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Analyzation of Micro Vague Generalized Semi Continuous Mappings in Micro Vague Topological Spaces

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Abstract: - A function is said to be continuous if it preserves the idea of closeness between the points in the domain and their corresponding points in the range. Unveiling the Micro Vague continuous mapping and Micro Vague Generalized Semi Continuous mapping is the ultimate purpose of this article. Many different types of Micro Vague Continuous mappings are introduced along with their relationships to other existing Micro Vague Continuous mappings with the help of suitable examples. They have applications in many areas of science and engineering where smooth, well-behaved functions are needed to model real-world phenomena.

Keywords: Micro Vague Continuous Mappings, Micro Vague Semi Continuous Mappings, Micro Vague Generalized Semi Continuous Mappings.

1. Introduction

An approach that delivers just one outcome for every input parameter is called a function. The input to the function is the independent variable also referred to as the argument of the function. The output of the function is the dependent variable. Continuity is a basic concept in computational mathematics and calculus. In mathematics, a continuous function is one in which there are no sudden shifts in value and a continuous variation of the function's value due to ongoing changes in the input. If a function's graph is continuous throughout the entire interval the function qualifies as continuous in that range. In real analysis, continuous functions are basic ideas that serve as a basis for learning about limit points, partial integrals, higher order derivatives and other topics.

Continuity was introduced by Augustin Louis Cauchy in 1821 in his famous textbook Cours d'Analyse. In 1965 and 1986 respectively, Zadeh [17] and Atanassov [2] proposed the ideas of Fuzzy sets and Intuitionistic Fuzzy sets. Gau and Buehrer [16] initially suggested the study of Vague sets as a continuation of fuzzy sets. There are different kind of continuous functions. Point wise continuity, Pair wise continuity, Uniform continuity, Lipschitz continuity are some of among them. Not all functions are continuous. There are various types of discontinuous functions such as Removable discontinuity, Jump discontinuity, Essential discontinuity and Infinite discontinuity.

Semi continuous functions are versatile tools in various fields allowing for more flexible and realistic modelling phenomena that may not be adequately captured by strictly continuous functions. Levine. N [4] introduced the semi open and semi continuity in 1963. Their ability to handle discontinuities and abrupt changes in data or systems makes them valuable in a wide range of applications. Generalized continuous functions are mathematical objects that generalizes the concept of functions. They often used in distribution theory and are not always traditional functions in the sense of having specific values at each point in their domain. Instead, they are defined through their action on test functions.

Micro topology is an enlargement of nano topology which has been established by Chandrasekar S [12]. By combining Micro topological space and Vague topological space, Vargees Vahini T and Trinita Pricilla M [15] have introduced the new topological space called Micro Vague Topological Space. Mashour. A. S [6] introduced pre continuous mappings and weakly pre continuous mappings in 1982. R. Devi and H. Maki and K. Balachandran [3] have introduced the generalized semi continuous and generalized semi homomorphism in 1995.

Generalized semi continuous sets are the combined idea of semi continuous and generalized continuous functions. It is often used in more abstract mathematical settings and it encompasses functions that may not be directly associated with real numbers. It is a broader concept that can apply to the functions defined on more general topological spaces. The notion of upper and lower semi continuity is adapted to these more general spaces. Generalized Semi continuous function is not necessarily be continuous but exhibits some degree continuity when defined on topological spaces.

In this article, many types of continuous functions and generalized continuous functions in Micro Vague Topological Spaces are introduced. Particularly, Micro Vague Generalized Semi Continuous function is introduced and the relationship between Micro Vague Generalized Semi Continuous mappings and the existing Micro Vague Continuous functions are presented and investigated with the suitable examples.

