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Pairwise Generalized$_{\mathcal{F}}$ −Contra Continuous Maps

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Abstract: In this paper the concept of pairwise generalized$_{\mathcal{F}}$ −contra continuous maps in generalized$_{\mathcal{F}}$ −bi-topological space has been introduced and several results have been proved by making the use of some counter examples.

Keywords: Generalized$_{\mathcal{F}}$ −bi-topological space, generalized$_{\mathcal{F}}$ contra-continuous maps, generalized$_{\mathcal{F}}$ semi-contra continuous maps

1. Introduction

Bin Shahana [3-4] has introduced the concept of fuzzy pre-open sets and fuzzy α-open sets in fuzzy topological space. Thakur [20] has introduced the concept of fuzzy semi pre-open sets in fuzzy topological spaces. Beceren [2] introduced and studied the concept of strongly α-continuous functions, strong semi-continuity and fuzzy pre-continuity and investigate various characterizations. Further the author verified that fuzzy strongly α-continuous map is the stronger form of fuzzy α-continuous map. Csaszar [6] introduced the notions of generalized topological spaces. He also introduced the notions of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. Palani Cheety [14] introduced the concept of generalized fuzzy topology and investigates various properties. Chang [5] has introduced the concept of fuzzy topological space as a generalization of topological space. Kandil [11] introduced fuzzy bi-topological spaces in 1989. Nazir Ahmad Ahengar, et.al [12] introduced the concept of generalized$_{\mathcal{F}}$ − topology in which they characterise the several results in the context of generalized$_{\mathcal{F}}$ − topology. Further the authors [13] have given the concept of generalized$_{\mathcal{F}}$ − closure and interior.

In this paper we have introduced the concept of pairwise generalized$_{\mathcal{F}}$ contra continuous map and studied its various relationships of these maps. The results have been shown by several counter examples.

Organization: Section 2 deals with the basic concepts and definitions related to generalized$_{\mathcal{F}}$ −bi-topological space. In section 3, we introduce the concept of pairwise generalized$_{\mathcal{F}}$ contra continuous maps and studied various results in this context. Section 4 concludes the paper.

2. Preliminaries

Definition 2.1: Let $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a universal set X with the generalized$_{\mathcal{F}}$ − $\mathcal{T}$opologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called generalized$_{\mathcal{F}}$ −Bi-topological space

Definition 2.2: A fuzzy set A of generalized$_{\mathcal{F}}$ −bi-topological space $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ $(\mathrm{i}, \mathrm{j}) - \mathrm{semi} - \mathrm{open\ set}$ if $\mathrm{A} \subseteq \mathcal{T}_\mathrm{j} - \mathrm{Cl}_{\mathcal{F}}(\mathcal{T}_\mathrm{i} - \mathrm{Int}_{\mathcal{F}}(\mathrm{A}))$

Definition 2.3: A fuzzy set A of generalized$_{\mathcal{F}}$ −bi-topological space $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ $(\mathrm{i}, \mathrm{j}) - \mathrm{semi} - \mathrm{closed\ set}$ if $\mathrm{A}^\mathrm{c} = 1 - \mathrm{A}$ is generalized$_{\mathcal{F}}$ $(\mathrm{i}, \mathrm{j}) - \mathrm{semi} - \mathrm{open\ set}$

Definition 2.4: A fuzzy set A of generalized$_{\mathcal{F}}$ −bi-topological space $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ $(\mathrm{i}, \mathrm{j}) - \mathrm{pre} - \mathrm{open\ set}$ if $\mathrm{A} \subseteq \mathcal{T}_\mathrm{i} - \mathrm{Int}_{\mathcal{F}}(\mathcal{T}_\mathrm{j} - \mathrm{Cl}_{\mathcal{F}}(\mathrm{A}))$

Definition 2.5: A fuzzy set A of generalized$_{\mathcal{F}}$ −bi-topological space $(\mathrm{X}, \mathcal{T}_1, \mathcal{T}_2)$ is calledgeneralized$_{\mathcal{F}}$ $(\mathrm{i}, \mathrm{j}) -$

pre − closed set if $A^c = 1 - A$ isgeneralized$_{\mathcal{F}}$ (i, j) − pre − open set.

