Investigating the Optimality of Existing Hubs in Terms of Coverage Capacity and Locating New Hubs for the Green Supply Chain of Medical and Pharmaceutical Equipment

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Abstract:-The problem of locating the hub is one of the most important and widely used problems in network design. The deployed hubs can fail over time for various reasons, including disruptions in the processes, in which case excessive costs are imposed on the operating companies. Therefore, proper planning is needed to reduce the harmful effects of environmental disorders. In this regard, we seek to examine the optimization of existing hubs in terms of the capacity of the new hubs and location of the hubs, with the aim of reducing the cost of coverage to reduce the cost and maximizing the level of green hubs. To this end, the economic benefits of the measures were carried out: the establishment of a new hub, the removal of the previous or current hub, the increase and decrease of the capacity of the hubs. Eventually a new structure of hubs was introduced where the location and capacity of some hubs have changed, but in contrast, the amount of demand coverage has increased.

Keywords: hub locating, coverage capacity, green supply chain medical and pharmaceutical equipment

Introduction
Locating can be used as one of the most commonly used names that are applicable to many of the supply chain research [1]. Deciding about locating has taken a great deal of attention from the university and business society. Hence, in terms of credit value, the value of the hubs is more likely to grow [2]. Selection is the appropriate place to manage the management of pharmaceutical equipment companies [3]. In recent years, various decisions and policies have been taken in order to reduce the costs of medical equipment manufacturing companies, which is one of the most important issues in reducing costs, choosing the location and distribution centers. One of the challenges in locating facilities is considering facilities in different places in the same way and with similar services and capacity. This is despite the fact that the features of the located points, limitations, type and amount of needs in each area are not necessarily similar to each other, and these differences make the uniformity of the facilities not to be efficient enough [4]. Among these differences, for example, the following cases can be mentioned: in an area with a high population density, a facility with a high capacity is needed, or in an area
where there are suitable and wide access roads, the coverage radius of a facility can be greater than other facilities. Alternatively, the types of needs are different in different places and require facilities with different services. However, in order to achieve an optimal answer, facilities with different features and capacities must be considered. The fierce competition in the global markets forces companies to design better supply chains. There are generally three different levels of decision-making in the supply chains: strategic level, tactical level, and operational level. Decisions at different levels are generally considered separately. Organizational structures are such that strategic decisions that are made by senior managers, while operational and tactical decisions are delegated to lower level managers. This structure creates a lot of contradictions and inconsistencies in the decisions made. The provision of cheap transportation services also has many benefits for the growing development of countries. One of the main branches of the supply chain is the hub networks, which move the flow between the source and destination points through a middle node called the hub. In addition, in hub network nodes, when the flow rate is higher than the service rate, we encounter a hub location system that has a hub capacity and an imbalance in planning. In the field of location in various areas, including health, uncertainty is one of the most important factors. In the case of various facilities, there are many parameters such as the amount, type, and timing of uncertainty, and considering this uncertainty in the parameters can increase the applicability of the problem [5]. Among the most important restrictions in locating a facility hub are consider; the limitations are on the capacity of the facilities and the means of communication so that each facilitation has a limited capacity due to the various factors that no more capacity can provide. Therefore, it is necessary to consider the limited capacity for facilities and if these restrictions are not considered we will move away from, the actual situation and the model will not be practical. Uncertainty in determining parameters is another factor we face in the real world and will affect the model. Providing a model that provides an answer to this uncertainty be of paramount importance; therefore, presenting a model that, while developing all-out, is a more accurate translation of the facts. Therefore, in this study, the problem of hub locating will be discussed. In this study, a new multipurpose hub allocation-concessions model is developed. In the model, several features and perspectives are applied along with the basic mathematical model in the articles. planning and optimization. Jafari and Hadianpour [6] in their research with a MOHLAP hub location single-objective model, after examining the definitions of hub location and types of location models and their solution algorithms, solve a case study example of the classical model algorithm. Locating Hub has added a fixed annual fee to determine the best place to ship goods from one country to another among the 5 most requested cities, with the aim of minimizing shipping and distribution costs. Nakhjirkan et al. [7] developed a supply chain decision-making model under the demand for uncertainty using genetic algorithm and network data envelopment analysis. Husseinzadeh Kashan et al. [8, 9] modeled and solved single-capacity hub locating algorithm with hub distribution capability and considered a small number of capacity transportation devices as a parameter and solved it as a two-step random problem with meta-algorithms. Wu et al. [1] introduced several non-random hub locating problems in a multipurpose transportation and distribution system and formulated it with uncertain transportation cost and demand and solved it using Benders algorithm using Turkish shipping network data and AP data. In his research, Erdogan et al. [10] addressed the issue of hubing and location hubs in a NP- Hard and MIP problem by linking the hub locating problem on a network and defining the cost of transportation between hubs and considering two levels of decision makers, one for transport suppliers and the other for customers. The proposed model is a multilevel programming formula that is first linear and then solved by meta-healing. Olivieria et al. [11] designed a mathematical model of the hub with a mathematical model and solved it with several versions of the Benders approach, which evaluated the results better than CPLEX and stated that up to 150 nodes can be solved by this method. Maharejan et al. [12] in a study presented a model for the allocation of a temporary multipurpose logistics-based logistics allocation to provide and distribute relief under uncertainty. Accidental operations are about uncertainty. While accidental uncertainties can be avoided for the post-disaster location problem, there may be inadequate cases long after the disaster. In spite of the uncertainty and insufficient information on the extent of the damage, the disaster should be provided promptly after the disaster. In addition, decisions about the opening, location, and timing of disaster response facilities are based on the amount and quality of information available during the decision period. To address these issues, a multipurpose location allocation model has been developed for the provision and distribution of relief that takes into account
the inaccurate and variable nature of different parameters and time-varying coverage, while considering the mental characteristics necessary to establish and operate temporary logistics centers (TLH). A Credit-based Fuzzy Possible Planning Model is used to calculate the inherent inaccuracy of parameter values when responding to disasters. The results show where, when, and several TLHs are suitable for opening and allocating relief resources. At the same time, sensitivity analysis provides a broader understanding of the impact of limiting the number of TLHs as well as the level of reliability and expansion of symmetrical triangular fuzzy numbers in achieving the model's goals.

