A Comprehensive Study of New Fuzzy Logic Metric Based on Interval Type-2 Fuzzy Sets Using Real Application

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Abstract

One of the crucial subjects used in operations research is decision-making processes. Multi-Criteria Decision-Making (MCDM) is one of the topics of this problem that is utilized rather often. In terms of ease and effectiveness, Technique for Order Preference by Similarity to Ideal Solutions (TOPSIS) is one of the most used MCDM techniques. To increase the applicability and simplicity of MCDM methods, Interval Type-2 Fuzzy Numbers (IT2FN), a specific type of Type-2 Fuzzy Sets (T2FS), are widely employed in combination with them. Defuzzification methods are used to find the best solution before the problem's ultimate step of resolution, however. Before the last phase of the process, switching to crisp numbers diminishes the use of the IT2FNs. A brief overview of fuzzy IT2FNs applications is given in this study. The review is based on 25 studies carried out between 2015 and 2023. Supply chains, the environment, energy sources, business, and healthcare are just a few of the application areas that were reviewed and segregated from fuzzy logic articles of various sorts that were the most pertinent and often discussed. The analysis and comparison of fuzzy logic implementations takes into account a wide range of alternatives and criteria, including the use of fuzzy logic mixed with additional techniques, such as the fuzzy Analytic Hierarchical Process (AHP), or improvements for group decision-making, fuzzy sets, hesitant fuzzy sets, intuitionistic fuzzy sets, or improvements for fuzzy logic. The merits and limitations of a variety of FLS types that have been employed for system identification are evaluated. Their effectiveness in various applications is also evaluated.

Keywords: *Type-2 fuzzy sets (T2FS), Fuzzy metric, Type-2 fuzzy logic and multi-Criteria Decision-Making*

1. Introduction

Everyday decisions are often made by people. With institutionalization, decision-making science has developed throughout time as a result of the issue of decision-making. As a result, mathematically based decision-making methods have now become essential to daily living. Decisions are now based on the views of subject matter experts rather than subjective person judgments as a result of the development of these methodologies [1, 2]. In order to address decision issues more realistically and to improve the reliability of the findings, MCDM techniques have been developing. To address concerns based on verbal or ambiguous data, Zadeh (1965) first developed the fuzzy set theory, connecting real-life and mathematics. Later, fuzzy sets were brought to decision theory by Bellman and Zadeh (1970). As a result, a variety of decision-making techniques have been developed that may employ both verbal and numerical data [3]. The first fuzzy logic controller (FLC) was introduced by Mamdani and Baaklini. T1f sets, which handle uncertainty by expressing it with integers that are typically in the range of (0, 1), can be used to regulate some of this uncertainty. Additionally, when a single numerical value can adequately describe a membership function for a fuzzy set, they are very beneficial in absorbing uncertainty. The exact numerical value of an entity's membership or providing a specific membership value for any ambiguous entity, however, can be exceedingly difficult in more complex situations. This undertaking is difficult in and of itself since it is uncertain what information is required to create the rules used in fuzzy systems [4]. Because of incorrectly constructed fuzzy rules, membership functions may also experience ambiguity. It is reasonable to conclude that despite type-1 fuzzy logic's usefulness in some applications, type-1 is often not fast enough or accurate enough to solve many problems. This results in the need for a variety of tools and methodologies that can represent larger degrees of uncertainty.A group of items whose members aren't clearly specified is referred to as a fuzzy set. In comparison to the traditional mathematical binary representation, fuzzy sets provide a more accurate depiction of reality. The progressive nature of being a member of fuzzy sets gives the idea essential for illustrating the finite degree of accuracy in mental representations. Although the historical development of fuzzy logic of type-2 and its introduction by Zadeh, often referred to as the "father of fuzzy logic" (FL), began in the same year, virtually little was written about it until the late 1990s. Researchers first concentrated on T1f logic and its uses, but when more difficult issues arose, it was only natural for people to become more interested in a technique that could handle more complicated circumstances. This is how we got to where we are with type-2 fuzzy logic today [5]. A less complicated variant of T2 FLSs, known as IT2 FLS, was developed due to the mathematical complexity of T2 FLSs. The value of the membership function (MF) for each point in a two-dimensional domain is additionally mapped by an IT2 FLS, which transforms the uncertainty into a third dimension [6].

