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Thermal Behavior of a Thermosensitive FG Plate

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Abstract:- The present investigation deals with the study of temperature and thermal profile of a functionally graded (FG) rectangular plate using thermosensitive material properties and internal heat source. The material properties are graded along x-direction in accordance with exponential law functions. Kirchhoff'transformation and integral transform methodology is used to solve the heat conduction equation. The plane stress and strain have been obtained using displacement and harmonic functions. The temperature distribution and stresses have been obtained for temperature independent as well as dependent material. A mathematical model is prepared for a FG composite material of ceramic and metal and numerical computations are performed.

Keywords: *Functionally graded rectangular plate, Heat conduction, Plane stress, Plane strain, Heat generation*

1. Introduction

FG Materials (FGMs) are nonhomogeneous materials that are portrayed by the variety in creation and design progressively over volume, bringing about relating changes in the properties of the material. Inhomogeneity in material structure in many cases occurs due to high- and low-level temperatures. Engineering application gives preference to the construction of solution of thermosensitive problems which is useful in production of stress bearing materials under high temperature heating. Therefore theeffect of thermosensitivity should be considered for investigation of thermoelastic behavior of different solids.

Noda [1] briefly described the thermoelastic behaviour of various solids with material properties depending on temperature. Bending moments of plates was analyzed by Tanigawa [2]. Popovych and Fedai [3] discussed the thermoelasticity of a multilayered tube. Morishita and Tanigawa [4] studied the three-dimensional elastic problem for nonhomogeneous medium for semi-infinite body. Popovych and Makhorkin [5] solved the heat conduction problems of thermosensitive bodies. Kawamura et al. [6] discussed the stress analyses of nonhomogeneous plate. Awaji et al. [7] evaluated the stresses of and FGM plate. The heat transfer and the corresponding thermal behavior of different solids were investigated by [8-18]. [19, 20] presented buckling FGM plates. Thermoelastic behavior of FGM solids was presented by [21, 22]. Yıldırım et al. [23] considered an FGM fin and studied its thermal behavior by considering the FGM properties. Manthena and Kedar [24] considered an FG plate with internal heat source and obtained the temperature and thermal stresses. Manthena [25] studied the effects of plane stress and strain field in an FG plate. Heat transfer in different solids was investigated by [26-31]. Following [25], here we studied the effect of heat source on heat conduction (HC) of a thermosensitive FG rectangular plate (FGRP) and obtained the plane stress and plane strain subjected to convective heating along x-direction.

2. Problem Formulation

2.1 Heat Conduction Equation and its Solution:

The transient HC equation (HCE) of a rectangular plate with initial and boundary conditions is:

$$\frac{\partial}{\partial x}\left(\lambda(x,\theta)\frac{\partial\theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda(x,\theta)\frac{\partial\theta}{\partial y}\right)+Q(x,y,t)=\rho C(x,\theta)\frac{\partial\theta}{\partial t} \tag{1}$$

$$\begin{aligned}
&\theta=\theta_0, && \text{at } t=0\\
&\lambda(x,\theta)\frac{\partial\theta}{\partial x}-k_1(\theta-\theta_0)=f(y,t), && \text{at } x=0\\
&\lambda(x,\theta)\frac{\partial\theta}{\partial x}+k_2(\theta-\theta_0)=f(y,t), && \text{at } x=a\\
&\lambda(x,\theta)\frac{\partial\theta}{\partial y}-k_3(\theta-\theta_0)=0, && \text{at } y=0\\
&\lambda(x,\theta)\frac{\partial\theta}{\partial y}+k_4(\theta-\theta_0)=0, && \text{at } y=b
\end{aligned} \tag{2}$$

where λ(x,θ) is the thermal conductivity, C (x,θ) is the specific heat capacity and Q(x, y, t) is the internal heat generation, p is the density which is a constant, θ0 is the temperature of the surrounding medium, and k1, k2 , k3 , k4 are the heat transfer coefficients. The following dimensionless parameters are used.

$$\begin{aligned}
&\theta^*=\frac{\theta}{\theta_{m0}},\ \theta_0{}^*=\frac{\theta_0}{\theta_{m0}},\ (x^*,y^*)=\frac{(x,y)}{a},\ (a^*,b^*,d^*,u^*,v^*)=\frac{(a,b,d,u,v)}{a},\ t^*=\frac{\kappa t}{a^2},\rho^*=\frac{\rho}{\rho_{m0}},\ k_i{}^*=\frac{k_i\,a}{\lambda_{m0}},\\
&(\phi^*,\varphi^*,\psi^*)=\frac{(\phi,\varphi,\psi)}{a},\ \lambda_{m0}{}^*=\frac{\lambda_{m0}}{\lambda_{c0}},\lambda_{c0}{}^*=\frac{\lambda_{c0}}{\lambda_{m0}},r_1{}^*=\frac{r_1a^2}{\kappa},\ \omega^*=\frac{\omega a^2}{\kappa},\ \varpi_\Omega{}^*=\frac{\varpi_\Omega a^2}{\kappa},\ G_{j0}{}^*=\frac{G_{j0}}{G_{m0}},\ \alpha_{j0}{}^*=\frac{\alpha_{j0}}{\alpha_{m0}},\\
&i=1,2,3,4,\quad \Omega=1,2,\quad j=m,c.
\end{aligned} \tag{3}$$