2. Preliminaries

Definition 2.1[15]

Let $\left(U, \tau_R(A)\right)$ be a Nano Vague Topological Space. Let $\eta_R(A) = \{S \cup (S' \cap \eta) : S, S' \in \tau_R(A) \text{ and } \eta \notin \tau_R(A)\}$. Then $\eta_R(A)$ is called the Micro Vague Topology ($shortly\ \mathcal{MVT}$) of $\tau_R(A)$ by η on U with respect to A. The triplet $(U, \tau_R(A), \eta_R(A))$ is called the Micro Vague Topological Space (shortly $\mathcal{MVTS}$). The elements of $\eta_R(A)$ are called Micro Vague open sets ($shortly\ \mathcal{MVOS}$) and the complement of $\mathcal{MVOS}$ is called Micro Vague Closed set ($shortly\ \mathcal{MVCS}$).

Definition 2.2[15]

Let U be the Universe and $X \subseteq U$. Let $\mathcal{G}$ and $\mathcal{H}$ be two $\mathcal{MV}$ sets in the $\mathcal{MVTS}$ $(U, \tau_R(X), \eta_R(X))$ of the form $\mathcal{G} = \left\{\langle x, [\mu_{\mathcal{G}}(x), \gamma_{\mathcal{G}}(x)]\rangle / x \in X\right\}$ and $\mathcal{H} = \{\langle x, [\mu_{\mathcal{H}}(x), \gamma_{\mathcal{H}}(x)]\rangle / x \in X\}$ respectively. Then the following conditions holds:

(i) $\mathcal{G} \subseteq \mathcal{H}\ iff\ \mu_{\mathcal{G}}(x) \le \mu_{\mathcal{H}}(x), \gamma_{\mathcal{G}}(x) \le \gamma_{\mathcal{H}}(x)\ \ \forall x \in U$

(ii) $\mathcal{G} = \mathcal{H}\ iff\ \mathcal{G} \subseteq \mathcal{H}\ and\ \mathcal{H} \subseteq \mathcal{G}$

(iii) $\mathcal{G}^C = \left\{\langle x, 1 - \gamma_{\mathcal{G}}(x), 1 - \mu_{\mathcal{G}}(x)\rangle / \forall x \in U\right\}$

(iv) $\mathcal{G} \cup \mathcal{H} = \left\{\langle x, \left(\mu_{\mathcal{G}}(x) \vee \mu_{\mathcal{H}}(x),\ \gamma_{\mathcal{G}}(x) \vee \gamma_{\mathcal{H}}(x)\right)\rangle / \forall x \in U\right\}$

(v) $\mathcal{G} \cap \mathcal{H} = \left\{\langle x, \left(\mu_{\mathcal{G}}(x) \wedge \mu_{\mathcal{H}}(x),\ \gamma_{\mathcal{G}}(x) \wedge \gamma_{\mathcal{H}}(x)\right)\rangle / \forall x \in U\right\}$

(vi) $0_{\mathcal{MV}} = \langle x, (0,0)\rangle$ and $1_{\mathcal{MV}} = \langle x, (1,1)\rangle\ \forall x \in U$.

3. Various types of Micro Vague Continuous Mappings

Definition 3.1

We define the image and preimage of $\mathcal{MV}\ Sets$. Let $\mathcal{F} = \{\langle x, [\mu_{\mathcal{F}}(x), \gamma_{\mathcal{F}}(x)]\rangle / x \in X\}$ and $\mathcal{G} = \left\{\langle x, [\mu_{\mathcal{G}}(x), \gamma_{\mathcal{G}}(x)]\rangle / x \in X\right\}$ be two $\mathcal{MV}\ Sets$ in $\left(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})\right)$ and $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ respectively. Let $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ be a function, then the following statements hold:

(i) The pre-image of $\mathcal{G}$ under $\hbar$ denoted by $\hbar^{-1}(\mathcal{G})$ is the $\mathcal{MV}$ set in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$ defined by $\hbar^{-1}(\mathcal{G}) = \{\langle x, [\hbar^{-1}(\mu_{\mathcal{G}})(x), \hbar^{-1}(\gamma_{\mathcal{G}})(x)] / x \in X\rangle\}$.

