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On Fermatean Neutrosophic Hypersoft σ- Baire Spaces

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Abstract:- Within this contribution the initiative of Fermatean Neutrosophic Hypersoft σ- Baireness in Fermatean Neutrosophic Hypersoft topological spaces are outlined. We have touted Fermatean Neutrosophic Hypersoft σ- nowhere dense set, Fermatean Neutrosophic Hypersoft σ- first category and Fermatean Neutrosophic Hypersoft σ- second category sets. Numerous descriptions of Fermatean Neutrosophic Hypersoft σ- Baire Spaces are also investigated.

***Keywords**:* Fermatean Neutrosophic Hypersoft σ-nowhere dense set, Fermatean Neutrosophic Hypersoft σ- first category sets, Fermatean Neutrosophic Hypersoft σ- second category sets and Fermatean Neutrosophic Hypersoft σ- Baire spaces.

1. Introduction

Almost every domain for the research has been impacted by the Fuzzy approach, to the extent that L. A. Zadeh[11] reveals a Fuzzy set. Fuzzy Topological Space: An imperative model emerged by C. L. Chang [4]. G. Thangaraj and S. Anjalmose[10] greeted and explored the idea of σ-Baire Spaces in a Fuzzy set. Atanassov[3] laid out the proposition of an Intuitionistic Fuzzy set for the first time. It was F. Smaradache[8] who initially pitched the formulation of the Neutrosophic set. Smarandache[9] reshaped the function into a multi-decision function, hence by prolonging the Soft to Hypersoft set. Senapati and Yager[7] launched the theoretical framework of Fermatean Fuzzy sets. Fermatean Neutrosophic set was purely stated by C.A.C. Sweety and R. Jansi[2]. D.Ajay[1] generated the thought process of Neutrosophic Hypersoft Topological Spaces. P.Reena Joice and M. Trinita Pricilla[5] introduced Fermatean Neutrosophic Hypersoft set by the extension of Fermatean Neutrosophic and Neutrosophic Hypersoft set. In this paper the concepts of Fermatean Neutrosophic Hypersoft σ- Baire spaces are laid out and functionalities of Fermatean Neutrosophic Hypersoft σ- Baire spaces are assessed.

2. Preliminaries

Definition 2.1 Let $\widetilde{\mathfrak{A}}_{\dot{\varsigma}}$ be the universal set and $P(\widetilde{\mathfrak{A}}_{\dot{\varsigma}})$ be the power set of $\widetilde{\mathfrak{A}}_{\dot{\varsigma}}$ and $\widehat{\mathfrak{E}}^{\bar{\mathfrak{e}}}_{Ŧ1}$, $\widehat{\mathfrak{E}}^{\bar{\mathfrak{e}}}_{Ŧ2}$,…,$\widehat{\mathfrak{E}}^{\bar{\mathfrak{e}}}_{Ŧ\mathfrak{B}}$ the pairwise disjoint sets of parameters. Let $\widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ\mathfrak{Q}}$ be the nonempty subset of the pair $\widehat{\mathfrak{E}}^{\bar{\mathfrak{e}}}_{Ŧ\mathfrak{Q}}$, for each $Ŧ\mathfrak{Q} = Ŧ1, Ŧ2, \ldots, Ŧ\mathfrak{B}$. A Fermatean Neutrosophic Hypersoft set over $\widetilde{\mathfrak{A}}_{\dot{\varsigma}}$ defined as the pair $\left(\underline{\mathfrak{K}}^{\curlyvee}, \widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ1} \times \widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ2} \times \ldots \widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ\mathfrak{B}}\right)$ where $\underline{\mathfrak{K}}^{\curlyvee}: \widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ1} \times \widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ2} \times \ldots \widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ\mathfrak{B}} \rightarrow P(\widetilde{\mathfrak{A}}_{\dot{\varsigma}})$ and $\underline{\mathfrak{K}}^{\curlyvee}\left(\widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ1} \times \widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ2} \times \ldots \widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{Ŧ\mathfrak{B}}\right) = \left\{\left\langle \mathfrak{U}_{\mathfrak{u}}{}^{\natural}, \widetilde{\mathfrak{R}}^{\underline{\flat}}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{\mathfrak{u}}{}^{\natural}), \widetilde{\mathfrak{I}}^{\underline{\sharp}}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{\mathfrak{u}}{}^{\natural}), \widetilde{\wp}^{\underline{\vartheta}}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{\mathfrak{u}}{}^{\natural}), \right\rangle : \mathfrak{U}_{\mathfrak{u}}{}^{\natural} \in \widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \right\}$

Here, $\widetilde{\mathfrak{R}}^{\underline{\flat}}$ is the membership value , $\widetilde{\mathfrak{I}}^{\underline{\sharp}}$ is the indeterminate value and $\widetilde{\wp}^{\underline{\vartheta}}$ is the non- membership value such that $\widetilde{\mathfrak{R}}^{\underline{\flat}}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{\mathfrak{u}}{}^{\natural})$, $\widetilde{\mathfrak{I}}^{\underline{\sharp}}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{\mathfrak{u}}{}^{\natural})$, $\widetilde{\wp}^{\underline{\vartheta}}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{\mathfrak{u}}{}^{\natural}) \in [0,1]$,

$0 \leq \left(\widetilde{\mathfrak{R}}^{\underline{\flat}}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{\mathfrak{u}}{}^{\natural})\right)^3 + \left(\widetilde{\wp}^{\underline{\vartheta}}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{\mathfrak{u}}{}^{\natural})\right)^3 \leq 1$ and $0 \leq \left(\widetilde{\mathfrak{I}}^{\underline{\sharp}}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{\mathfrak{u}}{}^{\natural})\right)^3 \leq 1$. Then

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$0 \leq \left(\widetilde{\mathfrak{R}}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural})\right)^{3} + \left(\widetilde{\mathfrak{I}}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural})\right)^{3} + \left(\widetilde{\wp}^{\vartheta}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural})\right)^{3} \leq 2$

We write the symbol $\widehat{\mathfrak{G}}^{\varepsilon}$ for $\widetilde{\mathfrak{E}}^{\bar{\varepsilon}}_{\mp 1} \times \widetilde{\mathfrak{E}}^{\bar{\varepsilon}}_{\mp 2} \times \ldots \times \widetilde{\mathfrak{E}}^{\bar{\varepsilon}}_{\mp \mathfrak{B}}$, $\widehat{\mathfrak{H}}^{\varrho}$ for $\widetilde{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{\mp 1} \times \widetilde{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{\mp 2} \times \ldots \widetilde{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{\mp \mathfrak{B}}$.