Specifications of the material λ(x,θ), C(x,θ) , heat generation Q(x, y, t) , and heat flow f (y, t) are taken as:

$$\begin{aligned}
&\lambda(x,\theta)=\lambda_{m0}\,\lambda^*(x^*,\theta^*)\\
&C(x,\theta)=C_{m0}\,C^*(x^*,\theta^*)\\
&Q(x,y,t)=q_0\,Q^*(x^*,y^*,t^*)\\
&f(y,t)=q_1 f^*(y^*,t^*)
\end{aligned} \tag{4}$$

where θm0 , λm0 , Cm0 , pm0 , Gm0 , am0 represent the temperature, thermal conductivity, specific heat capacity, density, shear modulus, coefficient of linear thermal expansion of the metal with dimensions, Φ, r1 ,πΩ are the frequency, K = λm0 /(Cm0 pm0), is the thermal diffusivity, u, v,φ,Q,Ψ are the displacement, and q0 , q1 are the strength of heat flow with dimensions, and λ* (x*,θ*), C * (x*,θ*), Q * (x*, y*,t*), f * (y*,t*) are the dimensionless quantities.

Using equations (3-4), equations (1-2) reduce to the following dimensionless form (ignoring asterisks for convenience).

$$\frac{\partial}{\partial x}\left(\lambda(x,\theta)\frac{\partial\theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda(x,\theta)\frac{\partial\theta}{\partial y}\right)+P_0Q(x,y,t)=\rho C(x,\theta)\frac{\partial\theta}{\partial t} \tag{5}$$

$$\begin{aligned}
&\theta=\theta_0, && \text{at } t=0\\
&\lambda(x,\theta)\frac{\partial\theta}{\partial x}-Bi_1(\theta-\theta_0)=Ki\,f(y,t), && \text{at } x=0\\
&\lambda(x,\theta)\frac{\partial\theta}{\partial x}+Bi_2(\theta-\theta_0)=Ki\,f(y,t), && \text{at } x=a\\
&\lambda(x,\theta)\frac{\partial\theta}{\partial y}-Bi_3(\theta-\theta_0)=0, && \text{at } y=0\\
&\lambda(x,\theta)\frac{\partial\theta}{\partial y}+Bi_4(\theta-\theta_0)=0, && \text{at } y=b
\end{aligned} \tag{6}$$

where $P_0 = (q_0 a^2)/(\lambda_{m0}\,\theta_{m0})$ is the dimensionless Pomerantsev reference number, $Ki = (q_1\, a)/(\lambda_{m0}\,\theta_{m0})$ is the dimensionless Kirpichev reference number, $Bi_e = (k_e\, a)/(\lambda_{m0})$, $e = 1,2,3,4$ is the Biot criteria.

Using Kirchhoff's variable [3, 5, 9, 10, 11, 12]

$$\Theta(\theta) = \int_{\theta_0}^{\theta} \lambda(x,\theta)\, d\theta \tag{7}$$

and considering material with simple thermal nonlinearity, equations (5-6) become:

$$\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + P_0 Q(x,y,t) = \frac{1}{\kappa}\frac{\partial \Theta}{\partial t} \tag{8}$$

$$\begin{aligned}
&\Theta = 0, && \text{at } t = 0\\
&\frac{\partial \Theta}{\partial x} - Bi_1 \Theta = Ki\, f(y,t), && \text{at } x = 0\\
&\frac{\partial \Theta}{\partial x} + Bi_2 \Theta = Ki\, f(y,t), && \text{at } x = a\\
&\frac{\partial \Theta}{\partial y} - Bi_3 \Theta = 0, && \text{at } y = 0\\
&\frac{\partial \Theta}{\partial y} + Bi_4 \Theta = 0, && \text{at } y = b
\end{aligned} \tag{9}$$

K* = K/ Kmo is the dimensionless thermal diffusivity (asterisk is ignored for convenience).
For the sake of simplicity, the internal heat generation Q(x, y, t) and heat supply f (y, t) are assumed as (x0,y0 being dimensionless constants)

$$Q(x,y,t) = \delta(x - x_0)\delta(y - y_0)\,\delta(t), \quad f(y,t) = \delta(y - y_0)\sinh(\omega t).$$

Applying integral transform method as per [24, 25, 32], Laplace transform and their inversion over the variable x, y, t, we obtain

$$\Theta(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \{S(\beta_n,x)\, S(\alpha_m,y)[E_1\exp(-\omega t) + E_2\exp(\omega t) + E_3\exp(-A_6 t)]\} \tag{10}$$

Where,

$$S(\beta_n,x) = A_1(\beta_n\cos\beta_n x + Bi_1\sin\beta_n x), \quad A_1 = \left[\sqrt{2}\Big/\sqrt{\left(\beta_n^2 + Bi_1^2\right)\left(a + \frac{Bi_2}{\beta_n^2 + Bi_2^2}\right) + Bi_1}\right],$$

Here β_n's are the (dimensionless) positive roots of the transcendental equation $\tan\beta_n a = \dfrac{\beta_n(Bi_1 + Bi_2)}{\beta_n^2 - Bi_1 Bi_2}$,

$A_2 = A_1\,\beta_n + A_1(\beta_n\cos\beta_n a + Bi_1\sin\beta_n a)$, n is the transformparameter,

$$S(\alpha_m,y) = A_3(\alpha_m\cos\alpha_m y + Bi_1\sin\alpha_m y), \quad A_3 = \left[\sqrt{2}\Big/\sqrt{\left(\alpha_m^2 + Bi_1^2\right)\left(a + \frac{Bi_2}{\alpha_m^2 + Bi_2^2}\right) + Bi_1}\right],$$

Here α_m's are the (dimensionless) positive roots of the transcendental equation $\tan\alpha_m b = \dfrac{\alpha_m(Bi_1 + Bi_2)}{\alpha_m^2 - Bi_1 Bi_2}$,

m is the transformparameter,

$A_4 = \kappa(\beta_n^2 + \alpha_m^2)$, $A_5 = A_2 A_3 y_0\,\kappa(\alpha_m\cos\alpha_m y_0 + Bi_1\sin\alpha_m y_0)$,
$A_6 = x_0 y_0\,\kappa\, A_1(\beta_n\cos\beta_n a + Bi_1\sin\beta_n a)$,
$E_1 = A_5/(2\omega - 2A_4)$, $E_2 = A_5/(2\omega + 2A_4)$, $E_3 = [A_5\omega/(A_4^2 - \omega^2)] + A_6$.