(ii) The image of $\mathcal{F}$ under f denoted by $\hbar(\mathcal{F})$ is the $\mathcal{MV}$ set in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ defined by $\hbar(\mathcal{F}) = \{\langle x, [\hbar(\mu_{\mathcal{F}})(x), \hbar(\gamma_{\mathcal{F}})(x)] / x \in X\rangle\}$.

Corollary 3.2

Let $\mathcal{C}_{\mathcal{MV}}$, $\mathcal{C}_{i_{\mathcal{MV}}}(i \in \mathcal{I})$ be $\mathcal{MV}$ sets in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$ and $\mathcal{D}_{\mathcal{MV}}$, $\mathcal{D}_{i_{\mathcal{MV}}}(j \in \mathcal{J})$ be $\mathcal{MV}$ sets in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$. Let us define a function $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$. Then the following properties holds

a) If $\mathcal{C}_{1_{\mathcal{MV}}} \subseteq \mathcal{C}_{2_{\mathcal{MV}}}$, then $\hbar(\mathcal{C}_{1_{\mathcal{MV}}}) \subseteq \hbar(\mathcal{C}_{2_{\mathcal{MV}}})$.

b) If $\mathcal{D}_{1_{\mathcal{MV}}} \subseteq \mathcal{D}_{2_{\mathcal{MV}}}$, then $\hbar(\mathcal{D}_{1_{\mathcal{MV}}}) \subseteq \hbar(\mathcal{D}_{2_{\mathcal{MV}}})$.

c) $\mathcal{C}_{\mathcal{MV}} \subseteq \hbar^{-1}(\hbar(\mathcal{C}_{\mathcal{MV}}))$.

d) $\mathcal{C}_{\mathcal{MV}} = \hbar^{-1}\big(\hbar(\mathcal{C}_{\mathcal{MV}})\big)$ if $\hbar$ is injective.

e) $\hbar(\hbar^{-1}(\mathcal{D}_{\mathcal{MV}})) \subseteq \mathcal{D}_{\mathcal{MV}}$.

f) $\hbar(\hbar^{-1}(\mathcal{D}_{\mathcal{MV}})) = \mathcal{D}_{\mathcal{MV}}$ if $\hbar$ is surjective.

g) $\hbar^{-1}\big(\cup \mathcal{D}_{i_{\mathcal{MV}}}\big) = \cup\, \hbar^{-1}\big(\mathcal{D}_{i_{\mathcal{MV}}}\big)$.

h) $\hbar^{-1}\big(\cap \mathcal{D}_{i_{\mathcal{MV}}}\big) = \cap\, \hbar^{-1}\big(\mathcal{D}_{i_{\mathcal{MV}}}\big)$.

i) $\hbar\big(\cup \mathcal{C}_{i_{\mathcal{MV}}}\big) = \cup\, \hbar\big(\mathcal{C}_{i_{\mathcal{MV}}}\big)$.

j) $\hbar\big(\cap \mathcal{C}_{i_{\mathcal{MV}}}\big) \subseteq \cap\, \hbar\big(\mathcal{C}_{i_{\mathcal{MV}}}\big)$.

k) If $\hbar$ is injective, then $\hbar\big(\cap \mathcal{C}_{i_{\mathcal{MV}}}\big) = \cap\, \hbar\big(\mathcal{C}_{i_{\mathcal{MV}}}\big)$.

l) $\hbar^{-1}(1_{\mathcal{MV}}) = 1_{\mathcal{MV}}$.

m) $\hbar^{-1}(0_{\mathcal{MV}}) = 0_{\mathcal{MV}}$.

n) $\hbar(1_{\mathcal{MV}}) = 1_{\mathcal{MV}}$, if $\hbar$ is surjective.

o) $\hbar(0_{\mathcal{MV}}) = 0_{\mathcal{MV}}$.

p) $\overline{\hbar(\mathcal{C}_{\mathcal{MV}})} \subseteq \hbar(\overline{\mathcal{C}_{\mathcal{MV}}})$, if $\hbar$ is surjective.

q) $\hbar^{-1}\big(\overline{\mathcal{D}_{\mathcal{MV}}}\big) = \overline{\hbar^{-1}(\mathcal{D}_{\mathcal{MV}})}$.