This particular IT2 FLS leverages the mathematical underpinnings of T1 FLS and is a more condensed variant of T2 FLS. The key difference is that an output processing block is used in place of the defuzzifier section in a T2 FLS. The type reduction (TR) function in this block converts a T2 FS to a T1 FS before performing defuzzification. As a consequence, it is possible to map the uncertainty using the space between the upper membership function (UMF) and the lower membership function (LMF). A T1 FLS typically struggles to handle challenges that a T2 FLS, especially IT2 FLS, can handle more effectively. Examining the T1 and IT2 FLSs' resilience, it was demonstrated that the IT2 FLS outputs had small approximation mistakes; nevertheless, whether a T1 or IT2 FLS should be designed depends largely on the application. Additionally, it has been shown in [7] that it is unfeasible to continue adding MFs and rules beyond a certain point. This has little to no effect on the output, but makes the FLC more challenging. However, IT2 FLSs are utilized in a variety of applications, including the management of robots, bandwidth usage, industrial systems, electrical systems, and aeroplanes. The fuzzy metric function is intended to be added to the fuzzy idea by this work. This will allow for the creation of fuzzy measurements in different fuzzy structures.

2. Background Study about Type- 2 Fuzzy Logic System

T-2 FLSs have mostly focused on managing information uncertainty to enhance the performance of the system for the specific purpose it was designed for. The design of T-2 fuzzy rules is the same as it is for type-1 scenarios, despite the fact that it has been demonstrated that a T2FL performs better than a T1FL in a number of ways. The sole distinction is in the way that membership duties are performed [8]. This section's goal is to enlighten the reader on the general principles and specifics of type-2 fuzzy systems. The output processor adds a second step to the type-2 scenario to transform T-2 FS into a T-1 FS counterpart. The Type-Reduction (TR) approach is used to carry out this operation. It has been argued that T-2 FLSs have a substantial potential to improve the performance of systems in a number of ways. The main arguments supporting this claim are as follows:

- The difficulties of simulating a rule-based system could be lessened with the use of T2f systems.
- A rule-based system may be fine-tuned and understood better with T-2 FS.
- A T2 FLS may also result in the production of complex input/output relationships, which are extremely difficult to get with T-1 FLS.
- Additionally, it is possible to represent these input/output interactions without making any changes to the number of rules.

The Mamdani fuzzy system's rule set for a model with two inputs and one output has a collection of Ψ rules $(\Psi = 1,2,3,.....n)$ having the following form.

$$R^{\Psi} : if\ \ i_1\ F_1^{\Psi}\ \ and\ i_2\ F_2^{\Psi}\ \ then\ \ O^{\Psi} = \lambda^{\Psi}$$

(1)

Here, $F_1^{\Psi} \in \{F_{1,1},......, F_{1,x1}\}$, $F_2^{\Psi} \in \{F_{1,1},......, F_{1,x2}\}$, and $\lambda^{\Psi} \in \{\lambda_{1,1},......, \lambda_{1,n,O}\}$ are language values, commonly referred to as fuzzy sets (FSs),, and yl is the as a result of the rule i_1 and i_2 are the language input variables, and O is the domains' output linguistic variable I_1, I_2 and Y.

A T-1F system uses T-1F sets, whereas a T-2F system requires at least one T-2F set. This obvious distinction calls for an extension of fuzzy set operations as well as fuzzy system inference processing.

2.1. Type-1 fuzzy (T-1F) systems

The following steps can be used to process a T-1F system with centroid defuzzification and fuzzy rules like in equation (1):-

- Use singleton membership functions, typically, to fuzzify the inputs [9].
- Using the input values (the known facts), determine the membership values of the rule's predecessor, in terms of compatibility.
- Calculate the firing strength of each rule F_R^{Ψ} using t-norm functions.
- Use of the implication operation O^{Ψ} to each rule should have a membership function that is induced consequent.
- To create the output T-1F set Ω, utilize an aggregation combination process for all induced membership functions.
- To obtain a precise output value, defuzzify Ω using the centroid method.