Definition 2.6: A fuzzy set A of generalized$_{\mathcal{F}}$ −bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ (i, j) − β − open set if $A \subseteq \mathcal{T}_j - Cl_{\mathcal{F}}(\mathcal{T}_i - Int_{\mathcal{F}}\left(\mathcal{T}_j - Cl_{\mathcal{F}}(A)\right))$.

Definition 2.7: A fuzzy set A of generalized$_{\mathcal{F}}$ −bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ (i, j) − β − closed set if $A^c = 1 - A$ isgeneralized$_{\mathcal{F}}$ (i, j) − β − open set.

Definition 2.8: A fuzzy set A of generalized$_{\mathcal{F}}$ bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ is called generalized$_{\mathcal{F}}$ (i, j) − α − open set if $A \subseteq \mathcal{T}_j - Int_{\mathcal{F}}(\mathcal{T}_i - Cl_{\mathcal{F}}\left(\mathcal{T}_j - Int_{\mathcal{F}}(A)\right))$

Definition 2.9: A mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ − continuous map if $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ − continuous maps

Definition 2.10: A mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ − semi − continuous map if inverse image of every ψ_i − generalized$_{\mathcal{F}}$ open set in Y is generalized$_{\mathcal{F}}$ (i, j) − semi − open set in X

Definition 2.11: A mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ − pre − continuous map if inverse image of every ψ_i − generalized$_{\mathcal{F}}$ open set in Y is generalized$_{\mathcal{F}}$ (i, j) − pre − open set in X

Definition 2.12: A mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ − irresolute map if inverse image of every generalized$_{\mathcal{F}}$ (i, j) − semi − open set in Y is generalized$_{\mathcal{F}}$ (i, j) − semi − open set in X

Definition 2.13: A mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ − α − continuous map if inverse image of every ψ_i − generalized$_{\mathcal{F}}$ open set in Y is generalized$_{\mathcal{F}}$ (i, j) − α − open set in X

Definition 2.14: A mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ − β − continuous map if inverse image of every ψ_i − generalized$_{\mathcal{F}}$ open set in Y is generalized$_{\mathcal{F}}$ (i, j) − β − open set in X

3. Pairwise GENERALIZED$_{\mathcal{F}}$ −Contra Continuous Maps

Definition 3.1: Consider the two generalized$_{\mathcal{F}}$ −topological space $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ on X and Y respectively. Then the mapping $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is said to be generalized$_{\mathcal{F}}$ − contra continuous map if inverse image of every $\mathcal{T}_2$ − generalized$_{\mathcal{F}}$ open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − closed set in X

Example 3.1: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Consider fuzzy sets $A = \{(x_1, 0.7), (x_2, 0.4)\}$, $B = \{(x_1, 0.5), (x_2, 0.6)\}$, $C = \{(x_1, 0.5), (x_2, 0.4)\}$, $D = \{(x_1, 0.3), (x_2, 0.6)\}$and $E = \{(x_1, 0.7), (x_2, 0.6)\}$on X. Also $P = \{(y_1, 0.3), (y_2, 0.6)\}$, $Q = \{(y_1, 0.5), (y_2, 0.4)\}$ and $R = \{(y_1, 0.5), (y_2, 0.6)\}$ on Y. Let $\mathcal{T}_1 = \{0, A, B, C, D, E, 1\}$, $\mathcal{T}_2 = \{0, P, Q, R, 1\}$ be the generalized$_{\mathcal{F}}$ − topologies defined on X and Y. Then we define a mapping $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ such that $\mathcal{F}(x_1) = y_1$ and $\mathcal{F}(x_2) = y_2$. Therefore $\mathcal{F}^{-1}(0) = 1'$, $\mathcal{F}^{-1}(1) = 0'$, $\mathcal{F}^{-1}(P) = \{(x_1, 0.3), (x_2, 0.6)\} = A'$, $\mathcal{F}^{-1}(Q) = \{(x_1, 0.5), (x_2, 0.4)\} = B'$ and $\mathcal{F}^{-1}(R) = \{(x_1, 0.5), (x_2, 0.6)\} = C'$. This shows that the inverse image of every generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − closed set in X. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is generalized$_{\mathcal{F}}$ − contra continuous map

Definition 3.2: Consider the two generalized$_{\mathcal{F}}$ −bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and (Y, ψ_1, ψ_2) on X and Y respectively. Then the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ − contra continuous map if $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ − contra continuous maps

