Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 2 (2024)

Ontology of Algebra Knowledge of Test Questions: A Construction of Instructional Design

Mohamad Rif'at [1*], Nurfadilah Siregar [2], Sudiansyah [3]
[1,2] *Universitas Tanjungpura*
[3] *Doctorate student, Universitas Islam Nusantara*

Abstract- This study aimed to construct an ontology design for the systematic representation of knowledge associated with a mathematics course based on hierarchical prerequisite relation. In addition, the extraction and inference methods were used by observing the clustering of knowledge with schema representing prerequisite concept dependency. The sample population comprised postgraduate (magister) students of mathematics education, and performance analysis was based on answer grading by 6 graders for a total of 24 questions. The questions were then categorized into high and low-average score groups, and the concept mapping led to clustering in ontology. A total of 90 concepts from 16 topics in 92 statements were collected in this study. The relationships in course ontology were kept to a minimum to enhance expressiveness and computability. The results showed the presence of small clusters of concepts between concepts 20-40. The design obtained was a composite and the characteristics of the elements (concept, principle, and basic theorem) were found to interplay in a very complex way. Furthermore, it accommodated various considerations, such as weight, size, semantic, and optimization objectives, including power, ratio, and rigorous. Based on the results, the design comprised a meta-knowledge of the elements, transcending the individual components.

Keywords*:* Course Ontology, Concept Space, Resource Space, Semantic Relation, Extraction

1. Introductıon

At present, there are two recurring themes, underscoring the assertion that algebra is the natural language for formulating a wide range of subjects in mathematics and the concept of intersection. Furthermore, the reformulation of known and established principles often leads to the identification of ideas and possibilities. The process of reformulation has been reported to produce novel and unusual insights. In other cases, a mathematical approach can lead to the development of a significant design. According to van der Wal et al.. 2019, various fundamental aspects are crucial for dealing with numbers. These aspects, which often lack precision and comprise measurement units, include proportional reasoning, representing and analysing data, as well as multi-step problem-solving. Several studies reported the concept of situated abstraction, recognizing that local mathematical models and ideas were only partially valid in a distant context. However, these models and concepts are intricately linked to anchors in the problem context (Hoyles et al.. 2005; Kartika & Hastari. 2022).

According to Atapattu et al., (2015) and Purohit et al., (2012), the design comprises the use of concept maps to evaluate educational resources, while ontology represents structured information and hierarchical knowledge containing semantic relationships and practical computation. This nuanced perspective gives rise to several critical pedagogical applications (Bargel et al., 2012). The existing methods necessitate a concrete knowledge representation but heavily rely on external cognitive, syntactical, and psychological parameters to discern difficulties. Several studies in cognition (Al-Aswadi et al.. 2020; Zulkipli et al.. 2022) identified other extrinsic parameters (Hunt et al.. 2007; Smith et al.. 1994) contributing to the perceived difficulty of a question, yet not some are knowledge-based. The limited existing knowledge-based methods (Alvaristo et al.. 2020; How & Hung. 2019; Romanov et al.. 2019) are constrained due to the reliance on incomplete, incomputable, and rigid representation methods, neglecting the knowledge structure.

Achieving effective testing (Andrews et al.. 2011; Manaswini & Rama Mohan Reddy. 2019) comprises a subjective design approach using specific parameters. During test design, lecturers aim to develop questions with diversity among the topics, maximum coverage of desired topics, relevance to the material taught, and good testing capabilities concerning student knowledge (Pham et al.. 2022). At present, this design process remains predominantly manual, relying on human experience and cognition. This adherence to basic design principles is guided by parameters that establish a standard for evaluating test problems (Chimalakonda & Nori. 2020; Pástor et al.. 2021).

The majority of existing finished materials are typically rough and granular, which makes them unsuitable for machine learning. To overcome this challenge, several studies proposed the use of ontology or representation methods, which can be hypothetically categorized into two description frameworks, such as resource space and concept space. Concept space comprises a graphical abstraction of concept space interlinked through semantic relations. In this context, knowledge is defined and specified through constructs for course ontology (Haendler & Neumann. 2020; Palomino et al.. 2023). (García-Peñalvo et al., 2014; Gruber R.. 1995; and Guarino et al., 1994), knowledge representation is an ontological commitment and a pragmatic medium of algorithm. Furthermore, it serves as a data model representing a domain, enabling reasoning about objects in the relations (Hudlická. 1988; Shih. 2006). The hierarchy in the representation is qualitatively different, showing the presence of varied cognitive demands. Consequently, the features of the task are considered in the design of ontology and assessment. This comprises a careful evaluation of the purpose, significant ideas, cognitive demands, other necessary knowledge, and standards (Hou et al.. 2023).

The representation of course ontology comprises a set of concept nodes, each endowed with both self-weight and prerequisite weight. These nodes are interconnected through prerequisite relationships, with the strength of the connection determined by the link weight (Wilson. 1997). Therefore, prerequisite concept holds numerical significance in completing the next concept. This framework describes associative structures with ontology overlaid on top of each other. A fundamental characteristic shared by these representations is the centrality of key concepts in the domain, referred to as the big ideas. These key concepts often manifest as nodes with several connections (Winter et al.. 2021).

To enable the machine processing of educational resources, the design of ontology provides the necessary contextual framework (Khan et al., 2005). In essence, when lecturers develop courseware, a mental map comprising concepts to be taught is created (Rif'at, 2018). The challenge of mapping a resource to concepts in ontology is a nontrivial problem, extensively addressed in natural processing and knowledge representation. This current study focuses specifically on using the concept mapping method by the schema (Aini et al.. 2021; Kordahi. 2022). The mathematical formulation of course ontology is in the form of concept space graph (Fiallos Ordonez et al., 2021; Nahhas et al., 2019). This graph can be envisioned as the projection of a net with vertices and links. Furthermore, each vertex describes a concept and each link with a weight shows the semantics as prerequisite concept for learning and the relative importance.

