

A Generalization of Decision Methods in Fuzzy Soft Environment

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Abstract:- The fuzzy soft set (n-parameters) is associated in this study with a subset of R^n space contained in a solid cube located in positive octant with one vortex at origin and edges on the axes. The class of all fuzzy soft sets in the same universe and set of parameters that is associated with the entire solid cube. We then consider the decision-making norm. An example is used to demonstrate all of the work.

Keywords: Fuzzy soft set, Decision norm, Pseudo metric, R^n -space.

1. Introduction

Soft set theory was recently introduced by Molodtsov[1] in 1999 as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. Maji et al.[4] defined the fuzzy soft sets. Afterwards, many researchers have worked on this concept. Roy and Maji[5] presented some results on an application of fuzzy soft sets in decision making problems. N. Cagman, S. Enginoglu[3] defined soft matrices i.e. the matrix representation of soft sets and their operations and constructed a decision making method. Awasthi et al.[8] applied decision column method on weighted fuzzy soft matrix to find the relative risk of Covid-19 in certain region. Many researchers provided many decision-making approaches in a fuzzy soft environment. In this study, we provide a generic theory that underpins several decision-making strategies.

We consider some basic definitions.

Definition [1] Let U be an initial universe, $P(U)$ be the power set of U , E be the set of all parameters and $A \subset E$. A soft set (f_A, E) on the universe U is defined by the set of ordered pairs

$$(f_A, E) = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\}$$

where $f_A : E \rightarrow P(U)$ such that $f_A(e) = \varphi$ if $e \notin A$.

Here, f_A is called an approximate function of the soft set (f_A, E) . The set $f_A(e)$ is called e -approximate value set or e -approximate set which consists of related objects of the parameter $e \in E$.

Definition [3] An fs -set Γ_A over U is a set defined by a function γ_A representing a mapping

$$\gamma_A : E \rightarrow F(U) \quad \text{such that } \gamma_A(x) = \varphi \text{ if } x \notin A.$$

Here, γ_A is called fuzzy approximate function of the fs -set Γ_A , and the value $\gamma_A(x)$ is a set called x -element of the fs -set for all $x \in E$. Thus, an fs -set Γ_A over U can be represented by the set of ordered pairs

$$\Gamma_E = \{(e, \gamma_E(e)) : e \in E, \gamma_E(e) \in F(U)\}$$

the set of all fs -sets over U will be denoted by $FS(U)$.

Definition [6] Let U be a non-empty universal set, X be a fuzzy set defined on U and E be a non-empty parameter set. Then a function $\epsilon : E \rightarrow X$ is said to be a fuzzy soft element of X . A soft element ϵ of X is said to belong to a fuzzy soft set A of X , denoted by $\epsilon \in A$ if $\epsilon(e) \in A(e), \forall e \in E$. Thus a soft set A of X with respect to the index set E can be expressed as $A(e) = \epsilon(e), \epsilon \in A, e \in E$.

2. Results and Discussions

2.1 Fuzzy Soft Set

Let U be a universe. A fuzzy set [9] X over U is a set defined by a function μ_X representing a mapping

$$\mu_X : U \rightarrow [0,1] \quad (\text{eqn. 1})$$

μ_X is called the membership function of X , and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. The value represents the degree of u belonging to the fuzzy set X . Thus, a fuzzy set X over U [3] can be represented as follows:

$$X = \left\{ \frac{\mu_X(u)}{u} : u \in U, \mu_X(u) \in [0,1] \right\} \quad (\text{eqn. 2})$$

A fuzzy set X over U can be considered as a fuzzy soft set Γ_E over U with singleton set of parameter, E . That is

$$\Gamma_E = \{(e, \gamma_E(e)) : e \in E, \gamma_E(e) \in F(U)\} \quad (\text{eqn. 3})$$

where, $\gamma_E(e) = X$.

The set Γ_E is singleton set of ordered pair $(e, \gamma_E(e))$ which can be represented on real line as each member $\frac{\mu_X(u)}{u}$ of $\gamma_E(e)$ associates to a real number $\mu_X(u)$. The parameter e associates to the real line itself.