Example 3.2: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$ .Consider fuzzy sets $A = \{(x_1, 0.7), (x_2, 0.4)\}$, $B = \{(x_1, 0.5), (x_2, 0.6)\}$, $C = \{(x_1, 0.5), (x_2, 0.4)\}$, $D = \{(x_1, 0.3), (x_2, 0.6)\}$, $E = \{(x_1, 0.7), (x_2, 0.6)\}$, $F = \{(x_1, 0.8), (x_2, 0.5)\}$, $G = \{(x_1, 0.4), (x_2, 0.7)\}$, $H = \{(x_1, 0.4), (x_2, 0.5)\}$, $I = \{(x_1, 0.8), (x_2, 0.7)\}$ and $J =$

$\{(x_1, 0.8), (x_2, 0.3)\}$ on X. Again $P = \{(y_1, 0.3), (y_2, 0.6)\}$, $Q = \{(y_1, 0.5), (y_2, 0.4)\}$, $R = \{(y_1, 0.5), (y_2, 0.6)\}$, $S = \{(y_1, 0.2), (y_2, 0.5)\}$, $T = \{(y_1, 0.6), (y_2, 0.3)\}$ and $U = \{(y_1, 0.6), (y_2, 0.5)\}$ on Y. Let $\mathcal{T}_1 = \{0, A, B, C, D, E, 1\}$, $\mathcal{T}_2 = \{0, F, G, H, I, J, 1\}$, $\psi_1 = \{0, P, Q, R, 1\}$ and $\psi_2 = \{0, S, T, U, 1\}$ be the generalized$_{\mathcal{F}}$ − topologies defined on X and Y. Then we define the mappings $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ such that $\mathcal{F}(x_1) = y_1$ and $\mathcal{F}(x_2) = y_2$. Therefore $\mathcal{F}^{-1}(0) = 1'$, $\mathcal{F}^{-1}(1) = 0'$, $\mathcal{F}^{-1}(P) = \{(x_1, 0.3), (x_2, 0.6)\} = A'$, $\mathcal{F}^{-1}(Q) = \{(x_1, 0.5), (x_2, 0.4)\} = B'$, $\mathcal{F}^{-1}(R) = \{(x_1, 0.5), (x_2, 0.6)\} = C'$, $\mathcal{F}^{-1}(S) = \{(x_1, 0.2), (x_2, 0.5)\} = F'$, $\mathcal{F}^{-1}(T) = \{(x_1, 0.6), (x_2, 0.3)\} = G'$ and $\mathcal{F}^{-1}(U) = \{(x_1, 0.6), (x_2, 0.5)\} = H'$. This shows that the inverse image of every generalized$_{\mathcal{F}}$ ψ_1 − open set and generalized$_{\mathcal{F}}$ ψ_2 − open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − closed set and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − closed set in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ − contra continuous maps. Thus the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ − contra continuous map

Definition 3.3: Consider the two generalized$_{\mathcal{F}}$ −topological space $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ on X and Y respectively. Then the mapping $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is said to be generalized$_{\mathcal{F}}$ −semi-contra continuous map if inverse image of every generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −semi-closed in X

Example 3.3: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Consider fuzzy sets $A = \{(x_1, 0.7), (x_2, 0.4)\}$, $B = \{(x_1, 0.5), (x_2, 0.6)\}$, $C = \{(x_1, 0.5), (x_2, 0.4)\}$, $D = \{(x_1, 0.3), (x_2, 0.6)\}$and $E = \{(x_1, 0.7), (x_2, 0.6)\}$on X. Also $P = \{(y_1, 0.3), (y_2, 0.6)\}$, $Q = \{(y_1, 0.5), (y_2, 0.4)\}$ and $R = \{(y_1, 0.5), (y_2, 0.6)\}$ on Y. Let $\mathcal{T}_1 = \{0, A, B, C, D, E, 1\}$, $\mathcal{T}_2 = \{0, P, Q, R, 1\}$ be the generalized$_{\mathcal{F}}$ − topologies defined on X and Y. Then we define a mapping $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ such that $\mathcal{F}(x_1) = y_1$ and $\mathcal{F}(x_2) = y_2$. Therefore $\mathcal{F}^{-1}(0) = 1'$, $\mathcal{F}^{-1}(1) = 0'$, $\mathcal{F}^{-1}(P) = \{(x_1, 0.3), (x_2, 0.6)\} = A'$, $\mathcal{F}^{-1}(Q) = \{(x_1, 0.5), (x_2, 0.4)\} = B'$ and $\mathcal{F}^{-1}(R) = \{(x_1, 0.5), (x_2, 0.6)\} = C'$. This shows that the inverse image of every generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −semi-closed in X. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is generalized$_{\mathcal{F}}$ −semi-contra continuous map