The effect of prerequisite node (concept) on the understanding of the next node through a specific path was computed in this study. When two concepts x_0 and x_t are linked through a pathway comprising nodes from the set $\{x_1.x_2.\dots.x_m.x_{m+1}.\dots.x_t\}$, then the pathway $\eta(x_0.x_t)$ between the two nodes with the condition $\eta \geq 1$ was: $\eta(x_0.x_t) = W_s(x_t)\prod_{m=t}^{1} l(x_{m-1}.x_m) * W_p(x_{m-1})$. (1)

The extraction method was employed as an effective method for processing the relevant information in ontology to obtain optimum results (Raud et al.. 2018). Furthermore, the projection graph was derived from the concept space graph, specifically focusing on an aspect of ontology using the threshold coefficient (λ). The coefficient was a virtual limit by controlling the projection size (see Nieto et al.. 2020). In this study (a context of education), the coefficient was a parameter, allowing for the adjustment of the depth at which a particular topic had been addressed.

Based on Equation (1), the projection graph $P(x_0.\lambda)$ was a subgraph of T with the root x_0 and all nodes x_t, where there existed at least one path from the root to the node in T such that node path weight $\eta\ (x_0.x_t)$ fulfilled the condition. $\eta \geq \lambda$. The projection set $\{x_1.x_2.\dots.x_n\}$ for a root concept x_0 was $P(x_0.\lambda) = P^{x_0} = \{x_1^{x_0}.x_2^{x_0}.x_3^{x_0}.\dots.x_n^{x_0}\}$, where the i[th] element was the projection set of node j. Test question and the parameters were used to quantify certain properties of the associated knowledge. The specific properties included coverage, diversity, and conceptual distance (Abdellatief et al., 2014). The assessment methods focused on accuracy and consistency (Kay & Holden, 2002) in line with the guidance provided by lecturers (Lee & Heyworth, 2000).

The question's coverage established a cumulative prerequisite effect of the projection graph on the knowledge necessary to respond to a specific question. The coverage of a node x_0 with respect to the root node r is the product of the sum of the node path weights for all nodes in the projection set $P(x_0.\lambda)$ for the concept x_0, and the incident path weight $\gamma(r.x_0)$ from the root r. When the projection set for concept node x_0. $P(x_0.\lambda)$, then the following equation is obtained:

$$\alpha(x_0) = \gamma\,(r.x_0) * \sum_{m=0}^{n} \eta\,(x_0.x_m). \quad (2)$$

Where $\alpha(x_0)$ are the coverage for node x_0 about ontology root r; $\eta\,(x_0.x_m)$ represents the node path weight, and $\gamma(r.x_0)$ indicates the incident path weight.

$$\gamma\,(x_0.x_n) = \frac{\eta\,(x_0.x_n)}{W_s\,(x_n)} = \frac{\eta\,(x_0.x_n)}{\eta\,(x_n.x_n)} \quad (3)$$

Total coverage of multiple concepts in a problem by the set. $\{C_1.C_2.\dots.C_n\}$ is:

$$\alpha(T) = \alpha(C_1) + \alpha(C_2) + \cdots + \alpha(C_n) \quad (4)$$

The node path weight delineates the prerequisite effect of a node to the designated root. The summation of the node path weights for all nodes in the projection set provides the cumulative prerequisite effect in the projection graph on the respective mapped concepts' roots. The coverage represents the quantity of knowledge necessary to respond to or comprehend a specific concept.

Diversity quantifies the effect of uncommon and common prerequisite concepts resulting from the projection of mapped concepts. This parameter represents the ratio of the summation of node path weights for all nodes in the non-overlapping set to their respective roots, the sum of the summation of node path weights for all nodes in the overlaps set, as well as the summation of node path weights for all nodes in the non-overlap set. The equation for diversity is written below:

$$\Delta = \frac{\sum_{m=1}^{p} \eta\,(i.N_m^i)}{\sum_{m=1}^{q} \eta\,\left(j.N_m^j\right) + \sum_{m=1}^{p} \eta\,(i.N_m^i)} \quad (5)$$

Where the non-overlapping and overlapping sets are $N = \{N_0.N_1.N_2.\dots.N_p\}$ and $O = \{O_0.O_1.O_2.\dots.O_q\}$, respectively. i and j indicate the local root of the next idea of any element from each of N and O. and $\forall\, i.j \in C$.

The conceptual distance serves as a measure of the distance between two concepts concerning the ontology root or the similarity between them by quantifying the distance. The parameter is expressed as the log of the inverse of the incident path weight minimum value (maximum value of threshold coefficient) necessary to cover all mapped concepts originating from the root concept r. The conceptual distance can be defined as:

$$\delta(C_0.C_1.C_2.\dots.C_n) = \log_2\left(\frac{1}{\min\{\gamma(r.C_0).\ \gamma(r.C_1)\dots.\ \gamma(r.C_n)\}}\right) \quad (6)$$

The parameters observed include coverage, diversity, and conceptual distance. The computation of values for coverage and diversity is contingent upon adjustments to threshold coefficients across a range of λ. The familiarity of students with concepts (or how well concepts have been taught) is a factor of the threshold coefficient. Furthermore, coverage provides an approximation of the knowledge necessary to respond to a specific question. Inferences are frequently drawn by examining the clustering of concepts within an ontology based on the projection graph of test questions distributed across various tests.