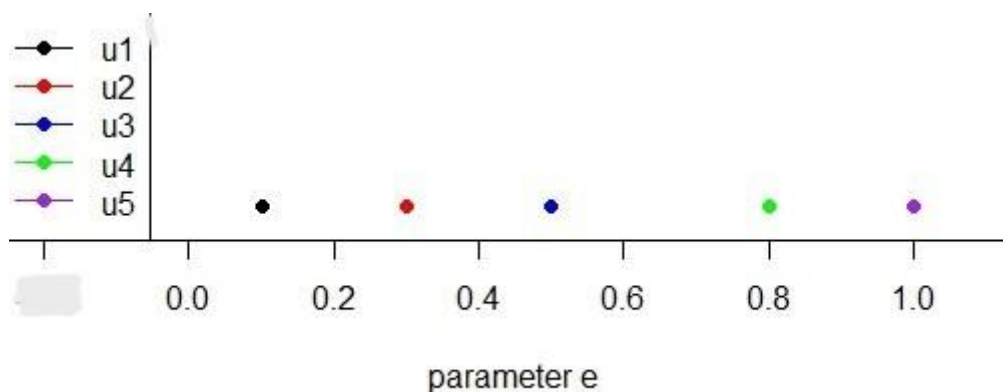
Example: Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ and

$$X = \left\{ \frac{0.1}{u_1}, \frac{0.3}{u_2}, \frac{0.5}{u_3}, \frac{0.8}{u_4}, \frac{1}{u_5} \right\}$$

be the fuzzy set over U . The fuzzy soft set Γ_E is given by

$$\Gamma_E = \left\{ \left(e, \left\{ \frac{0.1}{u_1}, \frac{0.3}{u_2}, \frac{0.5}{u_3}, \frac{0.8}{u_4}, \frac{1}{u_5} \right\} \right) \right\} \quad (\text{eqn. 4})$$

Which contains five distinct fuzzy soft elements. The set of all fuzzy soft elements, $P(\Gamma_E)$ is represented on real line.



Definition:(Decision norm) Suppose a pseudo metric $\|.\|_d$ on $P(\Gamma_E)$, the set of all fuzzy soft elements, defined as follows,

$$\|.\|_d : P(\Gamma_E) \rightarrow \mathbb{R}, \text{ such that}$$

$$\left\| \left(e, \left\{ \frac{\mu_X(u)}{u} \right\} \right), \left(e, \left\{ \frac{\mu_X(v)}{v} \right\} \right) \right\|_d = |\mu_X(u) - \mu_X(v)| \quad (\text{eqn. 5})$$

The distance of a fuzzy soft element $\left\{ \left(e, \left\{ \frac{\mu_X(u)}{u} \right\} \right) \right\}$ from the null fuzzy soft element $\left\{ \left(e, \left\{ \frac{\mu_X(u)}{u} \right\} \right) \right\}$ corresponds the distance of associated point on real line from the origin. We denote this distance by $\|\cdot\|_{d_0}$ and say as decision norm. That is

$$\begin{aligned} \|\cdot\|_{d_0} &= \left\| \left(e, \left\{ \frac{\mu_X(u)}{u} \right\} \right), \left(e, \left\{ \frac{0}{u_0} \right\} \right) \right\|_d \\ &= |\mu_X(u) - 0| \\ &= \mu_X(u) \end{aligned}$$

Hence, we have a structure $(P(\Gamma_E), \|\cdot\|_{d_0})$. The fuzzy decision set for the fuzzy soft set Γ_E is given by

$$D(\Gamma_E) = \left\{ \frac{\left\| \left(e, \left\{ \frac{\mu_X(u)}{u} \right\} \right) \right\|_{d_0}}{u} : \left(e, \left\{ \frac{\mu_X(u)}{u} \right\} \right) \in P(\Gamma_E) \right\} \quad (\text{eqn. 6})$$

Therefore, the decision set for the fuzzy soft set (eqn. 4) is

$$D(\Gamma_E) = \left\{ \left(e, \left\{ \frac{0.1}{u_1}, \frac{0.3}{u_2}, \frac{0.5}{u_3}, \frac{0.8}{u_4}, \frac{1}{u_5} \right\} \right) \right\} = X$$