In this script, we reveal the family of all Fermatean Neutrosophic Hypersoft sets over the universe set $\widetilde{\mathfrak{A}}_{\dot{\varsigma}}$ with $\mathcal{FNH}\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}\right)$.

Definition 2.2 Let us take two $\mathcal{FNH}$ set of the form $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}\right)$ and $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 2}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 2}\right)$

(1) A $\mathcal{FNH}$ set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}\right)$ over the universe set $\widetilde{\mathfrak{A}}_{\dot{\varsigma}}$ is said to be null $\mathcal{FNH}$ set if

$\widetilde{\mathfrak{R}}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}) = 0_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}FNH}, \widehat{\mathfrak{G}}^{\varepsilon})}$, $\widetilde{\mathfrak{I}}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}) = 1_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}FNH}, \widehat{\mathfrak{G}}^{\varepsilon})}$, $\widetilde{\wp}^{\vartheta}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}) = 1_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}FNH}, \widehat{\mathfrak{G}}^{\varepsilon})}$

It is denoted by $0_{([\underline{\mathfrak{X}}_{FNH}, \Xi_{u})}$.

(2) A $\mathcal{FNH}$ set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}\right)$ over the universe set $\widetilde{\mathfrak{A}}_{\dot{\varsigma}}$ is said to be absolute $\mathcal{FNH}$ set if

$\widetilde{\mathfrak{R}}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}) = 1_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}FNH}, \widehat{\mathfrak{G}}^{\varepsilon})}$, $\widetilde{\mathfrak{I}}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}) = 0_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}FNH}, \widehat{\mathfrak{G}}^{\varepsilon})}$, $\widetilde{\wp}^{\vartheta}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}) = 0_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}FNH}, \widehat{\mathfrak{G}}^{\varepsilon})}$

It is denoted by $1_{([\underline{\mathfrak{X}}_{FNH}, \Xi_{u})}$.

(3) $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}\right) \subseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 2}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 2}\right)$ if and only if $\widetilde{\mathfrak{R}}_{\dagger 1}{}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}) \leq \widetilde{\mathfrak{R}}_{\dagger 2}{}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural})$,

$\widetilde{\mathfrak{I}}_{\dagger 1}{}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}) \geq \widetilde{\mathfrak{I}}_{\dagger 2}{}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural})$ and $\widetilde{\wp}_{\dagger 1}{}^{\vartheta}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}) \geq \widetilde{\wp}_{\dagger 2}{}^{\vartheta}_{\mp 2\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural})$.

(4) $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}\right) = \left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 2}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 2}\right)$ if and only if $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}\right) \subseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 2}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 2}\right)$ and

$\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 2}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 2}\right) \subseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}\right)$

(5) $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}\right)^{c} = \left\{\left\langle \mathfrak{U}_{u}{}^{\natural}, \widetilde{\wp}^{\vartheta}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}), 1 - \widetilde{\mathfrak{I}}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}), \widetilde{\mathfrak{R}}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}), \right\rangle : \mathfrak{U}_{u}{}^{\natural} \in \widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \right\}$

(6) $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}\right) \cup \left(\underline{\mathfrak{K}}^{\curlyvee}_{\mp 2}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 2}\right)$ is defined as

$$\widetilde{\mathfrak{R}}^{\flat}((\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}) \cup (\underline{\mathfrak{K}}^{\curlyvee}_{\mp 2}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 2}))$$

$$= \begin{cases} \widetilde{\mathfrak{R}}_{\dagger 1}{}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}^{\varrho}_{\mp 1} - \widehat{\mathfrak{H}}^{\varrho}_{\mp 2} \\ \widetilde{\mathfrak{R}}_{\dagger 2}{}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}^{\varrho}_{\mp 2} - \widehat{\mathfrak{H}}^{\varrho}_{\mp 1} \\ max\{\widetilde{\mathfrak{R}}_{\dagger 1}{}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}), \widetilde{\mathfrak{R}}_{\dagger 2}{}^{\flat}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural})\} & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}^{\varrho}_{\mp 1} \cap \widehat{\mathfrak{H}}^{\varrho}_{\mp 2} \end{cases}$$

$$\widetilde{\mathfrak{I}}^{\sharp}((\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}) \cup (\underline{\mathfrak{K}}^{\curlyvee}_{\mp 2}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 2}))$$

$$= \begin{cases} \widetilde{\mathfrak{I}}_{\dagger 1}{}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}^{\varrho}_{\mp 1} - \widehat{\mathfrak{H}}^{\varrho}_{\mp 2} \\ \widetilde{\mathfrak{I}}_{\dagger 2}{}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}^{\varrho}_{\mp 2} - \widehat{\mathfrak{H}}^{\varrho}_{\mp 1} \\ min\{\widetilde{\mathfrak{I}}_{\dagger 1}{}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural}), \widetilde{\mathfrak{I}}_{\dagger 2}{}^{\sharp}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}(\mathfrak{U}_{u}{}^{\natural})\} & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}^{\varrho}_{\mp 1} \cap \widehat{\mathfrak{H}}^{\varrho}_{\mp 2} \end{cases}$$

$$\widetilde{\wp}^{\vartheta}((\underline{\mathfrak{K}}^{\curlyvee}_{\mp 1}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 1}) \cup (\underline{\mathfrak{K}}^{\curlyvee}_{\mp 2}, \widehat{\mathfrak{H}}^{\varrho}_{\mp 2}))$$

$$= \begin{cases} \widetilde{\wp}_{\dagger 1}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\vartheta}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} - \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} \\ \widetilde{\wp}_{\dagger 2}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\vartheta}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} - \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} \\ min\{\widetilde{\wp}_{\dagger 1}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\vartheta}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), \widetilde{\wp}_{\dagger 2}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\vartheta}(\mathfrak{U}_{\mathrm{u}}{}^{\natural})\} & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} \cap \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} \end{cases}$$