This study prompts a question: Can such characterization be employed in instructional design, especially in the context of a test? The characteristics of the elements in the design interact in intricate ways, demanding sophisticated additional senses and the capacity to respond effectively. In the design process, attention to detail is indispensable to fulfil diverse constraints and optimization objectives (AL-Aswadi et al., 2023; Watróbski, 2020). One of the main objectives of testing in education is to assess students' understanding level of a domain.

2. Method

The questions were objectively analyzed through a qualitative knowledge-based evaluation. The pedagogical focus comprised processing the testing based on the breadth, depth, and conceptual knowledge represented by a graph. The hypothesis was qualitatively examined using extraction methods (refer to Andreasen. 2003; Hao et al.. 2019; Yu et al.. 2022). The experimental setup comprised post-graduate-level courses on ontology of 60 concepts across 16 topics referencing standardized textbooks and guidance from the respective lecturers. A random set of

Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 2 (2024)

---

60 test questions from the course collection was then selected. The test targeted postgraduate (magister) students, and the scores were used for the performance analysis.

2.1 Validity and Reliability

A total number of 6 experts provided feedback and quality ratings, and the revision to ontology review process was repeated. The Alpha Cronbach was 0.635 and the Intra-class Correlation Coefficient, equal to 0.635 for Average Measures, reached a sufficiently high level. The validity evidence gathered (Sig. 0.012) showed expert-novice differences, sensitivity to instruction, as well as correlation with measures of conceptual knowledge ($p = 0.488 > \alpha$).

2.2 Analysis

The first method was a similarity measure, providing an index of the structures (see Koutsomitropoulos. 2019). This served as empirical support for evaluating the performance pattern on assessment items across students with diverse levels of knowledge. The second comprised evaluating the completeness of understanding using novices and experts. The third method validated the relationships among concepts, whereas the fourth involved a comparison of task evidence aimed at evaluating certain aspects of the ontology. Ultimately, the fifth method comprised randomized comparisons centered on the levels of interest.

The knowledge representation was synthesized as a domain and identified its core concepts by eliciting from a group of domains, focusing on important mathematical concepts in algebra. As a result the top nodes represented the highest level of abstraction, while nodes further down the tree increasingly manifested discrete knowledge. Furthermore, the pedagogy of ontology typically captured only the structure of the domain.

2.3 Criteria

The significance of the similarity between 2 objects diminished with a small distance, while a larger distance led to a lower degree of similarity. Each node in the Concept Space Graph ($\eta \geq \lambda$) had at least one path from the root, ensuring that the sum of self-weight, prerequisite, as well as link weights for all facts or conceptual equaled 1. Concept coverage from the root node comprised the product of the sum of path weight in the projection set and the incident path weight from the root. The node path weight calculated how prerequisite influenced the understanding of the next through a specific path. The projection, with a local root beginning concept and the threshold coefficient, formed a subgraph where there was at least one path between two points meeting the condition of node path weight. Diversity (Δ) was characterized as "the ratio of summation of path weights for all elements in the non-overlapping set to their respective roots, and the sum of the summation of path weights for all nodes in the overlap set and summation of path weights for all nodes in the non-overlap set."

3. Results

In this study, there were 16 nodes (big ideas) that pertained to standardized textbooks and assistance from related lecturers, as indicated in Table 1.

Table 1. Selected Concepts Arranged in Order from Facts to Abstractions

Distribution	Roots (A to P). The nodes and the Number of the Core Concepts and Questions																Total
	Logical Prerequisite	Numbers	Equations	Absolute Values	Sequences and Series	Variations	Logarithms	Polynomials	Set	Probability	Matrix	Algorithms	Algebraic Equations	Field	Vector	Linear Independence	
Nodes	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	16
Concepts	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	90

Questions	4	5	4	4	4	4	4	4	5	4	3	3	3	3	3	3	60

The Concept Space Graph with root A was depicted as T (A) in Figure 1.

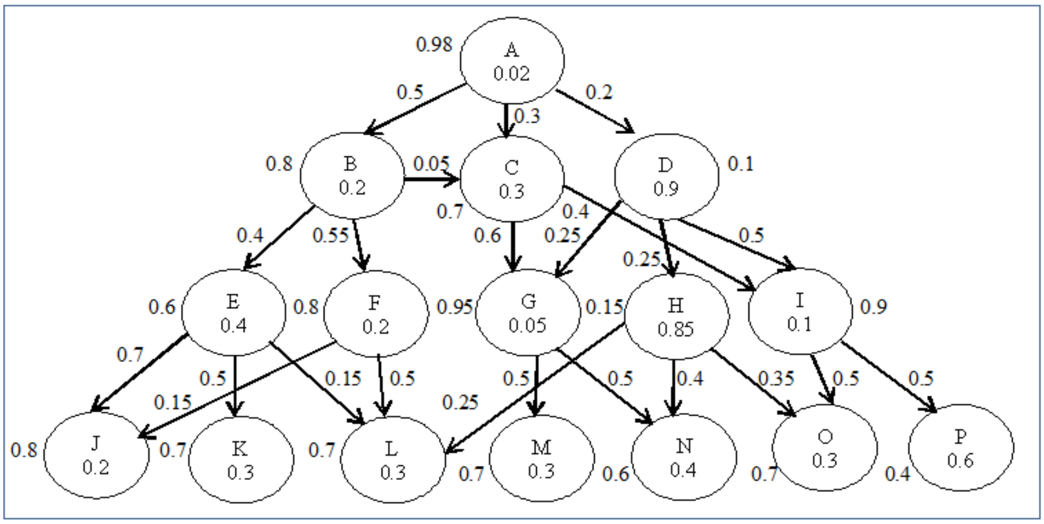

Figure 1. Concept Space Graph of T (A)

The summary was presented in Table 2.