Definition 3.4: Consider the two generalized$_{\mathcal{F}}$ −bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and (Y, ψ_1, ψ_2) on X and Y respectively. Then the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ −semi-contra continuous map if $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ − semi-contra continuous maps

Example 3.4: In Example 3.2, the inverse image of every generalized$_{\mathcal{F}}$ ψ_1 − open set and generalized$_{\mathcal{F}}$ ψ_2 − open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −semi-closed set and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −semi-closed set in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ −semi-contra continuous maps. Thus the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ −semi-contra continuous map

Proposition 3.1: Every pairwise generalized$_{\mathcal{F}}$ − contra continuous map in generalized$_{\mathcal{F}}$ − bitopological space is pairwise generalized$_{\mathcal{F}}$ − semi-contra continuous map

Proof: Let λ and β be generalized$_{\mathcal{F}}$ ψ_1 −open set and generalized$_{\mathcal{F}}$ ψ_2 −open set respectively in Y. Since $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ −contra continuous map, we have $\mathcal{F}^{-1}(\lambda)$ and $\mathcal{F}^{-1}(\beta)$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −closed sets in X and hence generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −semi-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −semi-closed in X. Hence we observe that $\mathcal{F}^{-1}(\lambda)$ and $\mathcal{F}^{-1}(\beta)$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −semi-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − semi-closed sets in X for every generalized$_{\mathcal{F}}$ ψ_1 − open set and generalized$_{\mathcal{F}}$ ψ_2 −open set in Y, therefore $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ − semi-contra continuous map

Remark 3.1: The converse of Proposition 3.1 is not true in general as shown in Example 3.5

Example 3.5: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$ .Consider fuzzy sets $A = \{(x_1, 0.7), (x_2, 0.4)\}$, $B = \{(x_1, 0.5), (x_2, 0.6)\}$, $C = \{(x_1, 0.5), (x_2, 0.4)\}$, $D = \{(x_1, 0.3), (x_2, 0.6)\}$, $E = \{(x_1, 0.7), (x_2, 0.6)\}$, $F = \{(x_1, 0.8), (x_2, 0.5)\}$, $G = \{(x_1, 0.4), (x_2, 0.7)\}$, $H = \{(x_1, 0.4), (x_2, 0.5)\}$, $I = \{(x_1, 0.8), (x_2, 0.7)\}$ and $J = \{(x_1, 0.8), (x_2, 0.3)\}$ on X. Again $P = \{(y_1, 0.3), (y_2, 0.6)\}$, $Q = \{(y_1, 0.5), (y_2, 0.4)\}$, $R = \{(y_1, 0.5), (y_2, 0.6)\}$, $S = \{(y_1, 0.4), (y_2, 0.6)\}$, $T = \{(y_1, 0.2), (y_2, 0.5)\}$, $U = \{(y_1, 0.6), (y_2, 0.3)\}$, $V = \{(y_1, 0.6), (y_2, 0.5)\}$and $W = \{(y_1, 0.3), (y_2, 0.5)\}$ on Y. Let $\mathcal{T}_1 = \{0, A, B, C, D, E, 1\}$, $\mathcal{T}_2 = \{0, F, G, H, I, J, 1\}$,

$\psi_1 = \{0, P, Q, R, S, 1\}$ and $\psi_2 = \{0, T, U, V, W, 1\}$ be the generalized$_{\mathcal{F}}$ $-$ topologies defined on X and Y.