2.2 Fuzzy Soft Set with two parameters

Let U be a universe and $E = \{x, y\}$ the set of parameters. A fuzzy soft set $(F, \tilde{E})$ over U is a family of parametrized fuzzy subsets X, Y of U . That is

$$(F, \tilde{E}) = \{(x, X), (y, Y)\}$$

where,

$$X = \left\{ \frac{\mu_X(u)}{u} : u \in U, \mu_X(u) \in [0, 1] \right\}, Y = \left\{ \frac{\mu_Y(u)}{u} : u \in U, \mu_Y(u) \in [0, 1] \right\}$$

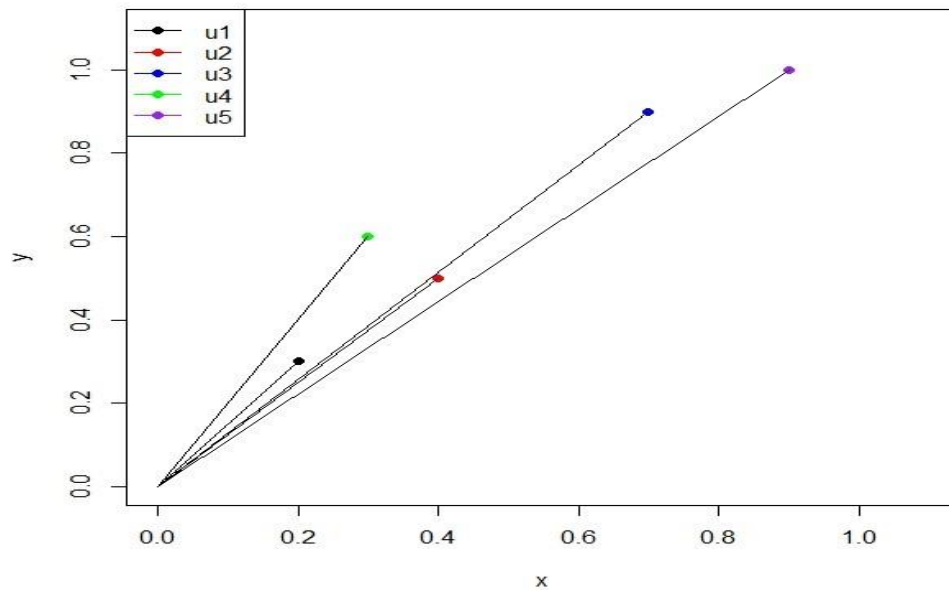
Example: Let $U = \{u_1, u_2, u_3, u_4, u_5\}$, $E = \{x, y\}$ and

$$X = \left\{ \frac{0.2}{u_1}, \frac{0.4}{u_2}, \frac{0.7}{u_3}, \frac{0.3}{u_4}, \frac{0.9}{u_5} \right\}, \quad Y = \left\{ \frac{0.3}{u_1}, \frac{0.5}{u_2}, \frac{0.9}{u_3}, \frac{0.6}{u_4}, \frac{1}{u_5} \right\}$$

be the fuzzy subsets over U . The fuzzy soft set $(F, \tilde{E})$ is given by

$$\left\{ \left(x, \left\{ \frac{0.2}{u_1}, \frac{0.4}{u_2}, \frac{0.7}{u_3}, \frac{0.3}{u_4}, \frac{0.9}{u_5} \right\} \right), \left(y, \left\{ \frac{0.3}{u_1}, \frac{0.5}{u_2}, \frac{0.9}{u_3}, \frac{0.6}{u_4}, \frac{1}{u_5} \right\} \right) \right\} \quad (\text{eqn. 7})$$

The complex lane representation of objects is given as follows.