(7) $\left(\underline{\mathfrak{K}}_{\dagger 1}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho}\right) \cap \left(\underline{\mathfrak{K}}_{\dagger 2}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho}\right)$ is defined as

$$\widetilde{\mathfrak{R}}^{\flat}((\underline{\mathfrak{K}}_{\dagger 1}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho}) \cap (\underline{\mathfrak{K}}_{\dagger 2}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho}))$$

$$= \begin{cases} \widetilde{\mathfrak{R}}_{\dagger 1}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\flat}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} - \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} \\ \widetilde{\mathfrak{R}}_{\dagger 2}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\flat}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} - \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} \\ min\{\widetilde{\mathfrak{R}}_{\dagger 1}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\flat}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), \widetilde{\mathfrak{R}}_{\dagger 2}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\flat}(\mathfrak{U}_{\mathrm{u}}{}^{\natural})\} & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} \cap \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} \end{cases}$$

$$\widetilde{\mathfrak{I}}^{\sharp}((\underline{\mathfrak{K}}_{\dagger 1}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho}) \cap (\underline{\mathfrak{K}}_{\dagger 2}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho}))$$

$$= \begin{cases} \widetilde{\mathfrak{I}}_{\dagger 1}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\sharp}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} - \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} \\ \widetilde{\mathfrak{I}}_{\dagger 2}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\sharp}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} - \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} \\ max\{\widetilde{\mathfrak{I}}_{\dagger 1}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\sharp}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), \widetilde{\mathfrak{I}}_{\dagger 2}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\sharp}(\mathfrak{U}_{\mathrm{u}}{}^{\natural})\} & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} \cap \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} \end{cases}$$

$$\widetilde{\wp}^{\underline{\vartheta}}((\underline{\mathfrak{K}}_{\dagger 1}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho}) \cap (\underline{\mathfrak{K}}_{\dagger 2}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho}))$$

$$= \begin{cases} \widetilde{\wp}_{\dagger 1}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\vartheta}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} - \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} \\ \widetilde{\wp}_{\dagger 2}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\vartheta}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} - \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} \\ max\{\widetilde{\wp}_{\dagger 1}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\vartheta}(\mathfrak{U}_{\mathrm{u}}{}^{\natural}), \widetilde{\wp}_{\dagger 2}{}_{\underline{\mathfrak{K}}^{\curlyvee}(\mathfrak{Y}^{\pounds})}^{\vartheta}(\mathfrak{U}_{\mathrm{u}}{}^{\natural})\} & \text{if} \mathfrak{Y}^{\pounds} \in \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho} \cap \widehat{\mathfrak{H}}_{\dagger 2}^{\varrho} \end{cases}$$

Definition 2.3 A $\mathcal{FNH}$ topology on a non empty set $\widetilde{\mathfrak{A}}_{\c{\varsigma}}$ is a family $\widehat{\mathfrak{V}}_{\underline{\eta}}$ of $\mathcal{FNH}$ sets in $\widetilde{\mathfrak{A}}_{\c{\varsigma}}$ satisfying the following axioms:

(i) $0_{(\widetilde{\mathfrak{A}}_{\c{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$, $1_{(\widetilde{\mathfrak{A}}_{\c{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ belongs to $\widehat{\mathfrak{V}}_{\underline{\eta}}$.

(ii) the union of any number of $\mathcal{FNH}$ sets in $\widehat{\mathfrak{V}}_{\underline{\eta}}$ belongs to $\widehat{\mathfrak{V}}_{\underline{\eta}}$.

(iii) the interesection of finite number of $\mathcal{FNH}$ sets in $\widehat{\mathfrak{V}}_{\underline{\eta}}$ belongs to $\widehat{\mathfrak{V}}_{\underline{\eta}}$.

Then $\left(\widetilde{\mathfrak{A}}_{\c{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{V}}_{\underline{\eta}}\right)$ is said to be $\mathcal{FNH}$ topological space over $\widetilde{\mathfrak{A}}_{\c{\varsigma}}$. Each members of $\widehat{\mathfrak{V}}_{\underline{\eta}}$ is said to be $\mathcal{FNH}$ open set.

Definition 2.4 Let $\left(\underline{\mathfrak{K}}_{\dagger 1}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho}\right)$ be a $\mathcal{FNH}$ set in a $\mathcal{FNH}$ topological space $\left(\widetilde{\mathfrak{A}}_{\c{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{V}}_{\underline{\eta}}\right)$. Then

(i) $\mathcal{FNH}int\left(\underline{\mathfrak{K}}_{\dagger 1}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho}\right) = \cup \{\left(\underline{\mathfrak{K}}_{\dagger n}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger n}^{\varrho}\right) \mid \left(\underline{\mathfrak{K}}_{\dagger n}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger n}^{\varrho}\right)$ is a FNH $open\ set\ and \left(\underline{\mathfrak{K}}_{\dagger n}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger n}^{\varrho}\right) \subseteq \left(\underline{\mathfrak{K}}_{\dagger 1}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho}\right)\}$ is called the $\mathcal{FNH}$ interior of $\left(\underline{\mathfrak{K}}_{\dagger 1}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 1}^{\varrho}\right)$.