Now we define the mappings $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ such that $\mathcal{F}(x_1) = y_1$ and $\mathcal{F}(x_2) = y_2$. Therefore $\mathcal{F}^{-1}(0) = 1'$, $\mathcal{F}^{-1}(1) = 0'$, $\mathcal{F}^{-1}(P) = \{(x_1, 0.3), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(Q) = \{(x_1, 0.5), (x_2, 0.4)\}$, $\mathcal{F}^{-1}(R) = \{(x_1, 0.5), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(S) = \{(x_1, 0.4), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(T) = \{(x_1, 0.2), (x_2, 0.5)\}$, $\mathcal{F}^{-1}(U) = \{(x_1, 0.6), (x_2, 0.3)\}$, $\mathcal{F}^{-1}(V) = \{(x_1, 0.6), (x_2, 0.5)\}$and $\mathcal{F}^{-1}(W) = \{(x_1, 0.3), (x_2, 0.5)\}$. This shows that the inverse image of every generalized$_{\mathcal{F}}$ $\psi_1 -$ open set and generalized$_{\mathcal{F}}$ $\psi_2 -$ open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1 -$ semi-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2 -$ semi-closed in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ $-$semi-contra continuous maps but not generalized$_{\mathcal{F}}$ $-$ contra continuous map because $\mathcal{F}^{-1}(S) = \{(x_1, 0.4), (x_2, 0.6)\}$ and $\mathcal{F}^{-1}(W) = \{(x_1, 0.3), (x_2, 0.5)\}$, where $\{(x_1, 0.4), (x_2, 0.6)\}$ and $\{(x_1, 0.3), (x_2, 0.5)\}$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1 -$ semi-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$semi-closed but not generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$closed in X

Definition 3.5: Consider the two generalized$_{\mathcal{F}}$ $-$bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and (Y, ψ_1, ψ_2) on X and Y respectively. Then the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ $-$ α-contra continuous map if $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ $-$ α-contra continuous maps

Example 3.6: In Example 3.2, the inverse image of every generalized$_{\mathcal{F}}$ $\psi_1 -$ open set and generalized$_{\mathcal{F}}$ $\psi_2 -$ open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1 -$ α-closed set and generalized$_{\mathcal{F}}$ $\mathcal{T}_2 -$ α-closed set in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ $-$ α-contra continuous maps. Thus the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ $-$ α-contra continuous map

Proposition 3.2: Every pairwise generalized$_{\mathcal{F}}$ $-$ contra continuous map in generalized$_{\mathcal{F}}$ $-$ bitopological space is pairwise generalized$_{\mathcal{F}}$ $-$ α-contra continuous map

Proof: Same as in Proposition 3.1

Remark 3.2: The converse of Proposition 3.2 is not true in general as shown in Example 3.7

Example 3.7: In Example 3.5, $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ $-$ α-contra continuous maps but not generalized$_{\mathcal{F}}$ $-$ contra continuous map because $\mathcal{F}^{-1}(S) = \{(x_1, 0.4), (x_2, 0.6)\}$ and $\mathcal{F}^{-1}(W) = \{(x_1, 0.3), (x_2, 0.5)\}$, where $\{(x_1, 0.4), (x_2, 0.6)\}$ and $\{(x_1, 0.3), (x_2, 0.5)\}$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1 -$ α-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2 -$ α-closed but not generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$closed in X

Definition 3.6: Consider the two generalized$_{\mathcal{F}}$ $-$topological space $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ on X and Y respectively. Then the mapping $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is said to be generalized$_{\mathcal{F}}$ $-$pre-contra continuous map if inverse image of every generalized$_{\mathcal{F}}$ $\mathcal{T}_2 -$ open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$pre-closed in X

Example 3.8: In Example 3.3, the inverse image of every generalized$_{\mathcal{F}}$ $\mathcal{T}_2 -$ open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$pre-closed in X. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is generalized$_{\mathcal{F}}$ $-$pre-contra continuous map

Definition 3.7: Consider the two generalized$_{\mathcal{F}}$ $-$bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and (Y, ψ_1, ψ_2) on X and Y respectively. Then the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ $-$pre-contra continuous map if $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ $-$pre-contra continuous maps

Example 3.9: In Example 3.2, the inverse image of every generalized$_{\mathcal{F}}$ $\psi_1 -$ open set and generalized$_{\mathcal{F}}$ $\psi_2 -$ open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$pre-closed set and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$pre-closed set in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \to (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ $-$ pre-contra continuous maps. Thus the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ $-$pre-contra continuous map

Proposition 3.3: Every pairwise generalized$_{\mathcal{F}}$ $-$ contra continuous map in generalized$_{\mathcal{F}}$ $-$ bitopological space is pairwise generalized$_{\mathcal{F}}$ $-$ pre-contra continuous map