2.2 Type-2 fuzzy (T-2F) systems

A T-2F set can be made using a tuple as shown below:

$$\gamma = \left\{ \begin{matrix} (i,x), \omega_{\gamma}(i,x) | \forall i \in I \\ \forall x \in Q_i \subseteq [0,1], \omega_{\gamma}(i,x) \in [0,1] \end{matrix} \right\}$$

(2)

Where, I is the fuzzy variable's domain, and x is a member of the interval designated as the primary membership i.e., $x \in Q_i \subseteq [0,1], \omega_{\gamma}(i,x)$ 2- membership in dimensions function is used to $0 \le \omega_{\gamma}(i,x) \le 1$ define the secondary membership [10].

Interval type-2 fuzzy sets (IT2FS) are a subclass of T-2F sets if for all $\omega_{\gamma}(i,x) = 1$. Then, the definition of an Interval T-2F set is as follows:

$$\omega = \{((i,x),1) | \forall i \in, \forall x \in Q_i \subseteq [0,1]\}$$

(3)

In this article, type-2 fuzzy systems are represented using IT2FSs. It is clear from equation (2) that the third dimension is superfluous and tha)))t a 2-dimensional tuple may adequately describe an IT2FS.

3. Review Based on Fuzzy Logic Sets

There are several academic articles using T2FSs. Even though it is impossible to include all the studies that have been conducted, a few of the most current articles on various T2FS-based study topics are highlighted.

Ruiz-Garcia et al., [11] have given a comprehensive framework for a T-2 FLS employing the most recent knowledge of IT2 FSs (the so-called generic forms of interval T-2F sets, gfIT2 FSs), whose secondary grades may be non-convex T1 FSs. This framework contains original equations for the meet and join operations of gfIT2 FSs as well as a cutting-edge type reduction technique for the T-2 FLS using gfIT2 FSs. Additionally demonstrated is the T-2 FLS process for singleton and non-singleton fuzzification. We will describe the many operations performed by a gfIT2 FLS, such as inference, type-reduction, defuzzification, singleton and non-singleton fuzzification.

Bernal et al., [12] have researched a T-2F logic controller design using the GSO and FA algorithms. A T-2 interval fuzzy controller, which is better at regulating uncertainty than a T-1F controller, is what the optimized fuzzy controller is, it should be mentioned. In this case, T-1F sets that represent the footprint of uncertainty serve as the limiting membership functions for the interval T-2F sets.

Chen et al., [13] have presented an interval T-2F sets and discussed the fundamentals of fuzzy reasoning, type-reduction, and defuzzification of interval T-2F logic systems. They did this by combining the Nagar-Bardini (NB) and Nie-Tan (NT) non-iterative algorithms for solving the centroids of output interval -2F sets. It has also been demonstrated that the continuous variant of NT (CNT) algorithms are accurate ones for doing type-reduction.

Moreno et al., [14] have developed a Mamdani based Interval T-2F Logic System (MAM-IT2FLS) with Center-Of-Sets defuzzification to better define the bounds of uncertainty inside the Interval T-2 Membership Functions (IT2MF) as retrieved from existing data. With the use of MAM-IT2FLS rules, we can use this to compute IT2MF parameters, build the fuzzy model using FCM grouping, and explain the uncertainty present throughout the entire T-2F Logic model.

Nagarajan et al., [15] have given two operators based on the Schweizer and Sklar triangular norms: a triangular interval T-2 Schweizer and Sklar weighted arithmetic (TIT2SSWA) operator and a triangular interval T-2 Schweizer and Sklar weighted geometric (TIT2SSWG) operator. Using a numerical example, the accuracy of these operators has been confirmed.

Li et al., [16] have created the improved artificial immune system (IAIS) technique to address a particular case of the flexible job shop scheduling issue (FJSP). In this situation, each task's processing time is represented by an IT2FS value for a non-symmetric triangular interval. First, a fresh approach for calculating affinity that takes into account IT2FS values is created. Then, in order to improve both quality and variety, four problem-specific starting strategies are developed. To increase the exploitation potential, six local search algorithms are used to the routing and scheduling vectors, respectively.

