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Pairwise Generalized$_{\mathcal{F}}$ −Strongly Contra-Continuous Maps

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Abstract: In this paper the concept of pairwise generalized$_{\mathcal{F}}$ strongly contra continuous maps in generalized$_{\mathcal{F}}$ −bi-topological space have been introduced and several results have been proved by making the use of some counter examples.

Keywords: Generalized$_{\mathcal{F}}$ −bi-topological space, generalized$_{\mathcal{F}}$ contra-continuous maps, generalized$_{\mathcal{F}}$ semi-contra continuous maps

1. Introduction

Csaszar [6] introduced the notions of generalized topological spaces. He also introduced the notions of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. Bin Shahana [3-4] has introduced the concept of fuzzy pre-open sets and fuzzy α-open sets in fuzzy topological space. Thakur [17] has introduced the concept of fuzzy semi pre-open sets in fuzzy topological spaces. Beceren [2] introduced and studied the concept of strongly α-continuous functions, strong semi-continuity and fuzzy pre-continuity and investigate various characterizations. Further the author verified that fuzzy strongly α-continuous map is the stronger form of fuzzy α-continuous map. Palani Cheety [11] introduced the concept of generalized fuzzy topology and investigates various properties. Chang [5] has introduced the concept of fuzzy topological space as a generalization of topological space.

Kandil [8] introduced fuzzy bi-topological spaces in 1989. Nazir Ahmad Ahengar, et.al [9] introduced the concept of generalized$_{\mathcal{F}}$ − topology in which they characterise the several results in the context of generalized$_{\mathcal{F}}$ − topology . Further the authors [10] have given the concept of generalized$_{\mathcal{F}}$ − closure and interior.

In this paper we have introduced the concept of pairwise generalized$_{\mathcal{F}}$ strongly contra continuous map and studied its various relationships of these maps. The results have been shown by several counter examples.

Organization: Section 2 deals with the basic concepts and definitions related to generalized$_{\mathcal{F}}$ −bi-topological space. In section 3, we introduce the concept of pairwise generalized$_{\mathcal{F}}$ strongly contra continuous maps and studied various results in this context. Section 4 concludes the paper.

2. Preliminaries

Definition 2.1: Let $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a universal set X with the generalized$_{\mathcal{F}}$ − $\mathcal{T}$opologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called generalized$_{\mathcal{F}}$ −Bi-topological space

Definition 2.2: A fuzzy set A of generalized$_{\mathcal{F}}$ −bi-topological space $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ $(\mathrm{i}, \mathrm{j}) -$ semi − open set if $\mathrm{A} \subseteq \mathcal{T}_{\mathrm{j}} - \mathrm{Cl}_{\mathcal{F}}(\mathcal{T}_{\mathrm{i}} - \mathrm{Int}_{\mathcal{F}}(\mathrm{A}))$ while as a fuzzy set A of generalized$_{\mathcal{F}}$ bi-topological space $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ $(\mathrm{i}, \mathrm{j}) - \alpha -$ open set if $\mathrm{A} \subseteq \mathcal{T}_{\mathrm{j}} - \mathrm{Int}_{\mathcal{F}}(\mathcal{T}_{\mathrm{i}} - \mathrm{Cl}_{\mathcal{F}}\left(\mathcal{T}_{\mathrm{j}} - \mathrm{Int}_{\mathcal{F}}(\mathrm{A})\right))$

Definition 2.3: A fuzzy set A of generalized$_{\mathcal{F}}$ −bi-topological space $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ $(\mathrm{i}, \mathrm{j}) -$ pre − open set if $\mathrm{A} \subseteq \mathcal{T}_{\mathrm{i}} - \mathrm{Int}_{\mathcal{F}}(\mathcal{T}_{\mathrm{j}} - \mathrm{Cl}_{\mathcal{F}}(\mathrm{A})$ while as a fuzzy set A of generalized$_{\mathcal{F}}$ −bi-topological space $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ $(\mathrm{i}, \mathrm{j}) - \beta -$ open set if $\mathrm{A} \subseteq \mathcal{T}_{\mathrm{j}} - \mathrm{Cl}_{\mathcal{F}}(\mathcal{T}_{\mathrm{i}} - \mathrm{Int}_{\mathcal{F}}\left(\mathcal{T}_{\mathrm{j}} - \mathrm{Cl}_{\mathcal{F}}(\mathrm{A})\right))$.