(ii)$\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = \cap\{\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger n}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger n}\right) \mid \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger n}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger n}\right)$ is a FNH $closed\ set\ and \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger n}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger n}\right) \supseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)\}$ is called the $\mathcal{FNH}$ closure of $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$.

Definition 2.5 A $\mathcal{FNH}$ set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ in a $\mathcal{FNH}$ topological space $\widetilde{\mathfrak{A}}_{\dot{\varsigma}}$ is called

(1) $\mathcal{FNH}$ open set if $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$and $\mathcal{FNH}$ closed set if

$\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$

(2) $\mathcal{FNH}$ regular open if $\mathcal{FNH}int\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ and $\mathcal{FNH}$ regular closed if $\mathcal{FNH}cl\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$.

(3) $\mathcal{FNH}$ pre- open if $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) \subseteq \mathcal{FNH}int\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ and $\mathcal{FNH}$ Pre- closed if $\mathcal{FNH}cl\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) \subseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$.

(4) $\mathcal{FNH}$ Semi- open if $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) \subseteq \mathcal{FNH}cl\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ and $\mathcal{FNH}$ semi- closed if $\mathcal{FNH}int\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) \subseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$.

(5) $\mathcal{FNH}$ β- open if $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) \subseteq \mathcal{FNH}cl\mathcal{FNH}int\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ and $\mathcal{FNH}$ β- closed if $\mathcal{FNH}int\mathcal{FNH}cl\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) \subseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$.

3. Fermatean Neutrosophic Hypetsoft σ- Nowhere Dense Sets

Definition 3.1 A $\mathcal{FNH}$ set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ in a $\mathcal{FNH}$topological space $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is called a *Fermatean Neutrosophic Hypetsoft dense* set if there exist no $\mathcal{FNH}$closed set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right)$ in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ such that $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) \subset \left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right) \subset 1_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. That is., $\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = 1_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$, in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$.

Definition 3.2 Let $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ be a $\mathcal{FNH}$ topological space. A $\mathcal{FNH}$set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is called a *Fermatean Neutrosophic Hypetsoft nowhere dense set* if there exists no non-zero $\mathcal{FNH}$open set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right)$ in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ such that $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right) \subseteq \mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$. That is., $\mathcal{FNH}int\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = 0_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$, in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$.

Definition 3.3 Let $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ be a $\mathcal{FNH}$ Topological space. A $\mathcal{FNH}$ set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is called *Fermatean Neutrosophic Hypetsoft σ- nowhere dense set* if $\left(\underline{K}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ is a $\mathcal{FNH}$ F_{σ}- set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ such that $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = 0_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$

Example 3.4 Let $\widetilde{\mathfrak{A}}_{\dot{\varsigma}} = \left\{\beth_{\mathfrak{P}_p}, \lambda_{\mathfrak{S}_s}\right\}$ and $\widehat{\mathfrak{E}}^{\bar{e}}_{\ddagger 1}, \widehat{\mathfrak{E}}^{\bar{e}}_{\ddagger 2}$ be the set of attributes. then the attributes are $\widehat{\mathfrak{E}}^{\bar{e}}_{\ddagger 1} = \{\widetilde{\mathfrak{K}}_{\S 1}, \widetilde{\mathfrak{K}}_{\S 2}, \widetilde{\mathfrak{K}}_{\S 3}\}$, $\widehat{\mathfrak{E}}^{\bar{e}}_{\ddagger 2} = \{\widetilde{\mathfrak{I}}_{\S 1}, \widetilde{\mathfrak{I}}_{\S 2}\}$

Suppose that $\widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{\ddagger 1} = \{\widetilde{\mathfrak{K}}_{\S 1}, \widetilde{\mathfrak{K}}_{\S 2}\}$, $\widehat{\mathfrak{F}}^{\bar{\mathfrak{f}}}_{\ddagger 2} = \{\widetilde{\mathfrak{I}}_{\S 1}, \widetilde{\mathfrak{I}}_{\S 2}\}$ are subset of $\widehat{\mathfrak{E}}^{\bar{e}}_{\ddagger i}$ for each i= 1,2.

Consider the $\mathcal{FNH}$ sets $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$, $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)$ and $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 3}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 3}\right)$ defined on $\widetilde{\mathfrak{A}}_{\dot{\varsigma}}$ as follows. $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = \left\{ \begin{matrix} \left\langle (\widetilde{\mathfrak{K}}_{\S 1}, \widetilde{\mathfrak{I}}_{\S 1}), \left\{\frac{\beth\mathfrak{P}_p}{[0.8,0.6,0.3]}, \frac{\lambda\mathfrak{S}_s}{[0.8,0.4,0.1]},\right\}\right\rangle, \left\langle (\widetilde{\mathfrak{K}}_{\S 1}, \widetilde{\mathfrak{I}}_{\S 2}), \left\{\frac{\beth\mathfrak{P}_p}{[0.9,0.5,0.6]}, \frac{\lambda\mathfrak{S}_s}{[0.8,0.1,0.2]},\right\}\right\rangle \\ \left\langle (\widetilde{\mathfrak{K}}_{\S 2}, \widetilde{\mathfrak{I}}_{\S 1}), \left\{\frac{\beth\mathfrak{P}_p}{[0.9,0.7,0.2]}, \frac{\lambda\mathfrak{S}_s}{[0.7,0.4,0.4]},\right\}\right\rangle, \left\langle (\widetilde{\mathfrak{K}}_{\S 2}, \widetilde{\mathfrak{I}}_{\S 2}), \left\{\frac{\beth\mathfrak{P}_p}{[0.8,0.5,0.3]}, \frac{\lambda\mathfrak{S}_s}{[0.7,0.5,0.3]},\right\}\right\rangle \end{matrix} \right\}$