Proof: Let λ and β be generalized$_{\mathcal{F}}$ ψ_1 $-$open set and generalized$_{\mathcal{F}}$ ψ_2 $-$open set respectively in Y. Since $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ $-$contra continuous map, we have $\mathcal{F}^{-1}(\lambda)$ and $\mathcal{F}^{-1}(\beta)$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$closed sets in X and hence generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$pre-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ $-$pre-closed in X. Hence we observe that $\mathcal{F}^{-1}(\lambda)$ and $\mathcal{F}^{-1}(\beta)$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ $-$pre-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2 -$ pre-closed sets in X for every generalized$_{\mathcal{F}}$ $\psi_1 -$ open set and generalized$_{\mathcal{F}}$ ψ_2 −open set in Y, therefore $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ − pre-contra continuous map

Remark 3.3: The converse of Proposition 3.3 is not true in general as shown in Example 3.10

Example 3.10: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$ .Consider fuzzy sets $A = \{(x_1, 0.7), (x_2, 0.4)\}$, $B = \{(x_1, 0.5), (x_2, 0.6)\}$, $C = \{(x_1, 0.5), (x_2, 0.4)\}$, $D = \{(x_1, 0.3), (x_2, 0.6)\}$, $E = \{(x_1, 0.7), (x_2, 0.6)\}$, $F = \{(x_1, 0.8), (x_2, 0.5)\}$, $G = \{(x_1, 0.4), (x_2, 0.7)\}$, $H = \{(x_1, 0.4), (x_2, 0.5)\}$, $I = \{(x_1, 0.8), (x_2, 0.7)\}$ and $J = \{(x_1, 0.8), (x_2, 0.3)\}$ on X. Again $P = \{(y_1, 0.3), (y_2, 0.6)\}$, $Q = \{(y_1, 0.5), (y_2, 0.4)\}$, $R = \{(y_1, 0.5), (y_2, 0.6)\}$, $S = \{(y_1, 0.3), (y_2, 0.3)\}$, $T = \{(y_1, 0.2), (y_2, 0.5)\}$, $U = \{(y_1, 0.6), (y_2, 0.3)\}$, $V = \{(y_1, 0.6), (y_2, 0.5)\}$and $W = \{(y_1, 0.2), (y_2, 0.2)\}$ on Y. Let $\mathcal{T}_1 = \{0, A, B, C, D, E, 1\}$, $\mathcal{T}_2 = \{0, F, G, H, I, J, 1\}$, $\psi_1 = \{0, P, Q, R, S, 1\}$ and $\psi_2 = \{0, T, U, V, W, 1\}$ be the generalized$_{\mathcal{F}}$ − topologies defined on X and Y.

Now we define the mappings $\mathcal{F}: (X, \mathcal{T}_1) \rightarrow (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \rightarrow (Y, \psi_2)$ such that $\mathcal{F}(x_1) = y_1$ and $\mathcal{F}(x_2) = y_2$. Therefore $\mathcal{F}^{-1}(0) = 1'$, $\mathcal{F}^{-1}(1) = 0'$, $\mathcal{F}^{-1}(P) = \{(x_1, 0.3), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(Q) = \{(x_1, 0.5), (x_2, 0.4)\}$, $\mathcal{F}^{-1}(R) = \{(x_1, 0.5), (x_2, 0.6)\}$, $\mathcal{F}^{-1}(S) = \{(x_1, 0.3), (x_2, 0.3)\}$, $\mathcal{F}^{-1}(T) = \{(x_1, 0.2), (x_2, 0.5)\}$, $\mathcal{F}^{-1}(U) = \{(x_1, 0.6), (x_2, 0.3)\}$, $\mathcal{F}^{-1}(V) = \{(x_1, 0.6), (x_2, 0.5)\}$and $\mathcal{F}^{-1}(W) = \{(x_1, 0.2), (x_2, 0.2)\}$. This shows that the inverse image of every generalized$_{\mathcal{F}}$ ψ_1 − open set and generalized$_{\mathcal{F}}$ ψ_2 − open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −pre-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −pre-closed in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \rightarrow (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \rightarrow (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ − pre-contra continuous maps but not generalized$_{\mathcal{F}}$ − contra continuous map because $\mathcal{F}^{-1}(S) = \{(x_1, 0.3), (x_2, 0.3)\}$ and $\mathcal{F}^{-1}(W) = \{(x_1, 0.2), (x_2, 0.2)\}$, where $\{(x_1, 0.3), (x_2, 0.3)\}$ and $\{(x_1, 0.2), (x_2, 0.2)\}$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − pre-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −pre-closed but not generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −closed in X