Definition 2.4: A mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is said to be pairwise $\text{generalized}_{\mathcal{F}}-$ continuous map if $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\psi_1)$ and $\mathcal{F}:(X,\mathcal{T}_2)\to(Y,\psi_2)$ are $\text{generalized}_{\mathcal{F}}-$ continuous maps

Definition 2.5: A mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is said to be pairwise $\text{generalized}_{\mathcal{F}}-\text{semi}-$ continuous map if inverse image of every $\psi_i-\text{generalized}_{\mathcal{F}}$ open set in Y is $\text{generalized}_{\mathcal{F}}$ $(i,j)-\text{semi}-$ open set in X

Definition 2.6: A mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is said to be pairwise $\text{generalized}_{\mathcal{F}}-\text{pre}-$ continuous map if inverse image of every $\psi_i-\text{generalized}_{\mathcal{F}}$ open set in Y is $\text{generalized}_{\mathcal{F}}$ $(i,j)-\text{pre}-$ open set in X

Definition 2.7: A mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is said to be pairwise $\text{generalized}_{\mathcal{F}}-\alpha-$ continuous map if inverse image of every $\psi_i-\text{generalized}_{\mathcal{F}}$ open set in Y is $\text{generalized}_{\mathcal{F}}$ $(i,j)-\alpha-$ open set in X

Definition 2.8: A mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is said to be pairwise $\text{generalized}_{\mathcal{F}}-\beta-$ continuous map if inverse image of every $\psi_i-\text{generalized}_{\mathcal{F}}$ open set in Y is $\text{generalized}_{\mathcal{F}}$ $(i,j)-\beta-$ open set in X

Definition 2.9: Consider the two $\text{generalized}_{\mathcal{F}}$ −topological space $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ on X and Y respectively. Then the mapping $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is said to be $\text{generalized}_{\mathcal{F}}-$ contra continuous map if inverse image of every $\mathcal{T}_2-\text{generalized}_{\mathcal{F}}$ open set in Y is $\text{generalized}_{\mathcal{F}}$ $\mathcal{T}_1-$ closed set in X

Definition 2.10: Consider the two $\text{generalized}_{\mathcal{F}}-$ topological space $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ on X and Y respectively. Then the mapping $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is said to be $\text{generalized}_{\mathcal{F}}$ −semi-contra continuous map if inverse image of every $\text{generalized}_{\mathcal{F}}$ $\mathcal{T}_2-$ open set in Y is $\text{generalized}_{\mathcal{F}}$ $\mathcal{T}_1$ −semi-closed in X

Definition 2.11: Consider the two $\text{generalized}_{\mathcal{F}}-$ topological space $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ on X and Y respectively. Then the mapping $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is said to be $\text{generalized}_{\mathcal{F}}$ −pre-contra continuous map if inverse image of every $\text{generalized}_{\mathcal{F}}$ $\mathcal{T}_2-$ open set in Y is $\text{generalized}_{\mathcal{F}}$ $\mathcal{T}_1$ −pre-closed in X

3. Pairwise $\text{GENERALIZED}_{\mathcal{F}}-$ Strongly Contra-Continuous Maps

Definition 3.1: Let $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ be two $\text{generalized}_{\mathcal{F}}-$ topological space and $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$. Then $\mathcal{F}$ is said to be $\text{generalized}_{\mathcal{F}}$ strongly continuous map if $\mathcal{F}^{-1}(\lambda)$ is $\text{generalized}_{\mathcal{F}}$ α-open in $(X,\mathcal{T}_1)$ for each $\text{generalized}_{\mathcal{F}}$semi-open set λ in $(Y,\mathcal{T}_2)$