Research problem

In this proposed model assumes that there are a number of facilitations at different levels in the study area. Given that existing facilities have been established over a long period of time, and given that changes in the level and concentration of subscriber demand for services have changed over the years, it is imperative that all existing hubs be monitored seamlessly. In this section, the hubs in the area are first classified into three levels for the transfer of medical equipment, and then the demand coverage is checked by these facilities and if there is a change in the capacity or location of the facility that is justified in terms of cost; These changes occur in the location and capacity of existing facilities. Finally, a new structure of hub locations is introduced where the location and capacity of some hubs have changed, in contrast the coverage of medical goods has increased. Improving demand coverage is not just about examining the capacity of existing hubs and modifying the location of these hubs. In this paper, there are three levels of capacity for hubs, and if needed, the hub will be upgraded to a higher capacity level or a new hub is positioned. In order to improve the quality of the coverage of medical equipment clients, the coverage radius for each hub is also considered. The coverage radius helps to allocate the demand to the hubs so that the distance from the demand point to the hub is optimized and acceptable. In this paper the radius of coating is considered variable. Given that the amount of demand coverage is directly related to the hub and the variable cost of the coverage; it is tried to minimize the coverage of each hub. In improving the current situation, it is not just about changes to existing hubs and if there is a need for new hub locating; the new hub is set up. In this regard, we seek to examine the optimization of existing hubs in terms of the capacity of the new hubs and the location of the new hubs, with the aim of reducing the cost of higher demand coverage and maximizing the level of green hub location. To this end, we will take these measures in our review with the following economic benefits: establishing a new hub, removing the previous or current hub, increasing and decreasing the capacity of the hubs.