Table 2. The Node Path Weight of Prerequisite of P (B) with λ=0.001

Vertices	Self-Weight	Specific Path	Effect of Prerequisite Nodes					Node Path Weight	η (r. n) ≥ λ
[B.E]	0.4	0.4	0.8	-	-	-	-	0.128	√
[B.F]	0.2	0.55	0.8	-	-	-	-	0.088	√
[B.C]	0.3	0.05	0.8	-	-	-	-	0.012	√
[B.E.J]	0.2	0.7	0.6	0.4	0.8	-	-	0.02688	√
[B.F.J]	0.2	0.5	0.8	0.55	0.8	-	-	0.0352	√
[B.E.K]	0.3	0.15	0.6	0.4	0.8	-	-	0.00864	√
[B.E.L]	0.3	0.15	0.6	0.4	0.8	-	-	0.00864	√
[B.F.L]	0.3	0.5	0.8	0.55	0.8	-	-	0.0528	√
[B.C.G]	0.05	0.6	0.7	0.05	0.8	-	-	0.00084	x
[B.C.I]	0.1	0.4	0.7	0.05	0.8	-	-	0.00112	√
[B.C.G.M]	0.3	0.5	0.95	0.6	0.7	0.05	0.8	0.00239	√
[B.C.G.N]	0.4	0.5	0.95	0.6	0.7	0.05	0.8	0.00319	√
[B.C.I.O]	0.3	0.5	0.9	0.4	0.7	0.05	0.8	0.002151	√
[B.C.I.P]	0.6	0.5	0.9	0.4	0.7	0.05	0.8	0.00302	√

Node L exhibited a more pronounced prerequisite effect on B through F rather than E. Specifically, L held greater importance for F (0.5) compared to E (0.15). Prerequisite significance of L to F was higher at 0.8 and exceeded the importance of E at 0.6, showing that F (0.55) played a more crucial role for B than E (0.4). Therefore, node path weights considered not only the individual impact it had on the immediate successor but also the cumulative prerequisite effect it had on (B) along a specific path.

In Figure 2, concept L was connected to B through E and F. Furthermore, prerequisite effect on B depended on the respective prerequisite effects of both E and F on B. The node path weight calculations were as follows: $\eta\ (B.E.L) = 0.3 * 0.15 * 0.6 * 0.4 * 0.8 = 0.00864$, and$\eta\ (B.F.L) = 0.3 * 0.5 * 0.8 * 0.55 * 0.8 = 0.0528$.

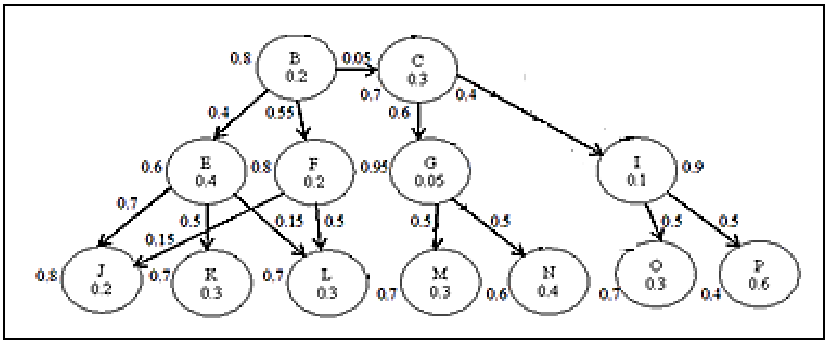

Figure 2. The Projection of P (B. 0.001)

The results showed that all node path weights exceeded the threshold coefficient. Node J and L had satisfying paths (J-E-B, J-F-B and L-E-B, L-F-B), while for O, only one path (O-I-D) met the condition. However, O was considered in the projection of D due to prerequisite effect on D.
Figures 2 and 3 show the projections and calculations.

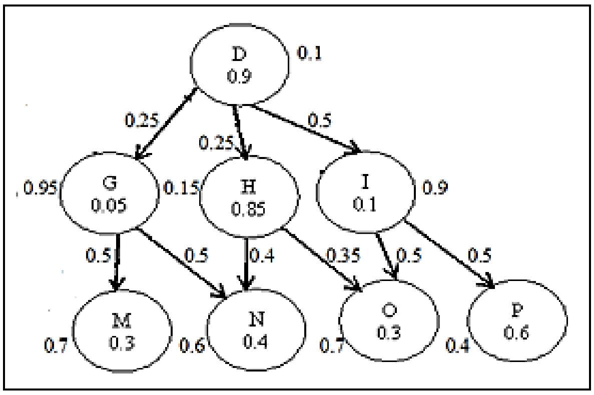

Figure 3. The Projection of P (D. 0.001)

Table 3 summarized the calculations of the local root D by the threshold coefficient of λ = 0.001.

Table 3. The Calculation of the Local Root r of P (D)

Nodes	Self-Weight	Specific Path	Effect of Prerequisite Nodes					Node Path Weight	η (r. n) ≥ λ
[D, G]	0.05	0.25	0.1	-	-	-	-	0.00125	√
[D, H]	0.85	0.25	0.1	-	-	-	-	0.02125	√
[D, I]	0.1	0.5	0.1	-	-	-	-	0.005	√
[D, G, M]	0.3	0.5	0.95	0.25	0.1	-	-	0.00356	√
[D, G, N]	0.4	0.5	0.95	0.25	0.1	-	-	0.00475	√
[D, H, N]	0.4	0.4	0.15	0.25	0.1	-	-	0.0006	x
[D, H, O]	0.3	0.35	0.15	0.25	0.1	-	-	0.00039	x
[D, H, L]	0.3	0.25	0.15	0.25	0.1	-	-	0.000281	x
[D, I, O]	0.3	0.5	0.9	0.5	0.1	-	-	0.00675	√
[D, I, P]	0.6	0.5	0.9	0.5	0.1	-	-	0.0135	√

The coverage represented the knowledge needed to understand a concept.