Applying inverse Kirchhoff's variable transformation as per [7, 24, 25], we obtain

$$\begin{aligned}\theta(x,y,t) \cong \theta_0 + [\{1/p(x)\exp(\varpi_1\theta_0)\}\{\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{S(\beta_n,x)\,S(\alpha_m,y)\\ \times[E_1\exp(-\omega t) + E_2\exp(\omega t) + E_3\exp(-A_6 t)]\}\}]\end{aligned} \tag{11}$$

where $p(x) = [f_m(x)(\lambda_{m0} - \lambda_{c0}) + \lambda_{c0}]$, $f_m(x) = 1 - x^d$.

3. THERMOELASTIC EQUATIONS

The equilibrium equations, stress-strain components are given by [8].

$$e_{xx} = \frac{\partial u}{\partial x},\ e_{yy} = \frac{\partial v}{\partial y},\ e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{12}$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} = 0\ ,\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \tag{13}$$

$$\begin{aligned}
\sigma_{xx} &= \frac{2}{1-\nu(x,\theta)} G(x,\theta)[e_{xx} + \nu(x,\theta) e_{yy} - (1+\nu(x,\theta))\alpha(x,\theta)\theta],\\
\sigma_{yy} &= \frac{2}{1-\nu(x,\theta)} G(x,\theta)[\nu(x,\theta) e_{xx} + e_{yy} - (1+\nu(x,\theta))\alpha(x,\theta)\theta],\\
\sigma_{xy} &= 2G(x,\theta)e_{xy}
\end{aligned} \qquad \textbf{plane stress field} \tag{14}$$

$$\begin{aligned}
\sigma_{xx} &= \frac{2}{1-2\nu(x,\theta)} G(x,\theta)[(1-\nu(x,\theta))e_{xx} + \nu(x,\theta) e_{yy}\\
&\quad - (1+\nu(x,\theta))\alpha(x,\theta)\theta],\\
\sigma_{yy} &= \frac{2}{1-2\nu(x,\theta)} G(x,\theta)[\nu(x,\theta) e_{xx} + (1-\nu(x,\theta))e_{yy}\\
&\quad - (1+\nu(x,\theta))\alpha(x,\theta)\theta],\\
\sigma_{xy} &= 2G(x,\theta)e_{xy}
\end{aligned} \qquad \textbf{plane strain field} \tag{15}$$

where G(x,θ), a(x,θ),v(x,θ) are respectively the shear modulus of elasticity, coefficient of linear thermal expansion and Poisson'sratio.

3.1 Plane Stress Field

Using (1) and (3) in (2), we get

$$\begin{aligned}
&\frac{2G(x,\theta)}{1-\nu(x,\theta)}\left[\begin{array}{l}\frac{\partial^2 u}{\partial x^2} + \nu(x,\theta)\frac{\partial^2 v}{\partial x\,\partial y} + \frac{\partial v}{\partial y}\frac{\partial}{\partial x}(\nu(x,\theta)) - (1+\nu(x,\theta))\frac{\partial}{\partial x}(\alpha(x,\theta)\theta)\\ - \alpha(x,\theta)\theta\frac{\partial}{\partial x}(\nu(x,\theta))\end{array}\right]\\
&+ \frac{\partial}{\partial x}\left[\frac{2G(x,\theta)}{1-\nu(x,\theta)}\right]\left[\frac{\partial u}{\partial x} + \nu(x,\theta)\frac{\partial v}{\partial y} - (1+\nu(x,\theta))\alpha(x,\theta)\theta\right]\\
&+ \frac{\partial}{\partial y}(G(x,\theta))\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + G(x,\theta)\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x\,\partial y}\right) = 0,\\
&\frac{2G(x,\theta)}{1-\nu(x,\theta)}\left[\begin{array}{l}\nu(x,\theta)\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial u}{\partial x}\frac{\partial}{\partial y}(\nu(x,\theta)) + \frac{\partial^2 v}{\partial y^2} + -(1+\nu(x,\theta))\frac{\partial}{\partial y}(\alpha(x,\theta)\theta)\\ - \alpha(x,\theta)\theta\frac{\partial}{\partial y}(\nu(x,\theta))\end{array}\right]\\
&+ \frac{\partial}{\partial y}\left[\frac{2G(x,\theta)}{1-\nu(x,\theta)}\right]\left[\nu(x,\theta)\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - (1+\nu(x,\theta))\alpha(x,\theta)\theta\right]\\
&+ \frac{\partial}{\partial x}(G(x,\theta))\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + G(x,\theta)\left(\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 v}{\partial x^2}\right) = 0
\end{aligned}$$