Mathematical modeling assumptions

In this study, three levels of hubs are considered: W-level hub, which is the lowest level of capacity and provides basic medical equipment services. C-level hub, that provides complementary medical equipment services. Finally, the H-level hub offers the most advanced medical equipment services as well as the C-level hub services. The study area consists of a number of areas. The areas themselves are also the subset of larger area called region, which is covered by hubs. Each point of demand must be covered by a W-level hub and a C-level hub as well as a H-level hub, but H-level hubs can also provide C-level hub services; Therefore, if a demand point is covered by the H-level hub, there is no need for that point of demand to be covered by C-level hub. There must be at least one W-level hub in each area. In one area we cannot have both C-level hubs and H-level hubs at the same time. In one area cannot be placed more than an H-level hub. Demand is considered as a point. Figure 1 shows the different states of coverage of a demand point based on assumptions. In the first case, the demand point is assigned to all three existing hub levels. In this case, the demand for each level is assigned to the same level hub. In the latter case, the demand point is assigned to two hubs at W and H levels. In this case, as described in the assumptions; the demand point of C-level hub is covered by H-level hub. Due to assumptions, H-level hub can also provide C-level hub services.
The proposed model

In this proposed model, it is assumed that there are a number of hubs at different levels in the study area and that the amount of demand coverage should be checked by these hubs and if there is a change in the capacity or location of the hubs that is justified in terms of cost; these changes occur in the location and capacity of existing hubs. Finally, a new structure of hubs is introduced where the location and capacity of some hubs have changed, in contrast, the amount of demand coverage has increased. This section presents the mathematical model of research. In this regard, we are looking at the optimization of existing hubs with the aim of reaching a higher demand level. To this end, we will take these measures in our review with the following economic benefits: establishing a hub, removing the hub, increasing and decreasing facilitating capacity. Sets, indexes, parameters, and decision variables are introduced in this section.

Collections and indexes

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Target functions and limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Network Sets</td>
<td>( \text{Max} \sum_i h_i z_i^0 ) (1)</td>
</tr>
<tr>
<td>B</td>
<td>Set of Areas in the Model</td>
<td>( \text{Min} \sum_{i,j,t} F_{ij}^t x_{ij}^t + \sum_i r_i^t (\varphi) ) (2)</td>
</tr>
<tr>
<td>R</td>
<td>Set of Areas in the Model</td>
<td>( \text{Max} \sum_{i,j,t} G_{ij}^t x_{ij}^t ) (3)</td>
</tr>
<tr>
<td>S</td>
<td>Set of uncertainties scenarios</td>
<td>( \sum_{j \in N_b} a_{ij}^{ls} \geq z_i^s ) (4) ( \forall i \in N_b, b \in B, l \in {w, h, c}, s )</td>
</tr>
<tr>
<td>F_r</td>
<td>Set of nodes in the area r</td>
<td>( \forall i \in F_r, r \in R, s )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>Index indicator of demand points ( i \in N )</td>
<td>( \sum_{j \in F_r} (a_{ij}^{cs} + a_{ij}^{hs}) \geq z_i^s ) (5) ( \forall i \in F_r, r \in R, s )</td>
</tr>
<tr>
<td>j</td>
<td>Index represents candidate points for establishing hubs ( j \in N )</td>
<td>( \forall b \in B )</td>
</tr>
<tr>
<td>l</td>
<td>Index corresponding to the hub level ( l \in {w, h, c} )</td>
<td>( \forall b \in B )</td>
</tr>
<tr>
<td>t</td>
<td>Index corresponding to the hub capacity level</td>
<td>( \forall b \in B )</td>
</tr>
<tr>
<td>b</td>
<td>Index corresponding to area</td>
<td>( \forall b \in B )</td>
</tr>
<tr>
<td>N_b</td>
<td>Nodes in the area ( b ), ( N_b \in N )</td>
<td>( \forall b \in B )</td>
</tr>
<tr>
<td>r</td>
<td>Index corresponding to the region</td>
<td>( \forall b \in B )</td>
</tr>
<tr>
<td>s</td>
<td>Index of each scenario</td>
<td>( \forall b \in B )</td>
</tr>
</tbody>
</table>

