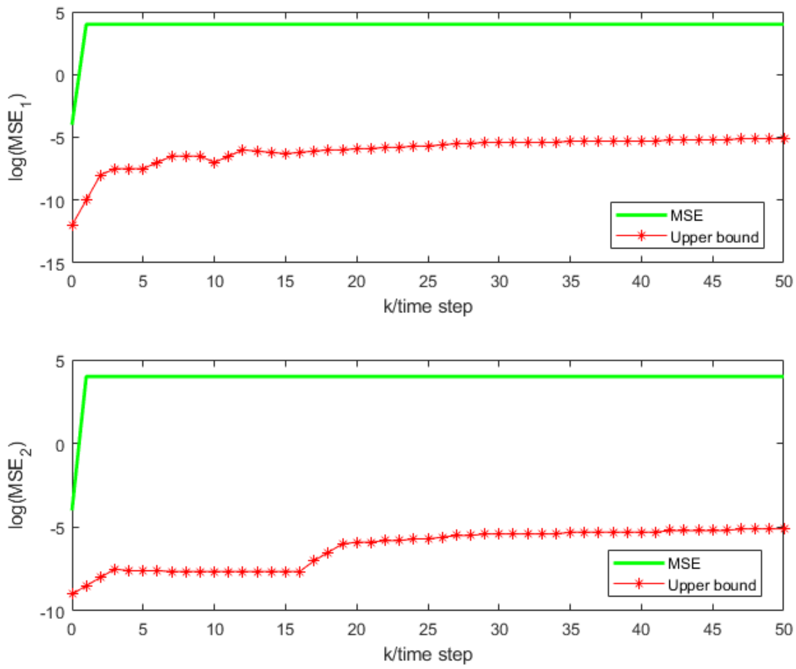

Fig. 3 MSE_i (i=1,2 with upper bounds)

The simulation results are shown in fig 1 and fig 2, its observed in fig. 1 that estimated state and the true state. The estimated state can track the originate state closely. Fig. 2, mean square error is plotted and found that its always under the upper bound limit. These results proofs that proposed filter scheme is verified and found estimation is valid.

5. Conclusion

This paper deals the problem of distributed filter model for a sensor network which is equivalent discrete time varying system with certain topological structure. System modelled with delays, stochastic nonlinearities and gaussian noises. Upper bound error covariance matrix is arrived by solving the Riccati type difference equation, filter gain is derived in virtue of minimizing the upper bound of filtering error covariance. Sparsity of the sensor network handled with new matrix simplification technique. Results are derived by considering and illustrative example, with sample values of time varying matrixes and nonlinear functions.The effectiveness of the modelled filter scheme is validated by comparing in a plot with original/true values and the estimated values. Mean square error is plotted and found that its always under the upper bound limit.

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