Definition 3.2: Let $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ be two $\text{generalized}_{\mathcal{F}}-$ topological space and $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$. Then $\mathcal{F}$ is said to be $\text{generalized}_{\mathcal{F}}$ strongly contra-continuous map if $\mathcal{F}^{-1}(\lambda)$ is $\text{generalized}_{\mathcal{F}}$ semi-open in $(X,\mathcal{T}_1)$ for each $\text{generalized}_{\mathcal{F}}$α-open set λ in $(Y,\mathcal{T}_2)$

Example 3.1: Let $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$ and $A=\{(x_1,0.1),(x_2,0.2)\}$, $B=\{(x_1,0.2),(x_2,0.1)\}$, $C=\{(x_1,0.2),(x_2,0.2)\}$, $D=\{(x_1,0.8),(x_2,0.9)\}$, $E=\{(x_1,0.9),(x_2,0.8)\}$, $F=\{(x_1,0.9),(x_2,0.9)\}$, $G=\{(y_1,0.7),(y_2,0.8)\}$, $H=\{(y_1,0.8),(y_2,0.7)\}$ and $I=\{(y_1,0.8),(y_2,0.8)\}$. Clearly $\mathcal{T}_1=\{0,A,B,C,D,E,F,1\}$ and $\mathcal{T}_2=\{0,G,H,I,1\}$ are $\text{generalized}_{\mathcal{F}}-$ topological space on sets X and Y respectively. Consider the $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ defined as $\mathcal{F}(x_1)=y_1$ and $\mathcal{F}(x_2)=y_2$. Therefore $\mathcal{F}^{-1}(G)=\{(x_1,0.7),(x_2,0.8)\}$, $\mathcal{F}^{-1}(H)=\{(x_1,0.8),(x_2,0.7)\}$ and $\mathcal{F}^{-1}(I)=\{(x_1,0.8),(x_2,0.8)\}$. This shows that the inverse image of every $\text{generalized}_{\mathcal{F}}$ $\mathcal{T}_2-\alpha-$ open set in Y is $\text{generalized}_{\mathcal{F}}$ $\mathcal{T}_1-\text{semi}-$ open set in X. Clearly $\mathcal{F}$ is $\text{generalized}_{\mathcal{F}}$ strongly contra-continuous map.

Definition 3.3: Consider the two $\text{generalized}_{\mathcal{F}}$ −bi-topological space $(X,\mathcal{T}_1,\mathcal{T}_2)$ and (Y,ψ_1,ψ_2) on X and Y respectively. Then the mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is said to be pairwise $\text{generalized}_{\mathcal{F}}-$ contra continuous map if $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\psi_1)$ and $\mathcal{F}:(X,\mathcal{T}_2)\to(Y,\psi_2)$ are $\text{generalized}_{\mathcal{F}}-$ contra continuous maps

Definition 3.4: Consider the two $\text{generalized}_{\mathcal{F}}$ −bi-topological space $(X,\mathcal{T}_1,\mathcal{T}_2)$ and (Y,ψ_1,ψ_2) on X and Y respectively. Then the mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is said to be pairwise $\text{generalized}_{\mathcal{F}}-$strongly contra continuous map if $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\psi_1)$ and $\mathcal{F}:(X,\mathcal{T}_2)\to(Y,\psi_2)$ are $\text{generalized}_{\mathcal{F}}$ strongly contra-continuous maps