}\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Different coverage states of a demand point}
\end{figure}
\[ h_i \quad \text{Point } i \text{ demand} \]
\[ F_{j,t}^l \quad \text{Fixed cost of establishing a hub at the level } l \text{ at the point } j \text{ with a capacity level } t \]
\[ G_{j,t}^l \quad \text{The green level of the hub at the level } l \text{ at the point } j \text{ with a capacity level } t \]
\[ \text{level}_t \quad \text{The amount of capacity associated with the level } t \]
\[ Q_{j,t}^l \quad \text{the capacity level } t \text{ is already present; } 0 \text{ Otherwise} \]
\[ p^l \quad \text{The number of hubs at the level } l \text{ that should be at the end} \]
\[ d_{i,j} \quad \text{The distance between two points } i \text{ and } j \]
\[ R_{\text{Max}} \quad \text{the maximum acceptable coverage radius} \]
\[ p_s \quad \text{An acceptable level in deviation from the optimal amount of each scenario in the original model} \]
\[ \rho^*_s \quad \text{The optimal amount of demand covered in each scenario} \]
\[ r^*_s \quad \text{Optimal amount of cost per scenario} \]
\[ z^*_i \in \{0,1\} \quad \text{In case the demand point } i \text{ is covered; } 0 \text{ Otherwise} \]
\[ x_{j,t}^l \in \{0,1\} \quad \text{1 if the hub at the level } l \text{ at the point } j \text{ with a capacity level } t \text{ has not already presented; } 0 \text{ otherwise} \]
\[ r_j \quad \text{the hub cover radius at point } j \]
\[ a_{i,j}^l \in \{0,1\} \quad \text{1 if the demand point of } i \text{ is assigned to the hub } j \text{ at the level } l \text{ in the scenario } s; \ 0 \text{ Otherwise} \]
\[ y_{j,t}^l \in \{0,1\} \quad \text{1 if the hub level } l \text{ is at the point } j \text{ and the capacity level } t; \ 0 \text{ Otherwise} \]

\[ \sum_{i \in F_r,i} y_{j,t}^l \leq 1 \quad \forall r \in R \tag{8} \]
\[ \sum_{l \in F} y_{j,t}^l \leq 1 \forall j \tag{9} \]
\[ \sum_{i} a_{i,j}^l h_i \leq (\sum_{l} \text{level}_t) y_{j,t}^l \quad \forall j, l, s \tag{10} \]
\[ r_j^l \geq \sum_{i} d_{i,j} a_{i,j}^l \quad \forall i, l, j \tag{11} \]
\[ r_j^l \leq R_{\text{Max}} \sum_{l} y_{j,t}^l \quad \forall l, j \tag{12} \]
\[ \sum_{i} y_{j,t}^l \leq p^l \quad \forall l \in L \tag{13} \]
\[ x_{j,t}^l = |y_{j,t}^l - Q_{j,t}^l| \quad \forall l, j, t \tag{14} \]
\[ \sum_{i} h_i z_i^s \geq (1 - p_s) \rho^*_s \quad \forall s \in S \tag{15} \]
\[ \sum_{i \in F_r,i} F_{j,t}^l y_{j,t}^l + \sum_{l \in F} r_j^l (\phi) \leq (1 + p_s) \tau_s^* \quad \forall s \in S/0 \tag{16} \]

\[ \textbf{Solving method} \]

According to Huang and Massoud [13], three main methods for solving multidisciplinary optimization problems are known based on the stage of decision-making preferences: previous methods, subsequent or productive methods, and interactive methods. In previous methods, the decision maker presents his preferences before trends. This method has the problem of quantifying preferences that are determined by the decision maker. The latter method is based on optimizing all target functions simultaneously. The decision-maker sequentially guides the search to the preferred solution. To solve this two-way model, the Epsilon precise solution method is used.
In this method, one of the target functions is maintained as a target function and the rest of the target functions are converted to unequal constraints due to the minimization or maximization of the target function as follows:

\[
\begin{align*}
\text{max} \ ax & \rightarrow \ ax \geq \varepsilon \\
\text{min} \ ax & \rightarrow \ ax \leq \varepsilon
\end{align*}
\]

(17)
(18)

In this model, the first limitation, which is the maximum demand, is kept as a target function, but the second target function, which is the minimization of cost and the amount of change, is considered as a limitation and is changed as follows:

\[
\sum_{j,t} F_{j,t}^I x_{j,t}^I + \sum_{j} r_j (\varphi) \leq \varepsilon
\]

(19)

In this method, the initial value is first assumed for the epsilon parameter (\(\varepsilon\)) and during the repetition the epsilon value (\(\varepsilon\)) is improved to obtain the optimal solution. In the Epsilon method, the model limit should be linear, but the model presented in this paper is a nonlinear mixed model, so we will describe the model linearization process.