Proof: Proof is obvious.

Definition 3.3

Let $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$ and $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ be any two $\mathcal{MVTS}$. A map $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is said to be

1. $\mathcal{MVO}$ mapping (*shortly* $\mathcal{MVOM}$) iff the image of each $\mathcal{MVO}$ sets in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVO}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$.

2. $\mathcal{MV}$ *Continuous mapping* (*shortly* $\mathcal{MVCM}$), if the inverse image $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVC}$ set $\mathcal{F}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVC}$ set in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

3. $\mathcal{MVS}$ *Continuous mapping* (*shortly* $\mathcal{MVSCM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVSCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

4. $\mathcal{MVP}$ *Continuous mapping* (*shortly* $\mathcal{MVPCM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVPCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

5. $\mathcal{MVSP}$ *Continuous mapping* (*shortly* $\mathcal{MVSPCM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVSPCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

6. $\mathcal{MV}\alpha$ *Continuous mapping* (*shortly* $\mathcal{MV}\alpha\mathcal{CM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MV}\alpha\mathcal{CS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

7. $\mathcal{MVR}$ *Continuous mapping* (*shortly* $\mathcal{MVRCM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVRCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

8. $\mathcal{MVG}$ *Continuous mapping* (*shortly* $\mathcal{MVGCM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVGCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

9. $\mathcal{MVGP}$ *Continuous mapping* (*shortly* $\mathcal{MVGPCM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVGPCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

10. $\mathcal{MVGSP}$ *Continuous mapping* (*shortly* $\mathcal{MVGSPCM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVGSPCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

11. $\mathcal{MV}\alpha\mathcal{G}$ *Continuous mapping* (*shortly* $\mathcal{MV}\alpha\mathcal{GCM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V},\mu_{\mathcal{R}}(\mathcal{S}),\zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MV}\alpha\mathcal{GCS}$ in $(\mathcal{U},\vartheta_{\mathcal{R}}(\mathcal{S}),\eta_{\mathcal{R}}(\mathcal{S}))$.
12. $\mathcal{MVG}\alpha$ *Continuous mapping* (*shortly* $\mathcal{MVG}\alpha\mathcal{CM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V},\mu_{\mathcal{R}}(\mathcal{S}),\zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVG}\alpha\mathcal{CS}$ in $(\mathcal{U},\vartheta_{\mathcal{R}}(\mathcal{S}),\eta_{\mathcal{R}}(\mathcal{S}))$.

Definition 3.4

Let $\hbar:(\mathcal{U},\vartheta_{\mathcal{R}}(\mathcal{S}),\eta_{\mathcal{R}}(\mathcal{S}))\longrightarrow(\mathcal{V},\mu_{\mathcal{R}}(\mathcal{S}),\zeta_{\mathcal{R}}(\mathcal{S}))$ is both one-to-one and onto mapping where $(\mathcal{U},\vartheta_{\mathcal{R}}(\mathcal{S}),\eta_{\mathcal{R}}(\mathcal{S}))$ and $(\mathcal{V},\mu_{\mathcal{R}}(\mathcal{S}),\zeta_{\mathcal{R}}(\mathcal{S}))$ are two $\mathcal{MVTS}s$. Then $\hbar$ is said to be $\mathcal{MV}$-*Homeomorphism* if $\hbar$ and $\hbar^{-1}$ are $\mathcal{MVCM}$.