Decision norm: We know that a fuzzy soft set can be represented as the union of all its fuzzy soft elements. Suppose $\left\{ \left(x, \left\{ \frac{\mu_X(u)}{u} \right\} \right), \left(y, \left\{ \frac{\mu_Y(u)}{u} \right\} \right) \right\}$ be a soft element. We associate this element to a point $(\mu_X(u), \mu_Y(u)) \in R^2$, the parameters x and y associate to horizontal and vertical axes respectively. Thus the set of all fuzzy soft elements, $P(F, \tilde{E})$ is represented on the cartesian plane. Further the distance from origin is directly proportional to the decision norm. We define the decision norm as follows:

$$\|\cdot\|_{d_0} : P(F, \tilde{E}) \rightarrow R, \text{ such that}$$

$$\left\| \left(x, \left\{ \frac{\mu_X(u)}{u} \right\} \right), \left(y, \left\{ \frac{\mu_Y(v)}{v} \right\} \right) \right\|_{d_0} = \sqrt{\frac{(\mu_X(u))^2 + (\mu_Y(v))^2}{2}}$$

Hence, we have a structure $(P(F, \tilde{E}), \|\cdot\|_{d_0})$. The fuzzy decision set for the fuzzy soft set $(F, \tilde{E})$ is given by

$$D(F, \tilde{E}) = \left\{ \frac{\left\| \left(x, \left\{ \frac{\mu_X(u)}{u} \right\} \right), \left(y, \left\{ \frac{\mu_Y(v)}{v} \right\} \right) \right\|_{d_0}}{u} : u \in U \right\} \quad (\text{eqn. 9})$$

Therefore, the decision set for the fuzzy soft set (7) is

$$D(F, \tilde{E}) = \left\{ \frac{0.25}{u_1}, \frac{0.45}{u_2}, \frac{0.8}{u_3}, \frac{0.47}{u_4}, \frac{0.95}{u_5} \right\} \quad (\text{eqn. 10})$$

2.3 Fuzzy Soft Set, n -parameters

Let U be a universe and $E = \{x_i, i = 1, 2, \dots, n\}$ the set of n - parameters. A fuzzy soft set $(F, \tilde{E})$ over U is a family of parametrized fuzzy subsets $X_i, i = 1, 2, \dots, n$ of U . That is

$$(F, \tilde{E}) = \{(x_i, X_i), i = 1, 2, \dots, n\} \quad (\text{eqn. 11})$$

where,

$$X_i = \left\{ \frac{\mu_{X_i}(u)}{u} : u \in U, \mu_{X_i} \in [0, 1] \right\}$$

Decision norm: Suppose $\left\{ \left(x_i, \left\{ \frac{\mu_{x_i}(u)}{u} \right\} \right), i = 1, 2, \dots, n \right\}$ be a soft element. We associate this element to a point $(\mu_{x_1}(u), \mu_{x_2}(u), \dots, \mu_{x_n}(u)) \in R^n$, the parameters x_i associate to n-axes. Thus the set of all fuzzy soft elements, $P(F, \tilde{E})$ is represented in the R^n -space. Further the distance from origin is directly proportional to the decision norm. We define the decision norm as follows:

$\|\cdot\|_{d_0} : P(F, \tilde{E}) \rightarrow R$, such that

$$\left\| \left\{ \left(x_i, \left\{ \frac{\mu_{x_i}(u)}{u} \right\} \right), i = 1, 2, \dots, n \right\} \right\|_{d_0} = \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1}^n (\mu_{x_i}(u))^2}$$

(eqn. 12)

The fuzzy decision set for the fuzzy soft set $(F, \tilde{E})$ is given by

$$D(F, \tilde{E}) = \left\{ \frac{\left\| \left\{ \left(x_i, \left\{ \frac{\mu_{x_i}(u)}{u} \right\} \right), i = 1, 2, \dots, n \right\} \right\|_{d_0}}{u} : u \in U \right\}$$

(eqn. 13)

3. Conclusion

We demonstrated that a fuzzy soft set with n-parameters may be represented in R^n -space by assigning the parameters to the independent axes and the objects to the points. The point-to-origin join is seen as a vector, and the norm of a vector is directly proportional to the grade of choice of the associated object. With this representation in mind, we define a function on the set of soft elements known as the decision norm. As a result, we anticipated a generalisation of a class of decision procedures in a fuzzy soft environment. We believe that this work will help to feel and grasp choice processes and choose the best way in a variety of circumstances in a fuzzy soft environment.

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