3.2 Plane Strain Field

Similarly using equations (1) and (4) in (2), we obtain

$$
\begin{aligned}
&\frac{2G(x,\theta)}{1-2\nu(x,\theta)}\left[\begin{array}{l}\frac{\partial}{\partial x}\left[(1-\nu(x,\theta))\frac{\partial u}{\partial x}\right]+\nu(x,\theta)\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial v}{\partial y}\frac{\partial}{\partial x}(\nu(x,\theta))\\ -(1+\nu(x,\theta))\frac{\partial}{\partial x}(\alpha(x,\theta)\theta)-\alpha(x,\theta)\theta\frac{\partial}{\partial x}(\nu(x,\theta))\end{array}\right]\\
&+\frac{\partial}{\partial x}\left[\frac{2G(x,\theta)}{1-2\nu(x,\theta)}\right]\left[\frac{\partial u}{\partial x}+\nu(x,\theta)\frac{\partial v}{\partial y}-(1+\nu(x,\theta))\alpha(x,\theta)\theta\right]\\
&+\frac{\partial}{\partial y}(G(x,\theta))\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+G(x,\theta)\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right)=0,\\
&\frac{2G(x,\theta)}{1-2\nu(x,\theta)}\left[\begin{array}{l}\nu(x,\theta)\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial}{\partial y}(\nu(x,\theta))+\frac{\partial}{\partial y}\left[(1-\nu(x,\theta))\frac{\partial v}{\partial y}\right]\\ -(1+\nu(x,\theta))\frac{\partial}{\partial y}(\alpha(x,\theta)\theta)-\alpha(x,\theta)\theta\frac{\partial}{\partial y}(\nu(x,\theta))\end{array}\right]\\
&+\frac{\partial}{\partial y}\left[\frac{2G(x,\theta)}{1-2\nu(x,\theta)}\right]\left[\nu(x,\theta)\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}-(1+\nu(x,\theta))\alpha(x,\theta)\theta\right]\\
&+\frac{\partial}{\partial x}(G(x,\theta))\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+G(x,\theta)\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right)=0
\end{aligned}
\tag{17}
$$

Following [24, 25], we define

$$
u=\frac{\partial\phi}{\partial x}+\frac{\partial\varphi}{\partial x}+2\frac{\partial\psi}{\partial y}\quad,\qquad v=\frac{\partial\phi}{\partial y}+\frac{\partial\varphi}{\partial y}-2\frac{\partial\psi}{\partial x}
\tag{18}
$$

in which the three functions must satisfy the conditions

$$
\nabla^2\phi=K(x,\theta)\tau,\ \ \nabla^2\varphi=0\ \ \text{and}\ \ \nabla^2\psi=0
\tag{19}
$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, $K(x,\theta)=\frac{(1+\nu(x,\theta))}{(1-\nu(x,\theta))}\alpha(x,\theta)$ is the restraint coefficient and $\tau=\theta-\theta_0$.

On using equation (7) in equations (3) and (4), one obtains the stress functions as

3.3 For Plane Stress Field

$$
\begin{aligned}
\sigma_{xx}&=\frac{2G(x,\theta)}{1-\nu(x,\theta)}\left\{\begin{array}{l}\left(\frac{\partial^2\phi}{\partial x^2}+\nu(x,\theta)\frac{\partial^2\phi}{\partial y^2}\right)+\left(\frac{\partial^2\varphi}{\partial x^2}+\nu(x,\theta)\frac{\partial^2\varphi}{\partial y^2}\right)\\ +2\frac{\partial^2\psi}{\partial x\partial y}(1-\nu(x,\theta))-(1+\nu(x,\theta))\alpha(x,\theta)\theta\end{array}\right\},\\
\sigma_{yy}&=\frac{2G(x,\theta)}{1-\nu(x,\theta)}\left\{\begin{array}{l}\left(\nu(x,\theta)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+\left(\nu(x,\theta)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)\\ -2\frac{\partial^2\psi}{\partial x\partial y}(1-\nu(x,\theta))-(1+\nu(x,\theta))\alpha(x,\theta)\theta\end{array}\right\},\\
\sigma_{xy}&=2G(x,\theta)\left\{\frac{\partial^2\phi}{\partial x\partial y}+\frac{\partial^2\varphi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right\}
\end{aligned}
\tag{20}
$$

3.4 For Plane Strain Field

$$
\begin{aligned}
\sigma_{xx}&=\frac{2G(x,\theta)}{1-2\nu(x,\theta)}\left\{\begin{array}{l}\left((1-\nu(x,\theta))\frac{\partial^2\phi}{\partial x^2}+\nu(x,\theta)\frac{\partial^2\phi}{\partial y^2}\right)+\left((1-\nu(x,\theta))\frac{\partial^2\varphi}{\partial x^2}+\nu(x,\theta)\frac{\partial^2\varphi}{\partial y^2}\right)\\ +2\frac{\partial^2\psi}{\partial x\partial y}(1-2\nu(x,\theta))-(1+\nu(x,\theta))\alpha(x,\theta)\theta\end{array}\right\},\\
\sigma_{yy}&=\frac{2G(x,\theta)}{1-2\nu(x,\theta)}\left\{\begin{array}{l}\left(\nu(x,\theta)\frac{\partial^2\phi}{\partial x^2}+(1-\nu(x,\theta))\frac{\partial^2\phi}{\partial y^2}\right)+\left(\nu(x,\theta)\frac{\partial^2\varphi}{\partial x^2}+(1-\nu(x,\theta))\frac{\partial^2\varphi}{\partial y^2}\right)\\ -2\frac{\partial^2\psi}{\partial x\partial y}(1-2\nu(x,\theta))-(1+\nu(x,\theta))\alpha(x,\theta)\theta\end{array}\right\},\\
\sigma_{xy}&=2G(x,\theta)\left\{\frac{\partial^2\phi}{\partial x\partial y}+\frac{\partial^2\varphi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right\}
\end{aligned}
\tag{21}
$$