Definition 3.8: Consider the two generalized$_{\mathcal{F}}$ −bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and (Y, ψ_1, ψ_2) on X and Y respectively. Then the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \psi_1, \psi_2)$ is said to be pairwise generalized$_{\mathcal{F}}$ − β-contra continuous map if $\mathcal{F}: (X, \mathcal{T}_1) \rightarrow (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \rightarrow (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ − β-contra continuous maps

Example 3.11: In Example 3.2, the inverse image of every generalized$_{\mathcal{F}}$ ψ_1 − open set and generalized$_{\mathcal{F}}$ ψ_2 − open set in Y is generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − β-closed set and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − β-closed set in X respectively. Hence $\mathcal{F}: (X, \mathcal{T}_1) \rightarrow (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \rightarrow (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ − β -contra continuous maps. Thus the mapping $\mathcal{F}: (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \psi_1, \psi_2)$ is pairwise generalized$_{\mathcal{F}}$ − β-contra continuous map

Proposition 3.4: Every pairwise generalized$_{\mathcal{F}}$ − contra continuous map in generalized$_{\mathcal{F}}$ − bitopological space is pairwise generalized$_{\mathcal{F}}$ − β-contra continuous map

Proof: Same as in Proposition 3.3

Remark 3.4: The converse of Proposition 3.4 is not true in general as shown in Example 3.12

Example 3.12: In Example 3.10, $\mathcal{F}: (X, \mathcal{T}_1) \rightarrow (Y, \psi_1)$ and $\mathcal{F}: (X, \mathcal{T}_2) \rightarrow (Y, \psi_2)$ are generalized$_{\mathcal{F}}$ − β-contra continuous maps but not generalized$_{\mathcal{F}}$ − contra continuous map because $\mathcal{F}^{-1}(S) = \{(x_1, 0.3), (x_2, 0.3)\}$ and $\mathcal{F}^{-1}(W) = \{(x_1, 0.2), (x_2, 0.2)\}$, where $\{(x_1, 0.3), (x_2, 0.3)\}$ and $\{(x_1, 0.2), (x_2, 0.2)\}$ are generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ − β-closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ − β-closed but not generalized$_{\mathcal{F}}$ $\mathcal{T}_1$ −closed and generalized$_{\mathcal{F}}$ $\mathcal{T}_2$ −closed in X

4. Conclusion

In this paper we have introduced the concept of pairwise generalized$_{\mathcal{F}}$ − contra continuous maps in generalized$_{\mathcal{F}}$bi-topological spaces and defined several maps like pairwisegeneralized$_{\mathcal{F}}$ semi-contra-continuous maps (pairwise generalized$_{\mathcal{F}}$ α-contra-continuous maps) and pairwise generalized$_{\mathcal{F}}$ pre-contra-continuous maps (pairwise generalized$_{\mathcal{F}}$ β-contra-continuous maps) and find the relationships of these maps with generalized$_{\mathcal{F}}$ contra-continuous maps in generalized$_{\mathcal{F}}$ bitopological spaces. The results have been supported by some suitable counter examples. We conclude the results in this paper given below:

Pairwisegeneralized$_{\mathcal{F}}$ − contra continuous map $\Rightarrow\nLeftarrow$ *pairwise generalized$_{\mathcal{F}}$ − semi-contra continuous map*

Pairwisegeneralized$_{\mathcal{F}}$ − contra continuous map $\Rightarrow\nLeftarrow$ *pairwise generalized$_{\mathcal{F}}$ − α-contra continuous map*

$$Pairwise generalized_{\mathcal{F}} - contra\ continuous\ map\ \Rightarrow\not\Leftarrow\ pairwise\ generalized_{\mathcal{F}} - \text{pre-contra continuous map}$$

$$Pairwise generalized_{\mathcal{F}} - contra\ continuous\ map\ \Rightarrow\not\Leftarrow\ pairwise\ generalized_{\mathcal{F}} - \beta\text{-contra continuous map}$$

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