Example 3.2: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$ .Consider fuzzy sets $A = \{(x_1, 0.7), (x_2, 0.4)\}$, $B = \{(x_1, 0.5), (x_2, 0.6)\}$, $C = \{(x_1, 0.5), (x_2, 0.4)\}$, $D = \{(x_1, 0.3), (x_2, 0.6)\}$, $E = \{(x_1, 0.7), (x_2, 0.6)\}$, $F = \{(x_1, 0.8), (x_2, 0.5)\}$, $G = \{(x_1, 0.4), (x_2, 0.7)\}$, $H = \{(x_1, 0.4), (x_2, 0.5)\}$, $I = \{(x_1, 0.8), (x_2, 0.7)\}$ and $J = \{(x_1, 0.8), (x_2, 0.3)\}$ on X. Again $P = \{(y_1, 0.3), (y_2, 0.6)\}$, $Q = \{(y_1, 0.5), (y_2, 0.4)\}$, $R = \{(y_1, 0.5), (y_2, 0.6)\}$, $S = \{(y_1, 0.2), (y_2, 0.5)\}$, $T = \{(y_1, 0.6), (y_2, 0.3)\}$ and $U = \{(y_1, 0.6), (y_2, 0.5)\}$ on Y. Let $\mathcal{T}_1 = \{0, A, B, C, D, E, 1\}$, $\mathcal{T}_2 = \{0, F, G, H, I, J, 1\}$, $\psi_1 = \{0, P, Q, R, 1\}$ and $\psi_2 = \{0, S, T, U, 1\}$ be the generalized$_{\mathcal{F}}$ $-$ topologies defined on X and Y.

Then we define the mappings $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ such that $\mathcal{F}(x_1) = y_1$ and $\mathcal{F}(x_2) = y_2$. Thus the inverse image of every generalized$_{\mathcal{F}}$ $\psi_1 - \alpha -$ open set and generalized$_{\mathcal{F}}$ $\psi_2 - \alpha -$ open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$semi-open and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$semi-open in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ $-$ strongly contra continuous maps. Thus the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ $-$strongly contra continuous map

Proposition 3.1: In a generalized$_{\mathcal{F}}$ bi-topological space every pairwise generalized$_{\mathcal{F}}$ $-$contra continuous map is pairwise generalized$_{\mathcal{F}}$ $-$strongly contra continuous map

Proof: Let λ and β be generalized$_{\mathcal{F}}$ $\psi_1 -$ open set (generalized$_{\mathcal{F}}$ $\psi_1 - \alpha$ -open set) and generalized$_{\mathcal{F}}$ $\psi_2 -$ open set (generalized$_{\mathcal{F}}$ $\psi_2 - \alpha$ -open set) respectively in Y. Since $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ $-$ contra continuous map, we have $\mathcal{F}^{-1}(\lambda)$ and $\mathcal{F}^{-1}(\beta)$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$closed sets in X and hence generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$semi-open and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$semi-open in X., therefore $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ $-$strongly-contra continuous map

Remark 3.1: The converse of Proposition 3.1 is not true in general as shown in Example 3.3

Example 3.3: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$ .Consider fuzzy sets $A = \{(x_1, 0.7), (x_2, 0.4)\}$, $B = \{(x_1, 0.5), (x_2, 0.6)\}$, $C = \{(x_1, 0.5), (x_2, 0.4)\}$, $D = \{(x_1, 0.4), (x_2, 0.6)\}$, $E = \{(x_1, 0.7), (x_2, 0.6)\}$, $F = \{(x_1, 0.8), (x_2, 0.5)\}$, $G = \{(x_1, 0.4), (x_2, 0.7)\}$, $H = \{(x_1, 0.4), (x_2, 0.5)\}$, $I = \{(x_1, 0.8), (x_2, 0.7)\}$ and $J = \{(x_1, 0.3), (x_2, 0.5)\}$ on X. Again $P = \{(y_1, 0.3), (y_2, 0.6)\}$, $Q = \{(y_1, 0.5), (y_2, 0.4)\}$, $R = \{(y_1, 0.5), (y_2, 0.6)\}$, $S = \{(y_1, 0.4), (y_2, 0.6)\}$, $T = \{(y_1, 0.2), (y_2, 0.5)\}$, $U = \{(y_1, 0.6), (y_2, 0.3)\}$, $V = \{(y_1, 0.6), (y_2, 0.5)\}$and $W = \{(y_1, 0.3), (y_2, 0.5)\}$ on Y. Let $\mathcal{T}_1 = \{0, A, B, C, D, E, 1\}$, $\mathcal{T}_2 = \{0, F, G, H, I, J, 1\}$, $\psi_1 = \{0, P, Q, R, S, 1\}$ and $\psi_2 = \{0, T, U, V, W, 1\}$ be the generalized$_{\mathcal{F}}$ $-$ topologies defined on X and Y.