The traction free conditions are

$$\sigma_{xx}\big|_{x=0} = \sigma_{xx}\big|_{x=a} = \sigma_{yy}\big|_{y=0} = \sigma_{yy}\big|_{y=b} = 0 \tag{22}$$

Using equation (11) in equation (19), we get

$$\begin{aligned}\phi = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} &\{\{K(x,\theta)l_1(x,y)/[l_2(x,y)(p(x))^3 \exp(\varpi_1\theta_0)]\}S(\beta_n,x)S(\alpha_m,y) \\ &\times[E_1\exp(-\omega t)+E_2\exp(\omega t)+E_3\exp(-A_6 t)]\}\end{aligned} \tag{23}$$

Where

$$\begin{aligned} &l_1(x,y) = S(\beta_n,x)\times S(\alpha_m,y), \\ &l_2(x,y) = S(\alpha_m,y)[p(x)l_3{}'(x) - 2l_3(x)p'(x)] + S(\beta_n,x)S''(\alpha_m,y)(p(x))^2, \\ &l_3(x) = p(x)S'(\beta_n,x) - p'(x)S(\beta_n,x).\end{aligned}$$

The Boussinesq harmonic functions Q and Ψ satisfying equation (19) are assumed as

$$\begin{aligned}\varphi = \psi = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} &\{\sinh(r_1 t)[B_n\sin(\beta_n x)+D_n\cos(\beta_n x)] \\ &\times[\sin(\alpha_m y)+\cos(\alpha_m y)]\}\end{aligned} \tag{24}$$

where B_n , D_n are dimensionless constants.

The displacement components from equation (18) are obtained using equations (23) and (24) as

$$\begin{aligned}u = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} &\{\phi_{,x} + \sinh(r_1 t)\beta_n[B_n\cos(\beta_n x) - D_n\sin(\beta_n x)][\sin(\alpha_m y)+\cos(\alpha_m y)] \\ &+ 2\alpha_m\sinh(r_1 t)[B_n\sin(\beta_n x)+D_n\cos(\beta_n x)][\cos(\alpha_m y)-\sin(\alpha_m y)]\} \\ v = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} &\{\phi_{,y} + \alpha_m\sinh(r_1 t)[B_n\sin(\beta_n x)+D_n\cos(\beta_n x)][\cos(\alpha_m y)-\sin(\alpha_m y)] \\ &- 2\sinh(r_1 t)\beta_n[B_n\cos(\beta_n x)-D_n\sin(\beta_n x)][\sin(\alpha_m y)+\cos(\alpha_m y)]\}\end{aligned} \tag{25}$$

Following [7, 25], we assume

$$\begin{aligned} &G(x,\theta) = G_m(\theta)f_m(x) + G_c(\theta)(1-f_m(x)), \\ &\alpha(x,\theta) = \alpha_m(\theta)f_m(x) + \alpha_c(\theta)(1-f_m(x)), \\ &\nu(x,\theta) = \nu_m(\theta)f_m(x) + \nu_c(\theta)(1-f_m(x))\end{aligned} \tag{26}$$

where G(θ), a(θ), v(θ) are assumed according to exponential law as follows [1]:

$$\begin{aligned} &G_j(\theta) = G_{j0}\exp(\varpi_1\theta), \quad \alpha_j(\theta) = \alpha_{j0}\exp(\varpi_2\theta), \quad \nu_j(\theta) = \nu_{j0}\exp(\varpi_2\theta), \\ &j = m, c; \quad \varpi_1 \le 0, \ \varpi_2 \ge 0\end{aligned} \tag{27}$$

Using equations (23-27) in equations (20) and (21), stresses are obtained.

4. Numerical Results and Discussions

The values of alumina (ceramic) and nickel (metal) are used from [7]. Figures (1 to 6) represent the graphs of temperature and stresses. Left side figures represent spatial variable and temperature independent case (TIC), while right figures represent spatial variable and temperature dependent case (TDC). Figs. 1 and 2 represent temperature along x andy axes. In both TIC and TDC sudden hike in the temperature is seen due to heat source. Figs. 3 and 4 represent plane stress field along both axes. In TIC, $\overline{\sigma}_{xy}$, $\overline{\sigma}_{yy}$ are tensile from the outer to middle part of the plate and compressive at the other end. In TDC, they are tensile near the outer region and zero towards the origin. Figs. 5 and 6 represent plane strain. In TIC, stresses are tensile. In TDC, σ_{xx} changes its nature from tensile to compressive.