$$\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right) = \left\{ \begin{matrix} \left\langle (\widetilde{\mathfrak{K}}_{\S 1}, \widetilde{\mathfrak{I}}_{\S 1}), \left\{\frac{\beth\mathfrak{P}_p}{[0.8,0.3,0.2]}, \frac{\lambda\mathfrak{S}_s}{[0.9,0.5,0.3]},\right\}\right\rangle, \left\langle (\widetilde{\mathfrak{K}}_{\S 1}, \widetilde{\mathfrak{I}}_{\S 2}), \left\{\frac{\beth\mathfrak{P}_p}{[0.8,0.2,0.2]}, \frac{\lambda\mathfrak{S}_s}{[0.7,0.7,0.1]},\right\}\right\rangle \\ \left\langle (\widetilde{\mathfrak{K}}_{\S 2}, \widetilde{\mathfrak{I}}_{\S 1}), \left\{\frac{\beth\mathfrak{P}_p}{[0.8,0.2,0.5]}, \frac{\lambda\mathfrak{S}_s}{[0.6,0.5,0.3]},\right\}\right\rangle, \left\langle (\widetilde{\mathfrak{K}}_{\S 2}, \widetilde{\mathfrak{I}}_{\S 2}), \left\{\frac{\beth\mathfrak{P}_p}{[0.8,0.5,0.3]}, \frac{\lambda\mathfrak{S}_s}{[0.7,0.5,0.2]},\right\}\right\rangle \end{matrix} \right\}$$

$$\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 3}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 3}\right) = \left\{ \begin{matrix} \left\langle (\widetilde{\mathfrak{K}}_{\S 1}, \widetilde{\mathfrak{I}}_{\S 1}), \left\{\frac{\beth\mathfrak{P}_p}{[0.2,0.6,0.9]}, \frac{\lambda\mathfrak{S}_s}{[0.1,0.6,0.8]},\right\}\right\rangle, \left\langle (\widetilde{\mathfrak{K}}_{\S 1}, \widetilde{\mathfrak{I}}_{\S 2}), \left\{\frac{\beth\mathfrak{P}_p}{[0.3,0.7,0.9]}, \frac{\lambda\mathfrak{S}_s}{[0.1,0.9,0.8]},\right\}\right\rangle \\ \left\langle (\widetilde{\mathfrak{K}}_{\S 2}, \widetilde{\mathfrak{I}}_{\S 1}), \left\{\frac{\beth\mathfrak{P}_p}{[0.1,0.7,0.9]}, \frac{\lambda\mathfrak{S}_s}{[0.2,0.6,0.7]},\right\}\right\rangle, \left\langle (\widetilde{\mathfrak{K}}_{\S 2}, \widetilde{\mathfrak{I}}_{\S 2}), \left\{\frac{\beth\mathfrak{P}_p}{[0.8,0.5,0.3]}, \frac{\lambda\mathfrak{S}_s}{[0.2,0.7,0.7]},\right\}\right\rangle \end{matrix} \right\}$$

$\widehat{\mathfrak{B}}_{\underline{\eta}} = \left\{0_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}, \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right), \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right), \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 3}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 3}\right), \left[\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)\right], \left[\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) \cap \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)\right], 1_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}\right\}$ is clearly a $\mathcal{FNH}$ topology on $\widetilde{\mathfrak{A}}_{\dot{\varsigma}}$.

Now we consider the $\mathcal{FNH}$ set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 4}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 4}\right) = \left[\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) \cup \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)\right]^{c} \cup \left[\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) \cap \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)\right]^{c}$ in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. Then $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 4}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 4}\right)$ is a $\mathcal{FNH}$ F_{σ}- set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ and $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 4}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 4}\right) = 0_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ and hence $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 4}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 4}\right)$ is a $\mathcal{FNH}$ σ- nowhere dense set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$.

Definition 3.5 A $\mathcal{FNH}$set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ in a $\mathcal{FNH}$topological space $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is called a *Fermatean Neutrosophic Hypetsoft* F_{σ}*- set* in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ if $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = \cup_{i=1}^{\infty} \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1i}\right)$, where $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1i}\right)^{c} \in \widehat{\mathfrak{B}}_{\underline{\eta}}$.

Definition 3.6 A $\mathcal{FNH}$ set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ in a $\mathcal{FNH}$ topological space $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is called a *Fermatean Neutrosophic Hypetsoft* G_{δ}*- set* in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ if $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = \cap_{i=1}^{\infty} \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1i}\right)$, where $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1i}\right) \in \widehat{\mathfrak{B}}_{\underline{\eta}}$.

Proposition 3.7 In a $\mathcal{FNH}$ topological space $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ a $\mathcal{FNH}$ set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ σ- nowhere dense in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ if and only if $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c}$ is a $\mathcal{FNH}$ dense and $\mathcal{FNH}$ G_{δ}- set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$.

Proof: Let $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ be a $\mathcal{FNH}$ σ- nowhere dense in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. Then $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = \cup_{i=1}^{\infty} \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)$ where $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)^{c} \in \widehat{\mathfrak{B}}_{\underline{\eta}}$, for i∈ I and $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = 0_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. then $\left(\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)\right)^{c} = 1_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ implies that $\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c} = 1_{(\widetilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. Also $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c} = \left(\cup_{i=1}^{\infty} \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)\right)^{c} = \cap_{i=1}^{\infty} \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)^{c}$ where $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)^{c} \in \widehat{\mathfrak{B}}_{\underline{\eta}}$, for i∈ I. Hence we have $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c}$ is a $\mathcal{FNH}$ dense and $\mathcal{FNH}$ G_{δ}- set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$.