Now we define the mappings $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ such that $\mathcal{F}(x_1) = y_1$ and $\mathcal{F}(x_2) = y_2$. Therefore $\mathcal{F}^{-1}(0) = 1'$, $\mathcal{F}^{-1}(1) = 0'$, $\mathcal{F}^{-1}(P) = \{(x_1, 0.3), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(Q) = \{(x_1, 0.5), (x_2, 0.4)\}$, $\mathcal{F}^{-1}(R) = \{(x_1, 0.5), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(S) = \{(x_1, 0.4), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(T) = \{(x_1, 0.2), (x_2, 0.5)\}$, $\mathcal{F}^{-1}(U) = \{(x_1, 0.6), (x_2, 0.3)\}$, $\mathcal{F}^{-1}(V) = \{(x_1, 0.6), (x_2, 0.5)\}$and $\mathcal{F}^{-1}(W) = \{(x_1, 0.3), (x_2, 0.5)\}$. This shows that the inverse image of every generalized$_{\mathcal{F}}$ $\psi_1 - \alpha -$ open set and generalized$_{\mathcal{F}}$ $\psi_2 - \alpha -$ open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$semi-open and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$semi-open in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ $-$strongly contra continuous maps but not generalized$_{\mathcal{F}}$ $-$ contra continuous map because $\mathcal{F}^{-1}(S) = \{(x_1, 0.4), (x_2, 0.6)\}$ and $\mathcal{F}^{-1}(W) = \{(x_1, 0.3), (x_2, 0.5)\}$, where $\{(x_1, 0.4), (x_2, 0.6)\}$ and $\{(x_1, 0.3), (x_2, 0.5)\}$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$ semi-open and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$semi-open but not generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$closed in X

Definition 3.5: Consider the two generalized$_{\mathcal{F}}$ $-$bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and (Y, ψ_1, ψ_2) on X and Y respectively. Then the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ $-$semi-contra continuous map (pairwise generalized$_{\mathcal{F}}$ $- \alpha$-contra continuous map) if $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ $-$ semi-contra continuous maps (generalized$_{\mathcal{F}}$ $- \alpha$-contra continuous maps)

Remark 3.2: In a generalized$_{\mathcal{F}}$ bi-topological space the concepts of pairwise generalized$_{\mathcal{F}}$ $-$semi-contra continuous map (pairwise generalized$_{\mathcal{F}}$ $- \alpha$-contra continuous maps) and pairwise generalized$_{\mathcal{F}}$ $-$strongly contra continuous maps are independent as shown in Example 3.4 and Example 3.5

Example 3.4: In Example 3.3, the mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous map but not pairwise generalized$_{\mathcal{F}}$ −semi-contra continuous map (pairwise generalized$_{\mathcal{F}}$ −α-contra continuous map) because . $\mathcal{F}^{-1}(S)=\{(x_1,0.4),(x_2,0.6)\}$, and $\mathcal{F}^{-1}(W)=\{(x_1,0.3),(x_2,0.5)\}$, where $\{(x_1,0.4),(x_2,0.6)\}$ and $\{(x_1,0.3),(x_2,0.5)\}$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −semi-open (generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − α-open) and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −semi-open (generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − α-open) but not generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −closed in X