Das et al., [17] have presented an inventive integrated mathematical model for a green solid transport system with dwell time to apply the carbon price, cap, and offset regulation. Due to market volatility, the supply and demand components are not always obvious. This article includes a twofold (T-2 intuitionistic) uncertainty to create a realistic transportation system. A innovative ranking defuzzification method is used to translate the data into a deterministic form. The stated problem's Pareto-optimal solution is then found by combining a non-fuzzy method with a fuzzy approach.

Cuevas et al., [18] investigated the use of T-2F logic for omnidirectional mobile robots. The invention of fuzzy logic has given the robotics community a solution to problems that required improving the capabilities of autonomous mobile robots. A crucial component of the development is the use of a human expert's experience in the creation of a fuzzy knowledge base that enables us to explain the connection between the inductive voltage, the separation of objects in an unknowable dynamic environment, and their linear and angular output velocities.

A firefly algorithm experiment by Melin et al., [19] produces an ensemble neural network design. Here, a weighted average integration method and the firefly ensemble neural network optimization technique are suggested for COVID-19 time series prediction. The suggested approach calculates the number of artificial neural networks needed to build an ensemble neural network and their design by merging the outputs of individual artificial neural networks to arrive at a final forecast.

A hazy spatiotemporal object based on interval T-2F sets has been created by Yin *et al.*, [20]. The fuzzy and ambiguous clustering technique was used to categorize the various plant cover types in China's Poyang Lake Plain in order to confirm the viability and superiority of the suggested approach. Additionally, field observations were used to confirm the classification accuracy, and Poyang Lake's yearly fluctuations in its unclear water area were used to confirm the classification's ability to recognize the region's rainy season.

Wozniak *et al.,* [21] implementation of a T-2F logic control module-based IoT system for driving assistance. In order to gather information regarding driving conditions and assess them in accordance with user demands, they have designed an IoT system. In order to adapt flexibly to the uncertainty of the appraisal of each driver's driving expectations, an applied module of fuzzy logic of the second kind was utilized in the analysis of accelerometer data.

Castillo *et al.,* [22] forecast of COVID-19 is based on the integration of interval T-3F logic and fractal theory. In this instance, the COVID-19 issue is used to evaluate the time series geometrical complexity level using the fractal dimension. Utilizing interval T-3F logic is primarily intended to handle uncertainty in predicting decision-making. The interval T-3F model of the hybrid technique uses the linear and non-linear values of the dimension as inputs, while the approach's outputs are predictions for COVID-19 cases. The contribution is an original, never-before-proposed method for accurate complex time series prediction using interval T-3F logic and the fractal dimension, especially for the COVID-19 instance. Using open data sources, they construct an interval T-3F system for a time series.

A mobile robot route planning approach employing a camera along with interval T-2F logic (IT2FIS) has been reported by Dirik *et al.,* [23]. They discussed a method for designing an obstacle-free route based on visual servoing. Identifying a mobile robot's position in the surrounding environment is required. An IT2FIS is made to construct a route in order to achieve the destination and avoid obstacles effectively in a variety of environments.

Dey et al. [24] have described a minimum spanning tree problem (FMST-IT2FS) with an undirected connected weighted interval T- 2F network. The interval T- 2F set is used in this instance to represent the arc lengths of a fuzzy graph. A unique evolutionary method is then used to add, rank, and defuzzify IT2FSs in order to resolve the FMST-IT2FS problem.

A covering-based approach for calculating the limits of the variance of an interval T-2F set is introduced by Figueroa *et al.,* [25], which also presents a fuzzy set. Unlike earlier techniques that rely on the centroid of an interval T-2F set to acquire a relative variance, which is shown to always be larger than the absolute variance, the proposed approach determines the boundaries of all potential variances of an interval T-2F set.

Table 1: Overall analysis of survey

Author	**Publishing Year**	**Types of Fuzzy Set**	**Problem**	**Application**
Ruiz-García *et al.,*	2019	Interval T-2F	Image thresholding	Mobile robot
Bernal *et al.,*	2021	Interval T-2F	Controlling	Autonomous mobile robot
Chen *et al.,*	2019	Interval T-2F	Controlling	Potential
Moreno *et al.,*	2020	MAM-IT2FLS	Highest computational cost	Many real-world
Nagarajan *et al.,*	2019	TIT2SSWA	Controlling	Traffic management
Li *et al.,*	2020	Triangular Interval T2FS	Job shop scheduling	Manufacturing
Das *et al.,*	2020	Interval T-2F	Solid transportation	Industrial