$\alpha(B) = \gamma(A.B) * \sum_i \eta(B.P^B) = (0.5 * 0.98) * (0.372031) = 0.182295$

$\alpha(D) = (0.2 * 0.98) * (0.05606) = 0.010988$

$\alpha(total) = 0.182295 + 0.010988 = 0.193283$

Diversity was computed by assessing the effect of both uncommon and common prerequisite concepts derived from the projection of mapped concepts. The equation is written as $\forall c \in P(B). \forall c \in P(\text{D}). and\ \forall c \in P(B) \cap P(C)$, then $\Delta = \frac{1.44714}{0.0448045+1.44714} = 0.97$ or 97%. This showed that concepts were very diverse in the concept space graph, as presented in Figure 4.

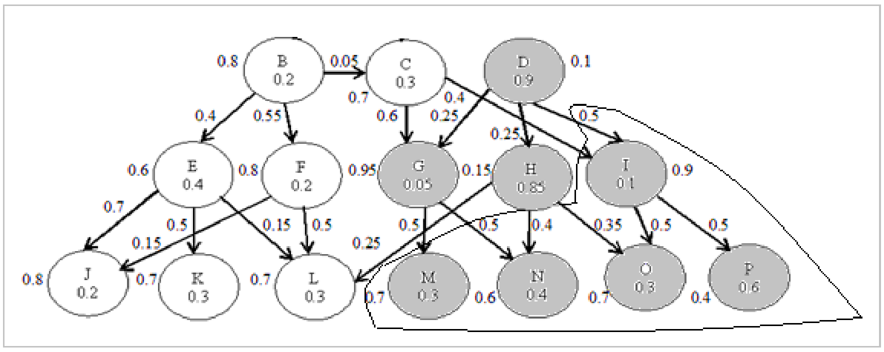

Figure 4. One of Diversity Calculation Example

Figure 5 illustrated the calculation of conceptual distance for the power set {E. M. F}. In cases involving multiple paths, such as M, consideration was given to the path with the lowest incident path weight value.

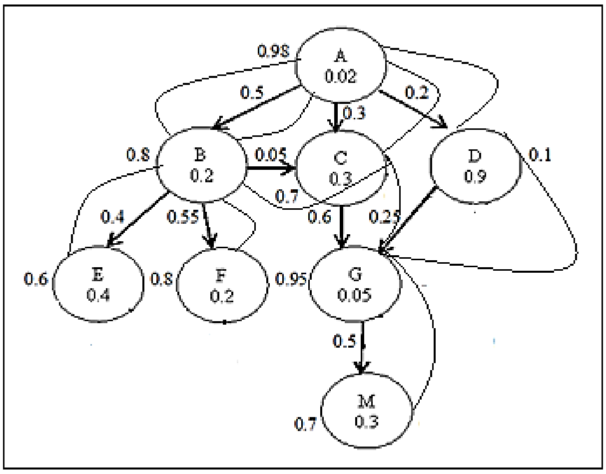

Figure 5. The Conceptual Distance Calculation

That is $\delta[\mathrm{E,F,M}] = \log_2\left(\frac{1}{\min(0.1568, 0.2156, 0.0023275)}\right) = 2.63$.

The graph depicted in Figure 6 elucidated the coverage analysis for each question under different threshold coefficients.

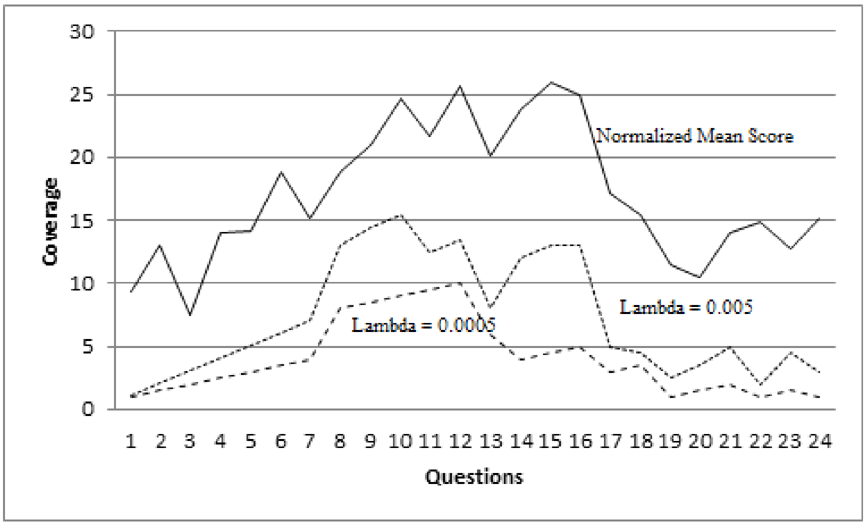

Figure 6. The Coverage versus the Average Score

The observation showed an inverse relationship between coverage and average score. With an increase in the average score, the coverage for a specific question decreased, and conversely. This relationship persisted across all values of λ, but it became increasingly apparent as λ decreased. Therefore, when the inverse correlation between the coverage and average score graphs was more pronounced for lower λ values, it was deduced that more concepts were needed to answer this particular question.

Among concepts that failed in the range of 20–60, there was a higher density of questions with high scores compared to those with low scores. This suggested that students perceived the problems associated with these concepts to be relatively easy to answer. However, for problems 19 and 24 related to the same concepts, low scores were observed. This showed that these specific problems were more challenging than the overall understanding of concepts. In conclusion, it could be asserted that additional explanation from lecturers' perspective was warranted for these concepts or segments of ontology.