Example 3.5: Let $X=\{x_1,x_2\}$ and $Y=\{y_1,y_2\}$ .Consider fuzzy sets $A=\{(x_1,0.7),(x_2,0.4)\}$, $B=\{(x_1,0.5),(x_2,0.6)\}$, $C=\{(x_1,0.5),(x_2,0.4)\}$, $D=\{(x_1,0.3),(x_2,0.6)\}$, $E=\{(x_1,0.7),(x_2,0.6)\}$, $F=\{(x_1,0.8),(x_2,0.5)\}$, $G=\{(x_1,0.4),(x_2,0.7)\}$, $H=\{(x_1,0.4),(x_2,0.5)\}$, $I=\{(x_1,0.8),(x_2,0.7)\}$ and $J=\{(x_1,0.8),(x_2,0.3)\}$ on X. Again $M=\{(y_1,0.3),(y_2,0.6)\}$, $N=\{(y_1,0.5),(y_2,0.4)\}$, $P=\{(y_1,0.5),(y_2,0.6)\}$, $Q=\{(y_1,0.4),(y_2,0.4)\}$, $R=\{(y_1,0.4),(y_2,0.6)\}$, $S=\{(y_1,0.2),(y_2,0.5)\}$, $T=\{(y_1,0.6),(y_2,0.3)\}$, $U=\{(y_1,0.6),(y_2,0.5)\}$, $V=\{(y_1,0.3),(y_2,0.3)\}$and $W=\{(y_1,0.3),(y_2,0.5)\}$ on Y. Let $\mathcal{T}_1=\{0,A,B,C,D,E,1\}$, $\mathcal{T}_2=\{0,F,G,H,I,J,1\}$, $\psi_1=\{0,M,N,P,Q,R,1\}$ and $\psi_2=\{0,S,T,U,V,W,1\}$ be the generalized$_{\mathcal{F}}$ − topologies defined on X and Y.

Now we define the mappings $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\psi_1)$ and $\mathcal{F}:(X,\mathcal{T}_2)\to(Y,\psi_2)$ such that $\mathcal{F}(x_1)=y_1$ and $\mathcal{F}(x_2)=y_2$. Therefore $\mathcal{F}^{-1}(0)=1'$, $\mathcal{F}^{-1}(1)=0'$, $\mathcal{F}^{-1}(M)=\{(x_1,0.3),(x_2,0.6)\}$, $\mathcal{F}^{-1}(N)=\{(x_1,0.5),(x_2,0.4)\}$, $\mathcal{F}^{-1}(P)=\{(x_1,0.5),(x_2,0.6)\}$, $\mathcal{F}^{-1}(Q)=\{(x_1,0.4),(x_2,0.4)\}$, $\mathcal{F}^{-1}(R)=\{(x_1,0.4),(x_2,0.6)\}$, $\mathcal{F}^{-1}(S)=\{(x_1,0.2),(x_2,0.5)\}$, $\mathcal{F}^{-1}(T)=\{(x_1,0.6),(x_2,0.3)\}$, $\mathcal{F}^{-1}(U)=\{(x_1,0.6),(x_2,0.5)\}$, $\mathcal{F}^{-1}(V)=\{(x_1,0.3),(x_2,0.3)\}$ and $\mathcal{F}^{-1}(W)=\{(x_1,0.3),(x_2,0.5)\}$. This shows that the inverse image of every generalized$_{\mathcal{F}}$ ψ_1 − open set and generalized$_{\mathcal{F}}$ ψ_2 − open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − semi-closed (generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − α -closed) and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − semi-closed (generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − α -closed) in X respectively. Hence $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\psi_1)$ and $\mathcal{F}:(X,\mathcal{T}_2)\to(Y,\psi_2)$ are generalized$_{\mathcal{F}}$ −semi-contra continuous map (generalized$_{\mathcal{F}}$ − α-contra continuous map) but not generalized$_{\mathcal{F}}$ − strongly contra continuous map because $\mathcal{F}^{-1}(Q)=\{(x_1,0.4),(x_2,0.4)\}$ and $\mathcal{F}^{-1}(V)=\{(x_1,0.3),(x_2,0.3)\}$, where $\{(x_1,0.4),(x_2,0.4)\}$ and $\{(x_1,0.3),(x_2,0.3)\}$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − semi-closed (generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − α -closed) and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − semi-closed (generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − α -closed) but not generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − semi-open and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − semi-open in X. Hence the mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is pairwise generalized$_{\mathcal{F}}$ −semi-contra continuous map (pairwise generalized$_{\mathcal{F}}$ − α-contra continuous map) but not pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous map