**Linearization**

In the model presented in this thesis are nonlinear relationships (14) and if these equations are linear, we achieve a linear integer (MIP) model. In the case of this nonlinear limitation, the presence of an absolute magnitude mark makes the model nonlinear, which, given the existence of an absolute magnitude variable in the target function, is linearized as follows:

First we introduce two binary variables \(x'_{j,t}\) and \(x''_{j,t}\). We rewrite (20) and (21) as follows:

\[
\begin{align*}
y_{j,t}^I - Q_{j,t}^I &= x_{j,t}^I - x''_{j,t} \\
x_{j,t}^I &= x_{j,t}^I + x''_{j,t}
\end{align*}
\]

(20)
(21)

As such, the model presented in this paper becomes a mixed integer linear model.

**Release of binary variables**

The presence of binary variables can increase the model solving time. In the second model presented in this study there are 4 binary variables; of these 4 categories of binary variables, we can release two categories of variables and reduce model solving time. The category of variables \(x'_{j,t}\), \(x''_{j,t}\), and \(z_s^I\) can be released and converted to positive variables. With the conversion of the above variables, no change in the final response of the model is made and the model is resolved in a shorter time than before. The final model after linearization and release is as follows:

\[
\begin{align*}
\text{Max} \sum_{i,s} p_i (h, z_i^I) \\
\text{Min} \sum_{i,j,t} F_{j,t}^I \left( x_{j,t}^I + x''_{j,t} \right) + \sum_{j} r_j (\varphi)
\end{align*}
\]

(22)
(23)

Limitations:

\[
\begin{align*}
y_{j,t}^I - Q_{j,t}^I &= x_{j,t}^I - x''_{j,t} \forall l,j,t \\
a_{l,j,t}^I, y_{l,j}^I &\in \{0,1\} \forall i, j, l, t, s \\
z_{j,t}^I, x_{j,t}^I, x''_{j,t}, r_j &\geq 0 \forall i, j, l, t, s
\end{align*}
\]

(24)
(25)
(26)

**Validation of the proposed model**

In this section, in order to confirm the correctness of the first mathematical model presented, we solve a small-scale experimental problem \((|N| \times |s| \times |l| \times |t| = 8 \times 2 \times 3 \times 2)\) and report the results. How to generate
different parameters is explained. Then, the proposed model is solved by GAMS software. It is done in this way that the two-objective model is considered as two single-objective models. The solution of the model was checked for each objective function and finally the solution of the equivalent single-objective model, which was converted to a single-objective model using the epsilon constraint method, will be examined. It is worth mentioning that in the following sections, several sensitivity analyzes are presented in order to further validate the model. In the considered numerical example, the network has 8 points. All the specified points are the demand points and at the same time, the candidate points for the establishment of various facilities are considered. The parameters of this numerical example are generated based on the following tables:

**Table 1. The way of generating parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_i$</td>
<td>$Uniform(20,80)$</td>
</tr>
<tr>
<td>$F_{j,t}$</td>
<td>$Uniform(100000,200000)$</td>
</tr>
<tr>
<td>level$_1$</td>
<td>100</td>
</tr>
<tr>
<td>level$_2$</td>
<td>150</td>
</tr>
<tr>
<td>$Q_{j,t}$</td>
<td>$Q_{i,2},Q_{i,1}^p = 1$ and others = 0</td>
</tr>
<tr>
<td>$p^i$</td>
<td>8</td>
</tr>
</tbody>
</table>

The distances of the considered points for this numerical example are according to Table 2.

**Table 2. Distances of considered points**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.707</td>
<td>1</td>
<td>1.414</td>
<td>2</td>
<td>2.236</td>
<td>2.55</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0.707</td>
<td>1.414</td>
<td>1</td>
<td>2.236</td>
<td>2</td>
<td>2.55</td>
</tr>
<tr>
<td>3</td>
<td>0.707</td>
<td>0.707</td>
<td>0</td>
<td>0.707</td>
<td>0.707</td>
<td>1.581</td>
<td>1.581</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.414</td>
<td>0.707</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1.414</td>
<td>1.581</td>
</tr>
<tr>
<td>5</td>
<td>1.414</td>
<td>1</td>
<td>0.707</td>
<td>1</td>
<td>0</td>
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</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2.236</td>
<td>1.581</td>
<td>1</td>
<td>1.414</td>
<td>0</td>
<td>1</td>
<td>0.707</td>
</tr>
<tr>
<td>7</td>
<td>2.236</td>
<td>2</td>
<td>1.581</td>
<td>1.414</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.707</td>
</tr>
<tr>
<td>8</td>
<td>2.55</td>
<td>2.55</td>
<td>2</td>
<td>1.581</td>
<td>1.581</td>
<td>0.707</td>
<td>0.707</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 2 shows the location of demand points and facilities that already exist in this network. In addition, the division of regions and areas is also shown in this figure.
Figure 2. The location of demand points, existing facilities, and the division of regions and areas