Questions with lower scores exhibited a greater dispersion of concepts throughout the ontology, contrasting with those displaying a high inverse correlation which tended to cluster more closely. Questions 1-6 were in Test 1, while 7-12, 13-18, and 19-24 were in Tests 2, 3, and 4, respectively. Furthermore, questions 13, 14, 19, 20, 23, and 24 addressed almost identical concepts. The results showed that questions 8, 9, 10, 11, and 12 achieved high scores, while 22 and 24 obtained low scores. This suggested that factors beyond understanding mapped concepts were essential for correctly answering questions. The inference was intriguing as it implied that lecturers opted to focus questions exclusively on selected topics from course ontology. Figure 7 illustrated the performance of conceptual distance versus average score.

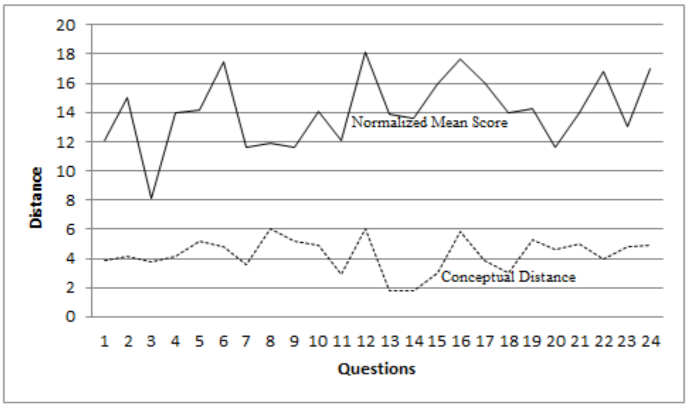

Figure 7. Conceptual Distance versus Average Score

4. Discussion

Although the distance was not directly reliant on the projection graph, it was inversely correlated with the average score. This showed that it served as a reliable indicator of the similarity between concepts. The results indicated that a decrease in the distance between two nodes corresponded to an increase in similarity. With increasing similarity, the dissimilarity in the knowledge required to answer concepts was reduced, thereby raising the average points scored. In all three assessment parameters, there was an inverse correlation between the average score and the parameters. This indicated that the parameters accurately reflected the perceived difficulty of the test problems. Furthermore, the clusters of concepts suggested that the small clusters represented mapped concepts, whereas the larger variants showed the projections of the mappings.

The subclass axiom aimed to define a property restriction that detailed an anonymous class. This restriction applied to all instances of class concepts that possessed prerequisite and belonged to the extension of the relation. The class encompassed all relationships between concepts, assigning values to the path within a range dictated by computational convenience. The remaining two data type properties were self-weight and prerequisite for assigning them to a node, respectively. Individual members of the Doing Math belonged to the class concept and held property values for path weight.

The projection graph constrained computation by limiting the propagated semantic significance. Extracting information served as a rationale for employing the Course Space Graph ontology. In contemporary settings, ontology comprised thousands to millions of concepts. Moreover, when mapped concepts were distant in ontology, it showed diverse knowledge requirements in the concept space.

This aspect played a crucial role in computability and semantic relevance, gained by implementing pruning through the introduction of a threshold coefficient.

The threshold coefficient varied as the projection graph needed prerequisite weight for the leaf nodes, even though it was zero for the leaf node. A larger coefficient resulted in a more rigorous screening process for nodes to be included in the projection, ultimately yielding a smaller graph. A lower coefficient showed more concepts in the projection. Within an educational context, the coefficient functioned as a parameter to determine the depth of topic coverage. For less detailed topics, a higher coefficient was assigned, thereby reducing the depth of the projection graph. Meanwhile, comprehensive themes had a lower threshold coefficient, creating a larger projection graph with more prerequisite concepts. Extracting the projection graph for concepts mapped to a resource allowed precise extraction of relevant parts from course ontology in line with the desired semantic significance. As an illustrative example, a test question served as a foundational resource for concept mapping. Diverse parameters facilitated the quantitative measurement of specific properties inherent in the knowledge related to test questions or similar resources. These notable properties included coverage, question diversity, and conceptual distance.

Ontology referred to standardized textbooks for the accuracy and consistency of education. Furthermore, observation showed that concepts higher up in ontology exhibited lower self-weight, but increased down ontology, reaching its maximum for leaf nodes. Preserving the extensibility of the ontology allowed for the incorporation of new results, studies, and concepts, thereby enhancing the inherent knowledge base. This transformed the course into a continually evolving and enhanced repository of knowledge. In this study, there was no necessity to disregard this space, as the probability of adding new links to higher-level concepts was compared to that of lower-level concepts.

The values for coverage and diversity shifted in response to the changing threshold coefficients. Hence, for these two parameters, the values were computed across a range of λ. By varying the coefficients, the size of the projection graph could be adjusted, thereby altering students' familiarity with concepts. From the findings, it was noted that coverage exhibited an inverse correlation with the average score. With an increase in the average score, the coverage for that specific question decreased, and vice versa. This showed that more concepts were required to address the inquiry.

In the case of diversity, the threshold coefficient decreased because the projection set of each concept expanded, leading to increased coverage of the overlap set and diminished diversity. In some cases, diversity rose concomitantly with a decrease in λ. The results indicated that as the threshold coefficient decreased, the projection expanded. Nevertheless, rather than having an increased number of overlapping nodes, there was an expansion in the non-overlapping node set, resulting in an augmentation of diversity.