Definition 3.6: Consider the two generalized$_{\mathcal{F}}$ −bi-topological space $(X,\mathcal{T}_1,\mathcal{T}_2)$ and (Y,ψ_1,ψ_2) on X and Y respectively. Then the mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ −pre-contra continuous map (pairwise generalized$_{\mathcal{F}}$ − β-contra continuous map) if $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\psi_1)$ and $\mathcal{F}:(X,\mathcal{T}_2)\to(Y,\psi_2)$ are generalized$_{\mathcal{F}}$ − pre-contra continuous maps (generalized$_{\mathcal{F}}$ − β-contra continuous maps)

Remark 3.3: In a generalized$_{\mathcal{F}}$ bi-topological space the concepts of pairwise generalized$_{\mathcal{F}}$ −pre-contra continuous map (pairwise generalized$_{\mathcal{F}}$ − β-contra continuous maps) and pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous maps are independent as shown in Example 3.6 and Example 3.7

Example 3.6: In Example 3.3, the mapping $\mathcal{F}:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\psi_1,\psi_2)$ is pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous map but not pairwise generalized$_{\mathcal{F}}$ −pre-contra continuous map (pairwise generalized$_{\mathcal{F}}$ − β-contra continuous map) because . $\mathcal{F}^{-1}(S)=\{(x_1,0.4),(x_2,0.6)\}$, and $\mathcal{F}^{-1}(W)=\{(x_1,0.3),(x_2,0.5)\}$, where $\{(x_1,0.4),(x_2,0.6)\}$ and $\{(x_1,0.3),(x_2,0.5)\}$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −pre-open (generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − β-open) and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − pre-open (generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − β -open) but not generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −closed in X

Example 3.7: Let $X=\{x_1,x_2\}$ and $Y=\{y_1,y_2\}$ .Consider fuzzy sets $A=\{(x_1,0.7),(x_2,0.4)\}$, $B=\{(x_1,0.5),(x_2,0.6)\}$, $C=\{(x_1,0.5),(x_2,0.4)\}$, $D=\{(x_1,0.3),(x_2,0.6)\}$, $E=\{(x_1,0.7),(x_2,0.6)\}$, $F=\{(x_1,0.8),(x_2,0.5)\}$, $G=\{(x_1,0.4),(x_2,0.7)\}$, $H=\{(x_1,0.4),(x_2,0.5)\}$, $I=\{(x_1,0.8),(x_2,0.7)\}$ and $J=\{(x_1,0.8),(x_2,0.3)\}$ on X. Again $M=\{(y_1,0.3),(y_2,0.6)\}$, $N=\{(y_1,0.5),(y_2,0.4)\}$, $P=\{(y_1,0.5),(y_2,0.6)\}$, $Q=\{(y_1,0.3),(y_2,0.3)\}$, $R=\{(y_1,0.2),(y_2,0.5)\}$, $S=\{(y_1,0.6),(y_2,0.3)\}$, $T=\{(y_1,0.6),(y_2,0.5)\}$ and $U=\{(y_1,0.2),(y_2,0.2)\}$ on Y. Let $\mathcal{T}_1=\{0,A,B,C,D,E,1\}$, $\mathcal{T}_2=\{0,F,G,H,I,J,1\}$,

$\psi_1 = \{0, M, N, P, Q, 1\}$ and $\psi_2 = \{0, R, S, T, U, 1\}$ be the generalized$_{\mathcal{F}}$ − topologies defined on X and Y.