Conversely, let $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ be a $\mathcal{FNH}$ dense and $\mathcal{FNH}$ G_{δ}- set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. then $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = \cap_{i=1}^{\infty} \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)$ where $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger i}\right) \in \widehat{\mathfrak{B}}_{\underline{\eta}}$, for i∈ I. Now $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c} = \left(\cup_{i=1}^{\infty} \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)\right)^{c} = \cup_{i=1}^{\infty} \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)^{c}$. Hence $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c}$ is a F_{σ}- set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ and $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c} = \left(\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)\right)^{c} =$

$0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \hat{\mathfrak{G}}^{\varepsilon})}$. [Since $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ dense]. therefore $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c}$ is a $\mathcal{FNH}$ σ- nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Proposition 3.8 If $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ such that $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right) \subseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c}$, where $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)$ is a $\mathcal{FNH}$ F_{σ}- set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$, then $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)$ is a $\mathcal{FNH}$ σ- nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Proof: Let $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ such that $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right) \subseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c}$. Now $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right) \subseteq \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c}$ implies that $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right) \subseteq \mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c} = \left(\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)\right)^{c} = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \hat{\mathfrak{G}}^{\varepsilon})}$ and hence $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \hat{\mathfrak{G}}^{\varepsilon})}$, therefore $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)$ is a $\mathcal{FNH}$ σ- nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Proposition 3.9 If $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ F_{σ}- set and $\mathcal{FNH}$ nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$, then $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ σ- nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Proof: Now $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) \subseteq \mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ for any $\mathcal{FNH}$ set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. then, $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) \subseteq \mathcal{FNH}int\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$. Since $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$, $\mathcal{FNH}int\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \hat{\mathfrak{G}}^{\varepsilon})}$ and hence $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \hat{\mathfrak{G}}^{\varepsilon})}$ and $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ F_{σ}- set implies that $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ σ- nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Definition 3.10 A $\mathcal{FNH}$ topological space $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is called a *Fermatean Neutrosophic Hypetsoft open hereditarily irresolvable space* if each non- zero open set is a $\mathcal{FNH}$ irresolvable set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Proposition 3.11 If $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ open hereditarily irresolvable space, any $\mathcal{FNH}$ σ- nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Proof: Let $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ be a $\mathcal{FNH}$ σ- nowhere dense set in a $\mathcal{FNH}$ open hereditarily irresolvable space $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. then $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ F_{σ}- set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ such that $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \hat{\mathfrak{G}}^{\varepsilon})}$. Since $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ open hereditarily irresolvable space, $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \hat{\mathfrak{G}}^{\varepsilon})}$ implies that $\mathcal{FNH}int\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \hat{\mathfrak{G}}^{\varepsilon})}$. Hence $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ is a $\mathcal{FNH}$ nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Definition 3.12 Let $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ be a $\mathcal{FNH}$topological space. A $\mathcal{FNH}$ set $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is called *Fermatean Neutrosophic Hypetsoft σ- first category* if $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) = \cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \hat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)$ where $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \hat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)$'s are $\mathcal{FNH}\sigma$- nowhere dense set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. Any other $\mathcal{FNH}$ set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is said to be *Fermatean Neutrosophic Hypetsoft σ- second category* in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \hat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Definition 3.13 Let $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$ be a $\mathcal{FNH}\sigma$- first category set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \mathfrak{G}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. then $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c}$ is called a *Fermatean Neutrosophic Hypetsoft σ- residual set* in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \mathfrak{G}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Definition 3.14 A $\mathcal{FNH}$ topological space $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \mathfrak{G}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is called a *Fermatean Neutrosophic Hypetsoft σ- first category space* if the $\mathcal{FNH}$ set $1_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \mathfrak{G}^{\varepsilon})}$ is a $\mathcal{FNH}$ σ- first category set in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \mathfrak{G}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. that is, $1_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \mathfrak{G}^{\varepsilon})} = \cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \hat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)$ where $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \hat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)$'s are $\mathcal{FNH}\sigma$- nowhere dense sets in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \mathfrak{G}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. Otherwise, $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \mathfrak{G}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ will be called a *Fermatean Neutrosophic Hypetsoft σ- second category space*.

4. Fermatean Neutrosophic Hypetsoft σ- Baire Space

Definition 4.1 Let $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \mathfrak{G}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ be a $\mathcal{FNH}$ topological space. then $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \mathfrak{G}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is called a *Fermatean Neutrosophic Hypersoft σ- Baire space* if $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \hat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \mathfrak{G}^{\varepsilon})}$ where $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger i}, \hat{\mathfrak{H}}^{\varrho}_{\dagger i}\right)$'s are $\mathcal{FNH}\sigma$- nowhere dense sets in $\left(\tilde{\mathfrak{A}}_{\dot{\varsigma}}, \mathfrak{G}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$.

Example 4.2 Let $\tilde{\mathfrak{A}}_{\dot{\varsigma}} = \left\{\beth_{\mathfrak{P}_p}, \lambda_{\mathfrak{S}_s}\right\}$ and $\tilde{\mathfrak{E}}^{\mathfrak{e}}_{\dagger 1}$, $\tilde{\mathfrak{E}}^{\mathfrak{e}}_{\dagger 2}$ be the set of attributes. then the attributes are $\tilde{\mathfrak{E}}^{\mathfrak{e}}_{\dagger 1} = \{\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{K}}_{\S 2}, \tilde{\mathfrak{K}}_{\S 3}\}$, $\tilde{\mathfrak{E}}^{\mathfrak{e}}_{\dagger 2} = \{\tilde{\mathfrak{I}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 2}\}$

Suppose that $\tilde{\mathfrak{F}}^{\mathfrak{f}}_{\dagger 1} = \{\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{K}}_{\S 2}\}$, $\tilde{\mathfrak{F}}^{\mathfrak{f}}_{\dagger 1} = \{\tilde{\mathfrak{I}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 2}\}$ are subset of $\tilde{\mathfrak{E}}^{\mathfrak{e}}_{\dagger i}$ for each i= 1,2.