It is assumed that all the points in the network have demand points and at the same time they are candidates for the establishment of the facility. In this numerical example, the network has 8 demand points. These 8 points are divided into 3 regions and these 3 regions are divided into 2 areas. The rectangles with blue lines and curved corners are the border of the regions, and the rectangles with red and dashed lines are also the borders of the areas. As you can see in the image, points 1, 2 and 3 are in zone one, points 4 and 5 are in zone two and points 6, 7 and 8 are in zone three. Region one includes area one and region two includes areas two and three. At point 1, there is h-level facilitation and at point 7, there is w-level facilitation.

Figure 3 shows the result of solving the model for the first objective function and the second objective function. In this figure, the triangles represent w-level facilities, the hexagons represent h-level facilities, the blue lines represent the coverage area of w-level facilities, and the red lines represent the coverage area of h-level facilities. In the first case, which shows the optimal solution considering the first objective function; considering that there is no limit for the cost and the function of the second objective, which is cost minimization, is not taken into account; facilitation has been established in all candidate points. According to the nature of the limitations of the model, two facilitations at h level have been established at points 1 and 4, and facilitation at w level has been established at the rest of the points. The allocation of demand is also such that the demand of points 1, 2 and 3 in level h is allocated to the facility established in point 1 and the demand of other points in level h is allocated to the facility established in point 4. The demand of level w has also been assigned to these points due to the establishment of facilitation at level w in all points except points 1 and 4. Demand level w points 1 and 4 are also assigned to facilities 2 and 5 respectively. In the second case, which shows the optimal solution considering the second objective function; with respect to only cost minimization is considered; only facilities have been established that are mandatory based on the limitations of the model. According to the limitations of the model, there should be at least one facility in the level w in each region. Considering that in the existing network, there are two facilities at h and w levels, respectively, at points 1 and 7 of the network; after solving the model, only two facilities have been established at the level of w at points 2 and 4. However, since considering the coverage radius for facilities requires variable cost, the coverage radius of all facilities is zero in the case of considering the second objective function. Therefore, no demand point is covered and the demand coverage becomes zero in this case. It is worth mentioning that the first and third objective functions have reached a consensus with each other using the weight method.

Table 2. The answer obtained from solving the model with the objective functions

<table>
<thead>
<tr>
<th>The first objective function(1)</th>
<th>The optimal answer considering the first objective function</th>
<th>The second objective function(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first objective function</td>
<td>375</td>
<td>0</td>
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</table>
The second objective function

2037861.816

219780

Figure 3. The result of solving the model for the first objective function, the second objective function

In Table 3, the proposed optimal solutions resulting from solving the equivalent single-objective model using the epsilon constraint method are displayed. As it is known, four answers have been proposed, and the choice of the right answer is based on the opinion of the decision maker.

Table 3. Optimal solutions of the equivalent single-objective model with the epsilon constraint method

<table>
<thead>
<tr>
<th>Answer No.</th>
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<td>The amount of first objective function</td>
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<td>154</td>
<td>154</td>
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<td>2.2</td>
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</table>

Figure 4 is the graph of the solutions obtained from solving the single-objective equivalent model using the epsilon constraint method:
As can be seen, the solutions obtained by the epsilon constraint method are logical solutions according to the solutions obtained from solving the model with each of the objective functions.