5. Graphical Results

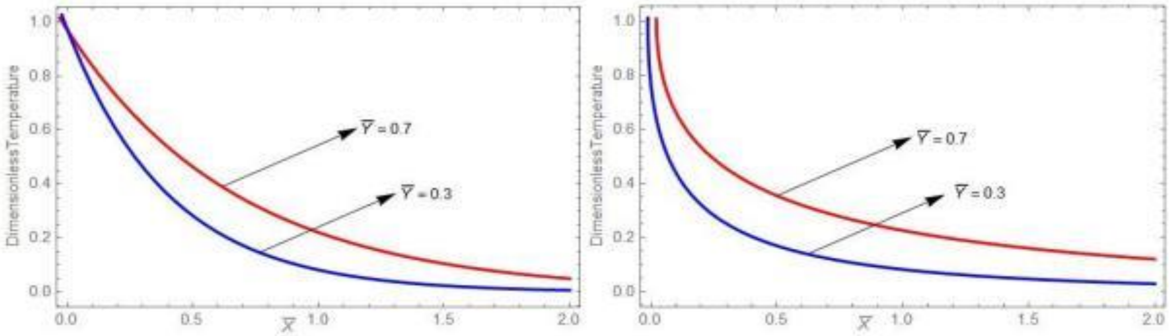

Fig. 1: Variation of dimensionless temperature along x-axis

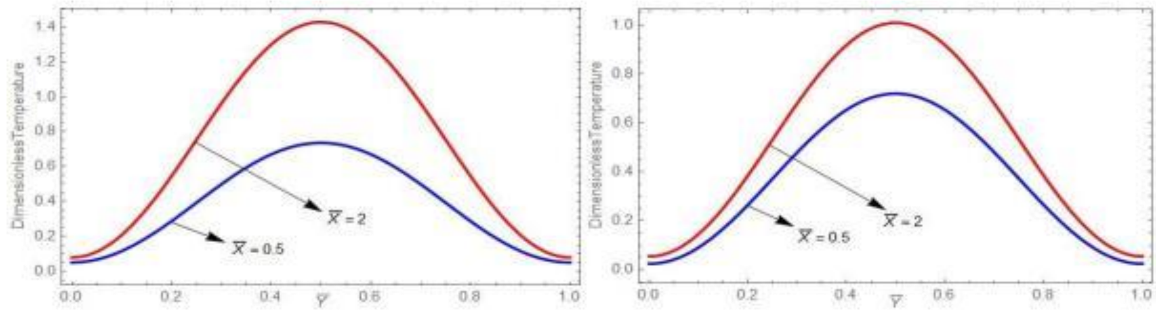

Fig. 2: Variation of dimensionless temperature along y-axis

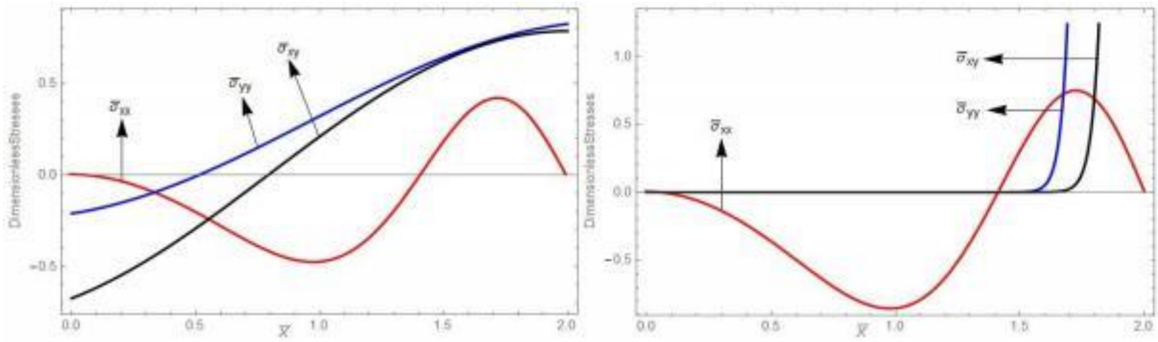

Fig. 3: Variation of dimensionless stresses (plane stress field) along x-axis

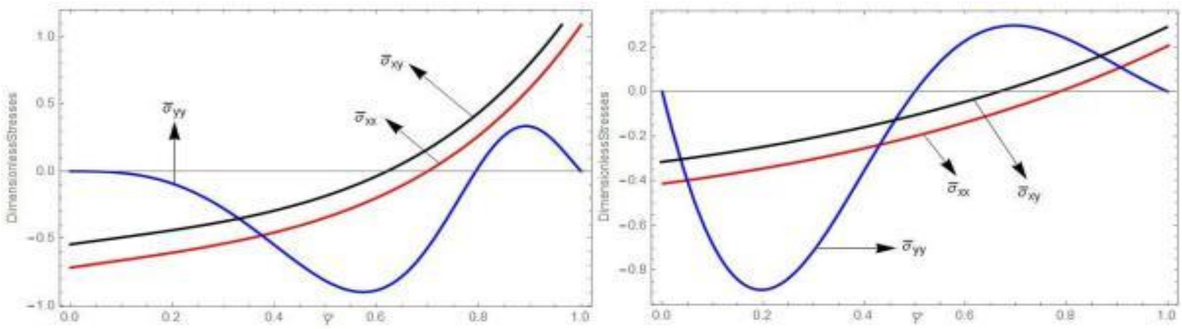

Fig. 4: Variation of dimensionless stresses (plane stress field) along y-axis

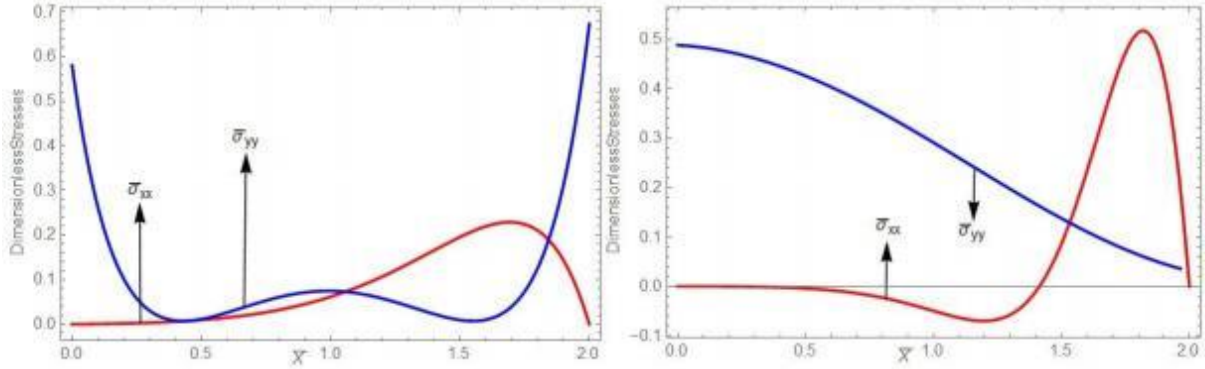

Fig. 5: Variation of dimensionless stresses (plane strain field) along x–axis

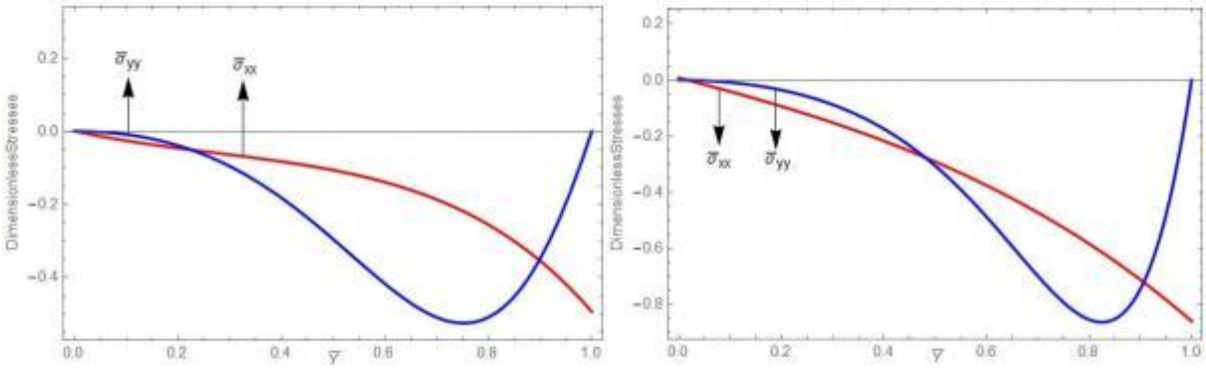

Fig. 6: Variation of dimensionless stresses (plane strain field) along y–axis

6. Conclusion

The temperature profile of a thermosensitive FGRP with heat source subjected to convective heat exchange has been obtained. The thermal profile is studied using plane stress strain field. The findings indicate significant effects on the behaviour of the transient temperature distribution due to the heat source. The temperature distribution of the FGRP is suddenly changing in TDC as compared to TDC along x direction. Due to convective heat exchange along x-axis, the stress profile is having tremendous change in the nonhomogenous case. Crest and trough are observed for the stress components σxx , σxy in the plane stress field. Since the material properties are dependent on temperature, the internal heat generation shows significant effect on the heat conduction of the FG rectangular plate. The proposed mathematical model may be useful for material physical characterization for the use of materials at high temperatures in advanced technology applications

Refrences

[1] N. Noda, “Thermal stresses in materials with temperature dependent properties”, Thermal Stresses I, North Holland, Amsterdam, pp. 391-483, 1986.

[2] Y. Tanigawa, Y. Ootao, and R. Kawamura, “Thermal bending of laminated composite rectangular plates and nonhomogeneous plates due to partial heating”, Journal of Thermal Stresses, vol. 14, no. 3,pp. 285-308, 1991.

[3] V. S. Popovych, and B. N. Fedai, “The axi-symmetric problem of thermoelasticity of a multilayer thermosensitive tube” , Journal of Mathematical Sciences, vol. 86, no. 2,pp. 2605–2610, 1997.

[4] H. Morishita, and Y. Tanigawa Y., “Formulation of three dimensional elastic problem for nonhomogeneous medium and its analytical development for semi-infinite body”, The Japan Society of Mechanical Engineers, vol. 97, pp. 97- 104, 1998.