The performance of conceptual distance compared to the average score was not directly influenced by the projection graph. Distance showed an inverse correlation with the average score, signifying its role as an indicator of concept similarity. Based on the results, the conceptual distance remained constant across all threshold coefficient values and was independent of the projection graph. Similar to coverage and diversity, it also exhibited an inverse correlation with 3 assessment parameters. The average score showed an inverse correlation with these parameters, and both projection graphs shared a significant number of common concepts. Moreover, instead of solely focusing on mapped concepts, the consideration extended to the projection of mapped concepts, offering a more comprehensive understanding regarding the complete set of prerequisite concepts necessary to respond to the question.

5. Conclusion

In conclusion, the results obtained in this study encompassed the following: 1) The difficulty and knowledge linked to a question demonstrated a close correlation, 2) Knowledge pertaining to a question tended to concentrate within specific sections of the course ontology, as well as 3) The clustering of knowledge not only offered insights into the developing area but also served as a valuable tool in guided instruction.

A representation schema was utilized to formally describe the course ontology, demonstrating independence and the ability to describe other concept hierarchies sharing identical properties. Furthermore, the relationships were minimized to enhance expressiveness and computability. The clustered knowledge related to test questions in course ontology made several inferences. The results showed the application of representation and extraction methods to classical learning theories, emphasizing the necessity to represent knowledge as prerequisite concept structures.

A finer-grained ontology contained more detailed concepts and implicit relationships provided the representation with increased knowledge and improved results. Hence, the critical factor was the depth of the represented knowledge, especially within the realm of mathematics.

6. Recommendation

Standards ought to delineate complex cognitive skills, requiring fewer standards of elevated complexity. Moreover, the focus should be on addressing cognitive demands and the domain-specific set of knowledge and skills as targeted parameters. Both ontology and assessment task design should confront the inherent demands within the content, including the specific cognitive outcomes expected from students, such as declarative or procedural knowledge, facts, problem-solving skills, and conceptual understanding. Assessments were required to encompass both the class of illustrations provided in instruction and several potential requirements for applying skills or knowledge in a different context. The task representation encompassed the domain model, comprising skills, knowledge, attitudes, behaviour, and other competency buildings. Performance was generalized to include the set of measures, the stimulus materials, the format of the task, the format of students' responses, as well as the associated scoring method. The domain representation served as a benchmark to assess relevance and reflected the universe performance in an assessment. The idea explicitly, precisely, and externally captured the essential elements tested in relation to the target. Represented as a graph, ontology utilized standard analytical methods derived from graph theory to examine the structure as well as address questions pertaining to the cognitive demands, content domain, instruction, and assessment.

Acknowledgments

This study was carried out by the Magister of Mathematics project with financial support from the Indonesian Ministry of Education, Culture, Report, and Technology through Tanjungpura University, under contract number 8963/UN22.6/PT.01.03/2023. The opinions, results, conclusions, or recommendations presented in this study were solely those of the authors and did not necessarily represent the position or policy endorsed by the funding agency. All authors equally contributed to this manuscript, having read and approved the final version. The authors declare no competing interests.

References

[1] Abdellatief, M., Sultan, A. B. M., Jabar, M. A., Abdullah, R., Achsan, H. T. Y., Wibowo, W. C., Agarwal, S., Tomar, D., Ahmadpour, A., Mirdamadi, M., Hosseini, J. F., Chizari, M., Al-Dosari, H., Al-Khanjari, Z. A., Kutti, N. S., Dorvlo, A. S. S., Ramadhan, H. A., AlQahtani, S. A., Al-Saadoon, G. M. W., … Srinath.