**Mathematical model sensitivity analysis**

Figure 5 shows the changes of the first objective function for the gradual increase in demand. As can be seen, with the increase in demand, the amount of covered demand also increases. In the last point, the amount of covered demand has decreased, and the reason is that the demand has exceeded the maximum capacity of the facility and it is not possible to establish a new facility, so one or a number of demand points are out of coverage. Figure 6 shows the changes in the second objective function, i.e. cost. Two types of changes can be seen in the diagram: minor changes due to the increase in coverage radius and the second jump between the fourth and fifth points due to the establishment of a new facility.
Figure 7 shows the changes of the first objective function with respect to the gradual increase in the facility's capacity level. As the capacity level increases, the amount of covered demand increases. This increase will either continue until there is no more demand for coverage, or there is a need to establish a new facility, but the establishment of a new facility is not cost-effective.

![Figure 7](image1.png)

**Figure 7. The changes of the first objective function in relation to the gradual increase of the facility capacity level**

Figure 8 shows the changes of the second objective function, i.e. cost, in relation to the increase in the facility's capacity level. As you can see, with the increase in the capacity level, the cost does not change significantly until point five, which has a significant decrease, and the reason for this is the reduction of a facility and as a result, the fixed cost of a facility decreases from the total cost, which is caused by the increase in the capacity of other facilities.

![Figure 8](image2.png)

**Figure 8. The changes of the second objective function, i.e. the cost, in relation to the increase in the capacity of the facilities**
Figure 9 shows the changes of the first objective function with the gradual increase of the maximum possible coverage radius of the facility. As can be seen, by increasing the maximum possible coverage radius of the facility, the amount of covered demand increases.

Figure 9. The changes of the first objective function with the gradual increase of the maximum possible coverage radius of the facility

Figure 10 shows the changes of the second objective function with the gradual increase of the maximum possible coverage radius of the facility. By increasing the coverage radius, it is possible to cover more demand with a smaller number of facilities and a larger coverage radius. By comparing the changes between the second and third points in Figure 4-8 and Figure 4-9, it can be seen that by increasing the maximum coverage radius, a fixed demand value can be covered with a lower number of facilities. The increases in Figure 9-4 are due to the increase in the coverage radius of the facilities.

Figure 10. Changes of the second objective function with the gradual increase of the maximum possible coverage radius of the facility
Calculation results

In Table 2, the values of the time to solve the model, the first objective function and the second objective function obtained for solving the model in the application mode and the solution mode with the GAMS solver are displayed. With the increase in the dimensions of the problems, the solving time with the GAMS solver increases significantly and the time difference of the solution increases exponentially.

Table 4. Comparison of the solution time and the values of the objective functions per solution with the GAMS solver

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<td>l</td>
<td>s</td>
<td>t</td>
<td>Solving time</td>
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Conclusion

In this research, first, the validation of the model is done by solving the sample problem. Then sensitivity analysis has been done on the main parameters of the model. Because it can never be stated definitively that the location of the hub will continue to function in any situation and one or a number of facilities may not be able to perform their services at any time, which may affect one of the possible reasons that are considered in the scenarios. Therefore, in this research, the possibility of the occurrence of scenarios has been considered, and this possibility is created for the decision makers to make decisions with a better and more realistic view and to have numbers and figures closer to the real values. One of the challenges in the issue of location of facilities is to consider facilities in different places in the same way and with similar services and capacity. This is despite the fact that the characteristics of the located points, the limitations, the type and amount of needs in each area are not necessarily similar to each other and these differences make the same facilities not efficient enough. In this research, the facilities can be different from various aspects. The facilities are considered hierarchically and at
three different levels. The facility capacity is not fixed and the capacity of each facility can be determined from three different levels by solving the model. The coverage radius of each facility is also different and is considered as a decision variable. Therefore, according to the conditions at each point of the network, facilities can have different characteristics. Considering the fact that considering the distance as a factor to improve the quality of demand coverage alone is not enough; therefore, considering limitations such as time, chain disturbances, along with paying attention to the distance criterion, can bring the model closer to the real world. Because the facilities considered in this research are health facilities and due to the importance of time to handle the demand in the health field, it can be useful to consider the queuing theory in this model.

Also considering that the important goals in improving a network of health facilities are not limited to considering the cost and the amount of covered demand, it is appropriate that other goals of maximizing the quality of receiving service or reducing the concentration of demand coverage by a facility are also considered. When facilities are considered as hub locations, it is possible that there are different relationships between different levels of facilities, and some of these relationships are considered in this research. Other types of these relationships can also be considered.

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References