Cuevas *et al.*,	2021	T-2F	Controlling	Autonomous mobile robot
Melin *et al.*,	2021	T-2F	Global COVID-19 pandemic prediction	Medicine
Yin *et al.*,	2022	Interval T-2F	Membership degree error	Geographic Information Systems (GIS)
Woźniak *et al.*,	2022	T-2F	Monitoring and road surface while driving	Internet of things (IoT)
Castillo *et al.*,	2023	Interval T-3F logic	COVID-19 prediction	Medicine
Dirik *et al.*,	2019	Interval T-2F logic	Path planning, localization, and motion control	Mobile robot
Dey *et al.*,	2020	Interval T-2F	Minimum spanning tree	Transportation and traffic planning
Figueroa *et al.*,	2023	Interval T–2F	Optimization	Industry

4. Summary of the Review

In this review, 15 publications have been thoroughly examined. Each study employs a unique fuzzy set and application. In this study, it was examined which fuzzy set respondents used, how much they classified the application and issue, what range they fell into, and rating metrics. When analyzing the current research papers listed in Table 1, some approaches are difficult, have a lot of controlling issues, and ineffectively divide the real-time application area. Many applications are employed in the real-time world. However, for the application of type-2 fuzzy sets, considerable enhancement is required.

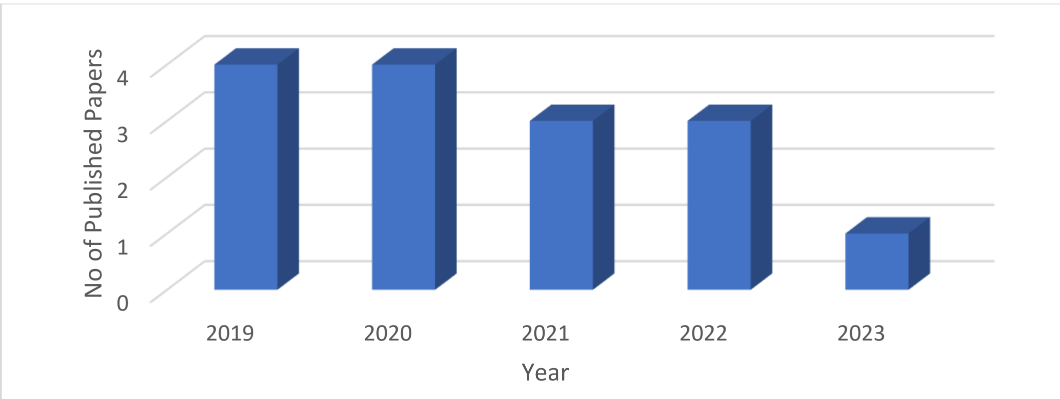

Figure.1: Review of published papers

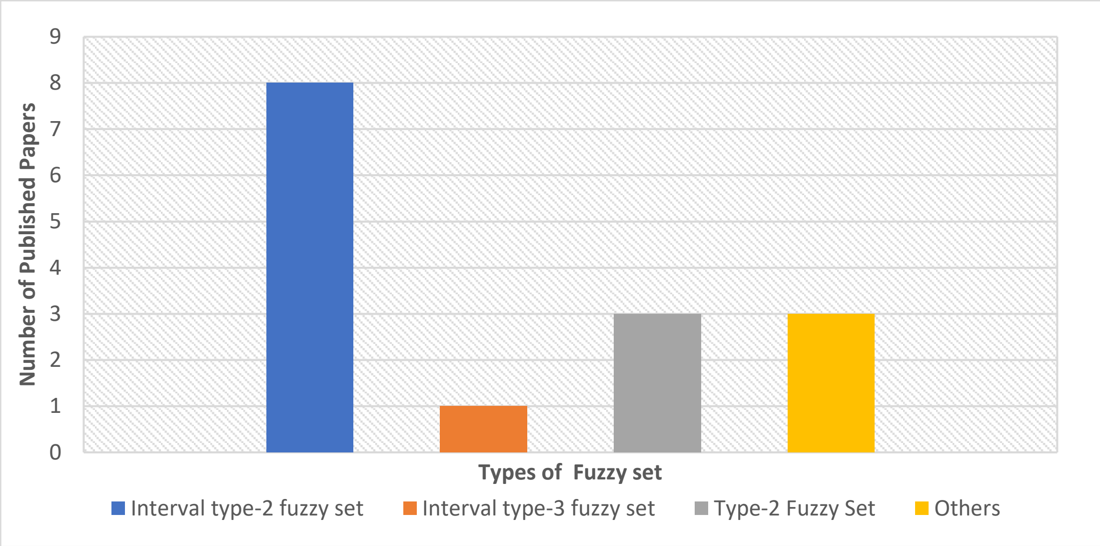

Figure.2: Review of Fuzzy sets

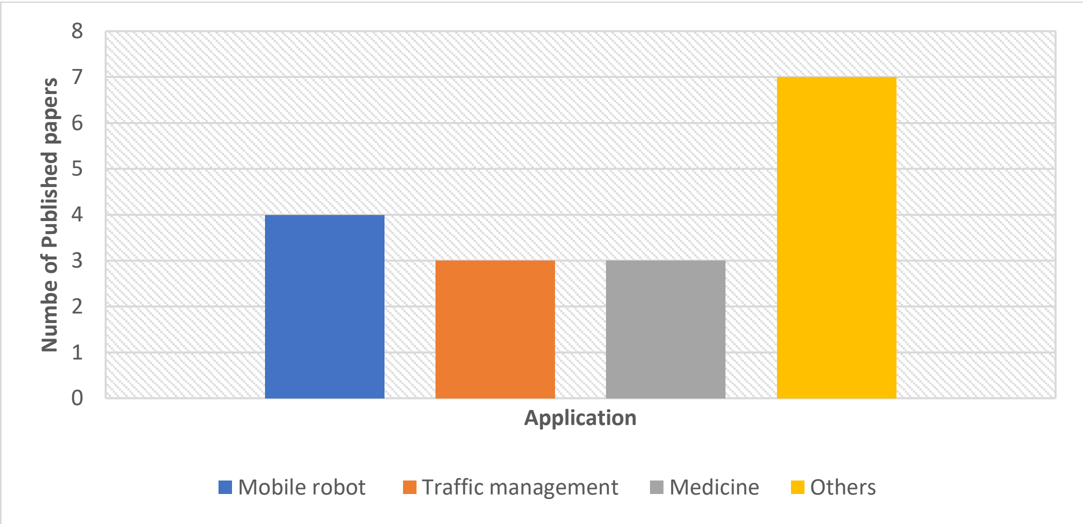

Figure.3: Review of Application

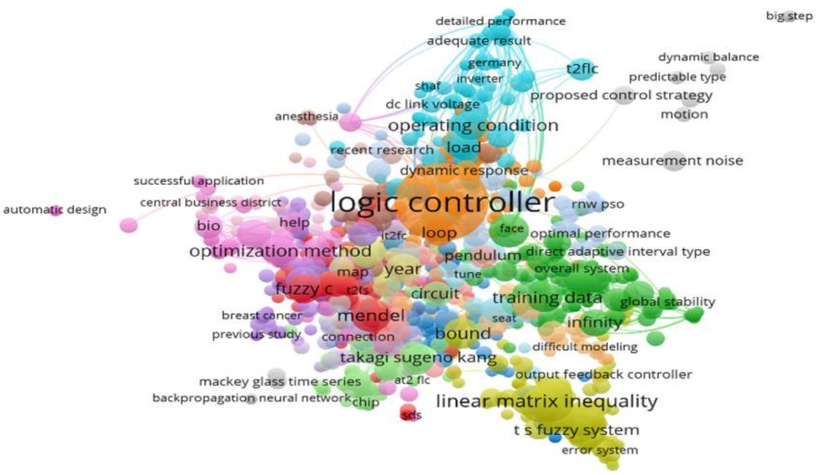

Figure.4: Type-2 fuzzy logic distribution by area and keyword density (data from Web of Science, 2015–2023)