Consider the $\mathcal{FNH}$ sets $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)$, $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)$, $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 3}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 3}\right)$ and $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 4}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 4}\right)$ defined on $\tilde{\mathfrak{A}}_{\dot{\varsigma}}$ $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) =$

$$\left\{\begin{array}{l}\left\langle\left(\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 1}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.7,0.5,0.1]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.8,0.4,0.1]},\right\}\right\rangle, \left\langle\left(\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 2}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.7,0.9,0.6]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.8,0.1,0.2]},\right\}\right\rangle \\ \left\langle\left(\tilde{\mathfrak{K}}_{\S 2}, \tilde{\mathfrak{I}}_{\S 1}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.7,0.5,0.6]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.7,0.4,0.4]},\right\}\right\rangle, \left\langle\left(\tilde{\mathfrak{K}}_{\S 2}, \tilde{\mathfrak{I}}_{\S 2}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.6,0.3]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.7,0.5,0.3]},\right\}\right\rangle\end{array}\right\}$$

$$\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right) = \left\{\begin{array}{l}\left\langle\left(\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 1}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.3,0.2]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.8,0.4,0.2]},\right\}\right\rangle, \left\langle\left(\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 2}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.2,0.2]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.7,0.2,0.1]},\right\}\right\rangle \\ \left\langle\left(\tilde{\mathfrak{K}}_{\S 2}, \tilde{\mathfrak{I}}_{\S 1}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.2,0.5]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.6,0.5,0.3]},\right\}\right\rangle, \left\langle\left(\tilde{\mathfrak{K}}_{\S 2}, \tilde{\mathfrak{I}}_{\S 2}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.4,0.3]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.7,0.5,0.2]},\right\}\right\rangle\end{array}\right\}$$

$$\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 3}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 3}\right) = \left\{\begin{array}{l}\left\langle\left(\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 1}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.3,0.1]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.8,0.3,0.1]},\right\}\right\rangle, \left\langle\left(\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 2}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.1,0.2]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.9,0.1,0.1]},\right\}\right\rangle \\ \left\langle\left(\tilde{\mathfrak{K}}_{\S 2}, \tilde{\mathfrak{I}}_{\S 1}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.2,0.4]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.7,0.3,0.3]},\right\}\right\rangle, \left\langle\left(\tilde{\mathfrak{K}}_{\S 2}, \tilde{\mathfrak{I}}_{\S 2}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.9,0.2,0.2]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.7,0.4,0.2]},\right\}\right\rangle\end{array}\right\}$$

$$\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 4}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 4}\right) = \left\{\begin{array}{l}\left\langle\left(\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 1}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.9,0.2,0.1]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.8,0.2,0.1]},\right\}\right\rangle, \left\langle\left(\tilde{\mathfrak{K}}_{\S 1}, \tilde{\mathfrak{I}}_{\S 2}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.1,0.1]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.9,0.1,0.1]},\right\}\right\rangle \\ \left\langle\left(\tilde{\mathfrak{K}}_{\S 2}, \tilde{\mathfrak{I}}_{\S 1}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.8,0.2,0.3]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.8,0.2,0.2]},\right\}\right\rangle, \left\langle\left(\tilde{\mathfrak{K}}_{\S 2}, \tilde{\mathfrak{I}}_{\S 2}\right), \left\{\frac{\beth_{\mathfrak{P}_p}}{[0.9,0.1,0.2]}, \frac{\lambda_{\mathfrak{S}_s}}{[0.8,0.3,0.1]},\right\}\right\rangle\end{array}\right\}$$

$\mathfrak{B}_{\underline{\eta}} = \Big\{0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \mathfrak{G}^{\varepsilon})}, \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right), \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right), \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 3}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 3}\right), \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 4}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 4}\right), \left[\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) \cup \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)\right], \left[\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) \cap \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)\right], 1_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \mathfrak{G}^{\varepsilon})}\Big\}$ is clearly a $\mathcal{FNH}$ topology on $\tilde{\mathfrak{A}}_{\dot{\varsigma}}$.

Now $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 5}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 5}\right) = \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right)^{c} \cup \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)^{c}$ and $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 5}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 5}\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \mathfrak{G}^{\varepsilon})}$ $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 6}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 6}\right) = \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 3}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 3}\right)^{c} \cup \left[\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) \cup \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)\right]^{c}$ and $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 6}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 6}\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \mathfrak{G}^{\varepsilon})}$ $\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 7}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 7}\right) = \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 4}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 4}\right)^{c} \cup \left[\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 1}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 1}\right) \cap \left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 2}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 2}\right)\right]^{c}$ and $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\curlyvee}_{\dagger 7}, \hat{\mathfrak{H}}^{\varrho}_{\dagger 7}\right) = 0_{(\tilde{\mathfrak{A}}_{\dot{\varsigma}_{FNH}}, \mathfrak{G}^{\varepsilon})}$

Then $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 5}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 5}\right)$, $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 6}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 6}\right)$ and $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 7}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 7}\right)$ are $\mathcal{FNH}$ σ- nowhere dense set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ and also $\mathcal{FNH}int\left[\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 5}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 5}\right) \cup \left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 6}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 6}\right) \cup \left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 7}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 7}\right)\right] = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ and therefore $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ- Baire space.

Proposition 4.3 Let $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ be a $\mathcal{FNH}$ topological space. then the following are equivalent:

(1) $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ- Baire Space.

(2) $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ for every $\mathcal{FNH}$ σ- first category set $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$.

(3) $\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right) = 1_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ for every $\mathcal{FNH}$ σ- residual set $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right)$ in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$.

Proof:. (1)⇒ (2)Let $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ be a $\mathcal{FNH}$ σ- first category set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. then $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = \cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)$, where $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)$'s are $\mathcal{FNH}$ σ- nowhere dense sets in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. then we have $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = \mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)\right)$. since $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ- Baire space, $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)\right) = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. Hence $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ for any $\mathcal{FNH}$ σ- first category set $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$.