(2014). What are Learning Analytics? Journal of Computer Science, 3(1).
[2] Aini, Q., Lutfiani, N., Santoso, N. P. L., Sulistiawati, S., & Astriyani, E. (2021). Blockchain For Education Purpose: Essential Topology. Aptisi Transactions on Management (ATM), 5(2). https://doi.org/10.33050/atm.v5i2.1506
[3] AL-Aswadi, F. N., Chan, H. Y., Gan, K. H., & Alma'aitah, W. Z. al. (2023). Enhancing relevant concepts extraction for ontology learning using domain time relevance. Information Processing and Management, 60(1). https://doi.org/10.1016/j.ipm.2022.103140
[4] Andreasen, T. (2003). An approach to knowledge-based query evaluation. Fuzzy Sets and Systems, 140(1). https://doi.org/10.1016/S0165-0114(03)00028-9
[5] Atapattu, T., Falkner, K., & Falkner, N. (2015). An evaluation methodology for concept maps mined from lecture notes: An educational perspective. Communications in Computer and Information Science, 510. https://doi.org/10.1007/978-3-319-25768-6_5
[6] Bargel, B. A., Schröck, J., Szentes, D., & Roller, W. (2012). Using Learning Maps for Visualization of Adaptive Learning Path Components. International Journal of Computer Information Systems and Industrial Management Applications., 4.
[7] Chimalakonda, S., & Nori, K. V. (2020). An ontology based modeling framework for design of educational technologies. Smart Learning Environments, 7(1). https://doi.org/10.1186/s40561-020-00135-6
[8] Fiallos Ordonez, A., Garcia, K. C., & Chehab, X. O. (2021). Curriculum Design Assistant System Based on Automatically-Generated Educational Ontologies. Revista Iberoamericana de Tecnologias Del Aprendizaje, 16(3). https://doi.org/10.1109/RITA.2021.3126583
[9] Haendler, T., & Neumann, G. (2020). Ontology-Based Analysis and Design of Educational Games for Software Refactoring. Communications in Computer and Information Science, 1220. https://doi.org/10.1007/978-3-030-58459-7_29
[10] Hao, J., Xu, L., Wang, G., Jin, Y., & Yan, Y. (2019). A Knowledge-based method for rapid design concept evaluation. IEEE Access, 7. https://doi.org/10.1109/ACCESS.2019.2933544
[11] Hou, Z., Cen, Y., Dong, Y., Zhang, J., & Tang, J. (2023). Automated Unsupervised Graph Representation Learning. IEEE Transactions on Knowledge and Data Engineering, 35(3). https://doi.org/10.1109/TKDE.2021.3115017
[12] Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2005). Statistical reasoning in the workplace: techno-mathematical literacies and learning opportunities. Proceedings of the British Society for Research into Learning Mathematics, 25(2).
[13] Kartika, H., & Hastari, R. C. (2022). Techno-mathematical literacies in digital age: Which categories are importance for teachers? Jurnal Kajian Pembelajaran Matematika, 6(2). https://doi.org/10.17977/um076v6i22022p76-83
[14] Kay, J., & Holden, S. (2002). Automatic extraction of ontologies from teaching document metadata. Proceedings - International Conference on Computers in Education, ICCE 2002. https://doi.org/10.1109/CIE.2002.1186343
[15] Khan, J. I., Hardas, M., & Ma, Y. (2005). A study of problem difficulty evaluation for semantic network ontology based intelligent courseware sharing. Proceedings - 2005 IEEE/WIC/ACM InternationalConference on Web Intelligence, WI 2005, 2005. https://doi.org/10.1109/WI.2005.15
[16] Kordahi, M. M. (2022). Ontology for the User-Learner Profile Personalizes the Search Analysis of Online Learning Resources: The Case of Thematic Digital Universities. Information Technology and Libraries, 41(2). https://doi.org/10.6017/ITAL.V41I2.13601
[17] Koutsomitropoulos, D. A. (2019). Semantic annotation and harvesting of federated scholarly data using ontologies. Digital Library Perspectives, 35(3–4). https://doi.org/10.1108/DLP-12-2018-0038
[18] Lee, F., & Heyworth, R. (2000). Problem complexity: A measure of problem difficulty in algebra by using computer. Educational Journal-Hong Kong, 28(1).
[19] Nahhas, S., Bamasag, O., Khemakhem, M., & Bajnaid, N. (2019). Bridging Education and Labor Skills by a Novel Competency-Based Course Linked-Data Model. IEEE Access, 7. https://doi.org/10.1109/ACCESS.2019.2937233
[20] Nieto, M. A. M., Mora, P. D. V., de la Calleja Mora, J., Vidal, M. T., Dominguez, E. L., Díaz, D. A., &

Patiño, I. E. B. (2020). Web service to retrieve and semantically enrich datasets for theses from open educational repositories. IEEE Access, 8. https://doi.org/10.1109/ACCESS.2020.3024614

[21] Palomino, P. T., Toda, A. M., Rodrigues, L., Oliveira, W., Nacke, L., & Isotani, S. (2023). An ontology for modelling user' profiles and activities in gamified education. Research and Practice in Technology Enhanced Learning, 18. https://doi.org/10.58459/rptel.2023.18018

[22] Pástor, D., Arcos-Medina, G., Bonito, V., & Cepeda, J. (2021). Design of an Adaptive Educational Application to Generate Customized Tests Based on Ontology. International Journal of Emerging Technologies in Learning, 16(3). https://doi.org/10.3991/ijet.v16i03.17805

[23] Purohit, V. K., Kumar, A., Jabeen, A., Srivastava, S., Goudar, R. H., Shivanagowda, & Rao, S. (2012). Design of adaptive question bank development and management system. Proceedings of 2012 2nd IEEE International Conference on Parallel, Distributed and Grid Computing, PDGC 2012. https://doi.org/10.1109/PDGC.2012.6449828

[24] Raud, Z., Vodovozov, V., Petlenkov, E., & Serbin, A. (2018). Ontology-based design of educational trajectories. 2018 IEEE 59th Annual International Scientific Conference on Power and Electrical Engineering of Riga Technical University, RTUCON 2018 - Proceedings. https://doi.org/10.1109/RTUCON.2018.8659893

[25] Rif'at, M. (2018). The Exploring of Visual Imagery: In their Relation to the Students' Mathematical Identity. Higher Education Research, 3(5). https://doi.org/10.11648/j.her.20180305.11

[26] van der Wal, N. J., Bakker, A., & Drijvers, P. (2019). Teaching strategies to foster techno-mathematical literacies in an innovative mathematics course for future engineers. ZDM - Mathematics Education, 51(6). https://doi.org/10.1007/s11858-019-01095-z

[27] Watróbski, J. (2020). Ontology learning methods from text -an extensive knowledge-based approach. Procedia Computer Science, 176. https://doi.org/10.1016/j.procs.2020.09.061

[28] Winter, R., Noé, F., & Clevert, D. A. (2021). Permutation-Invariant Variational Autoencoder for Graph-Level Representation Learning. Advances in Neural Information Processing Systems, 12.

[29] Yu, X., Hu, D., Li, N., & Xiao, Y. (2022). Comprehensive Evaluation on Teachers' Knowledge Sharing Behavior Based on the Improved TOPSIS Method. Computational Intelligence and Neuroscience, 2022. https://doi.org/10.1155/2022/2563210

Author Information

Mohamad Rif'at
https://orcid.org/0000-0001-7762-8072
Tanjungpura University
Jalan Prof. Dr. H. Hadari Nawawi. Pontianak-78124
Indonesia
mohammad.rifat@fkip.untan.ac.id

Nurfadilah Siregar
https://orcid.org/XXXX-XXXX-XXXX-XXXX
Tanjungpura University
Jalan Prof. Dr. H. Hadari Nawawi. Pontianak-78124
Indonesia
Contact e-mail: email@email.com