Figures 1, 2 and 3 show the review of published papers, types of fuzzy sets and its applications. It is clear from these figures that interval T-2F set is used for eight kinds of literature, T-2F set and others is used for 3 literature, T-3F set is one literature. The interval T-2F sets are mostly used in research papers. Similarly, application of mobile robot control system is used in four research articles, medicine filed is used in 2 literatures, traffic management system and other filed is used in seven articles. The distribution of publications with keyword density by geographic region is given in Figure. 4, and the information is gathered from the Database on the Web of Science between the years of 2015 and 2023. In terms of upcoming projects (trends), it can be stated that the transition from T-2F logic to T-3F logic seems to be a fresh and unexplored topic of study. In fact, T-3 is beginning to take shape; in fact, some work on it is now underway. This is necessary since there are still plenty of unsolved, more difficult issues that will need even more potent models. For instance, T-3F models might be very useful in the big data sector where there is a need for more sophisticated modelling tools due to the increasing degrees of uncertainty. A promising field of research is the fusion of T-2F logic with bio-inspired optimization methods. This may be accomplished in one of two ways: either by optimizing T-2F systems using bio-inspired metaheuristics, or by employing T-2F to achieve dynamic parameter adaptation in the metaheuristic to make them better optimizers.

4.1. Future of Type-2 Fuzzy Sets

Several potential areas for T-2F logic research will be discussed in this section, including both theory and applications. Th aforementioned literature scan highlights the huge interest that academicians from all over the world have in fuzzy set theory. Though some scientists are still resistant to formalizing fuzzy set theory as a science: One of the scathing criticisms voiced by these scientists was that "fuzzification is a kind of scientific permissiveness; it tends to result in socially appealing slogans unaccompanied by the discipline of hard scientific work and patient observation." Fuzzy theory also poses the risk of encouraging the kind of imprecise thinking that has caused us so much difficulty. The fuzzy set theory has many detractors, yet its advancement has substantially increased. Many important open issues with T-2F logic may be explored in future theoretical study, according to researchers:

Model Optimization

It is thought that there are still many unanswered questions about T2FLS optimization. Defuzzification algorithms, rules, membership functions, type reduction, and membership functions are still chosen by hand, while there are chances to make effective use of evolutionary algorithms and metaheuristics, as well as better learning algorithms.

Defuzzification

Although there are several defuzzification algorithms in use today, there is still much room for improvement.

Type reduction

Numerous different approaches have been proposed since the original Karnik-Mendel algorithm was first introduced, but type reduction still needs to be improved for effective real-world applications.

Computational Complexity

Although this problem has seen great progress, additional approaches and algorithms could still be investigated in order to improve T2FLS performance as a whole.

Researchers could investigate a variety of fresh real-world applications in their next application work:

- **Hardware Implementations:** More work has to be done in this area as it relates to the hardware implementation of T2FLS. Field programmable gate arrays (FPGAs) were mostly employed at first, but more modern and compact hardware components, including fuzzy controllers, are now also accessible for implementation.
- **Applications in Medical Diagnosis:** T-2F sets are very good at collecting and representing uncertainty in medical data. Because of this, T-2F applications in medical problems, like as diagnosis, are becoming more and more common.

- **Applications in Big Data:** Since uncertainty is a significant issue in this area, T-2F sets have become more common in big data. We believe that this pattern will persist.
- **Applications in Robotics:** T-2F sets are crucial for giving robots the knowledge and reasoning they require to navigate in uncertain and dynamic settings as robotic tasks get more complicated.

Research in each of these fields will help T-2F systems to be applied more readily to real-world issues and can also offer the mathematical foundation for generalizing T-2F logic to T-3F logic and even T-NF logic.

5. Conclusion

This paper presented an extensive review of every piece of literature, approach, fuzzy set type, issue, real-world application, and restrictions. It can be concluded that the most common applications for T2 FLCs are in the fields of automation, bandwidth control, industrial systems control, electrical control, and aviation control. The robotics and automotive industries show the greatest promise, since these industries have made advancements above conventional controllers. IT2 FLCs have been found to have a number of benefits over T1 FLCs, particularly in uncertain systems, and that IT2 FLCs often outperform conventional PID controllers. Future use of these controllers for uncertain systems is potential if more effective methods or better software/hardware can be used to compensate for the computational complexity of T2 FLCs. It is only natural to see more and more uses for these powerful controls as technology advances. For general and commercial uses, it is anticipated that several further applications may be created and found.

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