(2)⇒ (3)Let $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right)$ be a $\mathcal{FNH}$ σ- residual set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. then $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right)^{c}$ is a $\mathcal{FNH}$ σ- first category set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. By hypothesis, $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right)^{c} = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ which implies that $\left(\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right)\right)^{c} = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. Hence we have $\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right) = 1_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ for any $\mathcal{FNH}$ σ- residual set $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2}\right)$ in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$.

(3)⇒ (1)Let $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ be a $\mathcal{FNH}$ σ- first category set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. then $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = \left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)\right)$, where $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)$'s are $\mathcal{FNH}$ σ- nowhere dense sets in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. Since $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)$ is a $\mathcal{FNH}$ σ- first category set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$, $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)^{c}$ is a $\mathcal{FNH}$ σ- residual set in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. By hypothesis, we have $\mathcal{FNH}cl\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)^{c} = 1_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. Then $\left(\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right)\right)^{c} = 1_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$, which impies that $\mathcal{FNH}int\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 1}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 1}\right) = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. Hence $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)\right) = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ where $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)$'s are $\mathcal{FNH}$ σ- nowhere dense sets in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$. Hence $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ- Baire Space.

Proposition 4.4 If $\mathcal{FNH}cl\left(\cap_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)\right) = 1_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ where $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)$'s are $\mathcal{FNH}$ dense G_{δ}- sets in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$, then $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ- Baire Space.

Proof: Now $\mathcal{FNH}cl\left(\cap_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)\right) = 1_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$ implies that $\left(\mathcal{FNH}cl\left(\cap_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)\right)\right)^{c} = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. then we have $\mathcal{FNH}int\left(\cap_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)\right)^{c} = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$, which implie that $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)^{c}\right) = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. Let $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2i}\right) = \left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)^{c}$. then $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger 2i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger 2i}\right)\right) = 0_{(\widetilde{\mathfrak{A}}_{\varsigma_{FNH}}, \widehat{\mathfrak{G}}^{\varepsilon})}$. Since $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)$ is a $\mathcal{FNH}$ dense G_{δ}- sets in $\left(\widetilde{\mathfrak{A}}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \widehat{\mathfrak{B}}_{\underline{\eta}}\right)$, by proposition 3.7, $\left(\underline{\mathfrak{K}}^{\Upsilon}_{\ddagger i}, \widehat{\mathfrak{H}}^{\varrho}_{\ddagger i}\right)^{c}$ is a $\mathcal{FNH}$ σ- nowhere dense set in

$\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. Hence $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger 2i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 2i}^{\varrho}\right)\right)= 0_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$, where $\left(\underline{\mathfrak{K}}_{\dagger 2i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger 2i}^{\varrho}\right)$'s are $\mathcal{FNH}$ σ- nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. then $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ- Baire Space.

Proposition 4.5 If the $\mathcal{FNH}$ topological space $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ- Baire Space and $\mathcal{FNH}$ open hereditarily irresolvable space, then $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ Baire Space.

Proof: Let $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ be a $\mathcal{FNH}$ σ- Baire Space and $\mathcal{FNH}$ open hereditarily irresolvable space. then $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)\right)= 0_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$ where $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)$'s are $\mathcal{FNH}$ σ- nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. By proposition 3.11, $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)$'s are $\mathcal{FNH}$ nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. Hence, $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)\right)= 0_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$ where, $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)$'s are $\mathcal{FNH}$ nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. therefore $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ Baire Space.

Proposition 4.6 If the $\mathcal{FNH}$ topological space $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ Baire Space and if the $\mathcal{FNH}$ nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ are $\mathcal{FNH}$ F_{σ}- sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$, then $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ-Baire Space.

Proof: Let $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ be a $\mathcal{FNH}$ Baire Space such that every $\mathcal{FNH}$ nowhere dense set $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)$ is a $\mathcal{FNH}$ F_{σ} sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. then, $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)\right)= 0_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$ where $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)$'s are $\mathcal{FNH}$ nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. By proposition 3.9, $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)$ is a $\mathcal{FNH}$ σ- nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. Hence $\mathcal{FNH}int\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)\right)= 0_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$ where $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)$'s are $\mathcal{FNH}$ σ- nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. therefore $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ-Baire Space.

Proposition 4.7 Let $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ be a $\mathcal{FNH}$ topological space. If $\cap_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right) \neq 0_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$, where $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)$'s are $\mathcal{FNH}$ dense and $\mathcal{FNH}$ G_{δ}- sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$, then $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ-second category Space.

Proof:. Now $\cap_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right) \neq 0_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$ implies that $\left(\cap_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)\right)^{c} \neq 0_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}= 0_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$. Then we have $\left(\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)^{c}\right) \neq 1_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$. Since $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)$ is a $\mathcal{FNH}$ dense and $\mathcal{FNH}$ G_{δ}- sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$, by proposition 3.7, $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)^{c}$ is a $\mathcal{FNH}$ σ- nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. Hence $\cup_{i=1}^{\infty}\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)^{c} \neq 1_{(\mathfrak{A}_{\varsigma_{FNH}}, \mathfrak{G}^{\varepsilon})}$, where $\left(\underline{\mathfrak{K}}_{\dagger i}^{\curlyvee}, \widehat{\mathfrak{H}}_{\dagger i}^{\varrho}\right)^{c}$'s are $\mathcal{FNH}$ σ- nowhere dense sets in $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$. Hence $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is not a $\mathcal{FNH}$ σ-first category Space. therefore $\left(\mathfrak{A}_{\dot{\varsigma}}, \widehat{\mathfrak{G}}^{\varepsilon}, \mathfrak{B}_{\underline{\eta}}\right)$ is a $\mathcal{FNH}$ σ-second category Space.

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