[5] V. S. Popovych, and I. M. Makhorkin, “On the solution of heat-conduction problems for thermosensitive bodies”, Journal of Mathematical Sciences, vol. 88, no. 3,pp. 352-359, 1998.
[6] R. Kawamura, D. Huang, and Y. Tanigawa, “Thermoelastic deformation and stress analyses of an orthotropic nonhomogeneous rectangular plate”, Proceedings of Fourth International Congress on Thermal Stresses, pp. 189- 192, 2001.
[7] H. Awaji, H. Takenaka, S. Honda, and T. Nishikawa, "Temperature/Stress distributions in a stress-relief-type plate of functionally graded materials under thermal shock", JSME International Journal, vol. 44, pp. 1059- 1065, 2001.
[8] Y. Tanigawa, R. Kawamura, S. Ishida, “Derivation of fundamental equation systems of plane isothermal and thermoelastic problems for in-homogeneous solids and its applications to semi-infinite body and slab”, Theoretical and Applied Mechanics, vol. 51, pp. 267- 279, 2002.
[9] R. M. Kushnir, and V. S. Popovych, “Heat conduction problems of thermosensitive solids under complex heat exchange”, INTECH, London, 2011.
[10] R. M. Kushnir, V. S. Popovych, and V. V. Yanishevsky, “Thermal and thermoelastic state of thin walled thermosensitive structures subject to complex heat exchange”, Journal of Thermal Stresses, vol. 35, no. 1,pp. 91- 102, 2012.
[11] V. S. Popovych, “Methods for determination of the thermo-stressed state of thermally sensitive solids under complex heat exchange conditions”, Encyclopedia of Thermal Stresses, vol. 6, Springer, pp. 2997–3008, 2014.
[12] V. S. Popovych, and B. M. Kalynyak, “Mathematical modeling and methods for the determination of the static thermoelastic state of multilayer thermally sensitive cylinders”, Journal of Mathematical Sciences, vol. 215, no. 2,pp. 218-242, 2016.
[13] V. R. Manthena, N. K. Lamba, G. D. Kedar, and K. C. Deshmukh, “Effects of stress resultants on thermal stresses in a functionally graded rectangular plate due to temperature dependent material properties”, International Journal of Thermodynamics, vol. 19, no. 4, pp. 235-242, 2016.
[14] T. R. Mahapatra, V. R. Kar, S. K. Panda, and K. Mehar, “Nonlinear thermoelastic deflection of temperature-dependent FGM curved shallow shell under nonlinear thermal loading”, Journal of Thermal Stresses, vol. 40, no. 9, pp. 1184- 1199, 2017.
[15] V. R. Manthena, N. K. Lamba, and G. D. Kedar, “ Transient thermoelastic problem of a nonhomogeneous rectangular plate”, Journal of Thermal Stresses, vol. 40, no. 5,pp. 627-640, 2017.
[16] V. R. Manthena, and G. D. Kedar, “Transient thermal stress analysis of a functionally graded thick hollow cylinder with temperature dependent material properties”, Journal of Thermal Stresses, vol. 41,no. 5,pp. 568-582, 2018.
[17] V. R. Manthena, N. K. Lamba, and G. D. Kedar, “Thermoelastic analysis of a rectangular plate with nonhomogeneous material properties and internal heat source”, Journal of Solid Mechanics, vol. 10, no. 1,pp. 200-215, 2018.
[18] V. R. Manthena, G. D. Kedar, and K. C. Deshmukh, “Thermal stress analysis of a thermosensitive functionally graded rectangular plate due to thermally induced resultant moments”, Multidiscipline Modeling in Materials and Structures, vol. 14, no. 5,pp. 857-873, 2018.
[19] A. A. Daikh, and A. Megueni, "Thermal buckling analysis of functionally graded sandwich plates", Journal of Thermal Stresses, vol. 41,no. 2,pp. 139- 159, 2018.
[20] A. Hossein, and M. M. Lajevardi, "Buckling of functionally graded plates under thermal, axial, and shear in-plane loading using complex finite strip formulation", Journal of Thermal Stresses, vol. 41,no. 2,pp. 182-203, 2018.
[21] Y. Z. Wang, D. Liu, Q. Wang, and J. Z. Zhou, "Thermoelastic interaction in functionally graded thick hollow cylinder with temperature-dependent properties", Journal of Thermal Stresses, vol. 41, no. 4,pp. 399-417, 2018.
[22] A. S. Sayyad, and Y. M. Ghugal, "Effects of nonlinear hygrothermomechanical loading on bending of FGM rectangular plates resting on two-parameter elastic foundation using four-unknown plate theory", Journal of Thermal Stresses, vol. 42, no. 2, pp. 213-232, 2019.

[23] A. Yıldırım, D. Yarımpabuç, and Kerimcan Celebi, "Thermal stress analysis of functionally graded annular fin", Journal of Thermal Stresses, vol. 42, no. 4,pp. 440-451, 2019.
[24] V. R. Manthena, and G. D. Kedar, “On thermoelastic problem of a thermosensitive functionally graded rectangular plate with instantaneous point heat source”, Journal of Thermal Stresses, vol. 42,no. 7,pp. 849–862, 2019.
[25] V. R. Manthena, “Uncoupled thermoelastic problem of a functionally graded thermosensitive rectangular plate with convective heating”, Archive of Applied Mechanics, vol. 89, no. 8,pp. 1627– 1639, 2019.
[26] W. Yang, and Z. Chen, "Investigation of the thermal-elastic problem in cracked semi-infinite FGM under thermal shock using hyperbolic heat conduction theory", Journal of Thermal Stresses, vol. 42,no. 8,pp. 993-1010, 2019.
[27] V. R. Manthena, V. B. Srinivas, and G. D. Kedar, “Analytical solution of heat conduction of a multilayered annular disk and associated thermal deflection and thermal stresses”, Journal of Thermal Stresses, vol. 43, no. 5, pp. 563–578, 2020.
[28] N. S. Ruziboyevich, "Heat transfer during the pressure flow of liquid in pipes in heat supply systems", Solid State Technology, vol. 63, no.4, pp. 213-224, 2020.
[29] L. C. Bawankar, G. D. Kedar, and K. C. Deshmukh, "Effect of temperature dependency in a one dimensional two-temperature magneto-thermoelastic problem due to the influence of modified Ohm’s law", Solid State Technology, vol. 63, no.4, pp. 5870-5885, 2020.
[30] S. V. Mundhe, and R. S. Bindu, "Vortex Generator: An effective way of heat transfer enhancement for circular tube", Solid State Technology, vol. 63, no.5, pp. 67-78, 2020.
[31] V. B. Srinivas, V. R. Manthena, J. Bikram, and G. D. Kedar, “Fractional order heat conduction and thermoelastic response of a thermally sensitive rectangular parallelopiped”, International Journal of Thermodynamics, vol. 24, no. 1, pp. 62–73, 2021.
[32] M. N. Ozisik, Boundary value problems of heat conduction, Dover Publications, New York, 1989.