Now we define the mappings $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ such that $\mathcal{F}(x_1) = y_1$ and $\mathcal{F}(x_2) = y_2$. Therefore $\mathcal{F}^{-1}(0) = 1'$, $\mathcal{F}^{-1}(1) = 0'$, $\mathcal{F}^{-1}(M) = \{(x_1, 0.3), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(N) = \{(x_1, 0.5), (x_2, 0.4)\}$, $\mathcal{F}^{-1}(P) = \{(x_1, 0.5), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(Q) = \{(x_1, 0.3), (x_2, 0.3)\}$, $\mathcal{F}^{-1}(R) = \{(x_1, 0.2), (x_2, 0.5)\}$, $\mathcal{F}^{-1}(S) = \{(x_1, 0.6), (x_2, 0.3)\}$, $\mathcal{F}^{-1}(T) = \{(x_1, 0.6), (x_2, 0.5)\}$ and $\mathcal{F}^{-1}(U) = \{(x_1, 0.2), (x_2, 0.2)\}$. This shows that the inverse image of every generalized$_{\mathcal{F}}$ ψ_1 − open set and generalized$_{\mathcal{F}}$ ψ_2 − open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − pre-closed (generalized$_{\mathcal{F}}$ $\mathcal{T}_1 - \beta$ -closed) and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − pre-closed (generalized$_{\mathcal{F}}$ $\mathcal{T}_2 - \beta$ -closed) in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ −pre-contra continuous map (generalized$_{\mathcal{F}}$ − β-contra continuous map) but not generalized$_{\mathcal{F}}$ − strongly contra continuous map because $\mathcal{F}^{-1}(Q) = \{(x_1, 0.3), (x_2, 0.3)\}$ and $\mathcal{F}^{-1}(U) = \{(x_1, 0.2), (x_2, 0.2)\}$, where $\{(x_1, 0.3), (x_2, 0.3)\}$ and $\{(x_1, 0.2), (x_2, 0.2)\}$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −pre-closed (generalized$_{\mathcal{F}}$ $\mathcal{T}_1 - \beta$-closed) and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −pre-closed (generalized$_{\mathcal{F}}$ $\mathcal{T}_2 - \beta$-closed) but not generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −semi-open and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − semi-open in X. Hence the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ − pre-contra continuous map (pairwise generalized$_{\mathcal{F}}$ − β-contra continuous map) but not pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous map

4. Conclusion

In this paper we have introduced the concept of pairwise generalized$_{\mathcal{F}}$ strongly contra maps in generalized$_{\mathcal{F}}$bi-topological spaces and establish its relationships with other maps like pairwise generalized$_{\mathcal{F}}$contra continuous maps, pairwise generalized$_{\mathcal{F}}$ -semi-contra continuous maps (pairwise generalized$_{\mathcal{F}}$ − α-contra continuous maps) and pairwise generalized$_{\mathcal{F}}$ −pre-contra continuous maps (pairwise generalized$_{\mathcal{F}}$ − β-contra continuous maps). The results have been supported by some suitable counter examples. We conclude the results in this paper given below:

Pairwise generalized$_{\mathcal{F}}$ −contra continuous map ⇒⇍ pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous map

Pairwise generalized$_{\mathcal{F}}$ −semi-contra continuous map ⇎ pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous maps

Pairwise generalized$_{\mathcal{F}}$ − α-contra continuous maps ⇎ pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous maps

Pairwise generalized$_{\mathcal{F}}$ −pre-contra continuous map ⇎ pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous maps

Pairwise generalized$_{\mathcal{F}}$ − β-contra continuous maps) ⇎ pairwise generalized$_{\mathcal{F}}$ −strongly contra continuous maps

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