4. Characterizations and Properties of Micro Vague Generalized Semi Continuous Mappings

Definition 4.1

Let $(\mathcal{U},\vartheta_{\mathcal{R}}(\mathcal{S}),\eta_{\mathcal{R}}(\mathcal{S}))$ and $(\mathcal{V},\mu_{\mathcal{R}}(\mathcal{S}),\zeta_{\mathcal{R}}(\mathcal{S}))$ be any two $\mathcal{MVTS}$. A map $\hbar:(\mathcal{U},\vartheta_{\mathcal{R}}(\mathcal{S}),\eta_{\mathcal{R}}(\mathcal{S}))\longrightarrow(\mathcal{V},\mu_{\mathcal{R}}(\mathcal{S}),\zeta_{\mathcal{R}}(\mathcal{S}))$ is said to be $\mathcal{MVGS}$ *Continuous mapping* (*shortly* $\mathcal{MVGSCM}$), if $\hbar^{-1}(\mathcal{F})$ of every $\mathcal{MVCS}$ $\mathcal{F}$ in $(\mathcal{V},\mu_{\mathcal{R}}(\mathcal{S}),\zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVGSCS}$ in $(\mathcal{U},\vartheta_{\mathcal{R}}(\mathcal{S}),\eta_{\mathcal{R}}(\mathcal{S}))$.

Example 4.2: Let $\mathcal{U}=\{\alpha,\beta,\gamma\}$ be the Universe of discourse. $\mathcal{U}/\mathcal{R}=\{\{\alpha\},\{\beta,\gamma\}\}$ be the equivalence relation on $\mathcal{U}$. Let $\mathcal{S}=\{<\alpha,(0.2,0.5)>,<\beta,(0.2,0.7)>,<\gamma,(0.2,0.4)>\}$ be a subset of $\mathcal{U}$. Then, $\vartheta_{\mathcal{R}}(\mathcal{S})=\{0_{NV},1_{NV},\{<\alpha,(0.2,0.5)>,<\beta,(0.2,0.4)>,<\gamma,(0.2,0.4)>\},\{<\alpha,(0.2,0.5)>,<\beta,(0.2,0.7)>,<\gamma,(0.2,0.7)>\}\}$ is a $\mathcal{NVT}$ on $\mathcal{U}$. Let $\eta=\{<\alpha,(0.3,0.8)>,<\beta,(0.5,0.8)>,<\gamma,(0.3,0.8)>\}$. Then $\eta_{\mathcal{R}}(\mathcal{S})=\{0_{NV},1_{NV},\{<\alpha,(0.2,0.5)>,<\beta,(0.2,0.4)>,<\gamma,(0.2,0.4)>\},\{<\alpha,(0.2,0.5)>,<\beta,(0.2,0.7)>,<\gamma,(0.2,0.7)>\},\{<\alpha,(0.3,0.8)>,<\beta,(0.5,0.8)>,<\gamma,(0.3,0.8)>\}\}$ is a $\mathcal{MVT}$ and $(\mathcal{U},\vartheta_{\mathcal{R}}(\mathcal{S}),\eta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Let $\mathcal{V}=\{\delta,\sigma,\tau\}$ be another universe of discourse. $\mathcal{V}/\mathcal{R}=\{\{\delta,\tau\}\{\sigma\}\}$ be the equivalence relation on $\mathcal{V}$. Let $\mathcal{S}=\{<\delta,(0.1,0.7)>,<\sigma,(0.3,0.6)>,<\tau,(0.2,0.3)>\}$ be a subset of $\mathcal{V}$. Then, $\mu_{\mathcal{R}}(\mathcal{S})=\{0_{NV},1_{NV},\{<\delta,(0.1,0.3)>,<\sigma,(0.3,0.6)>,<\tau,(0.1,0.3)>\},\{<\delta,(0.2,0.7)>,<\sigma,(0.3,0.6)>,<\tau,(0.2,0.7)>\}\}$ is a $\mathcal{NVT}$ on $\mathcal{U}$. Let $\zeta=\{<\delta,(0.4,0.5)>,<\sigma,(0.1,0.4)>,<\tau,(0.2,0.3)>\}$. Then $\zeta_{\mathcal{R}}(\mathcal{S})=\{0_{NV},1_{NV},\{<\delta,(0.1,0.3)>,<\sigma,(0.3,0.6)>,<\tau,(0.1,0.3)>\},\{<\delta,(0.2,0.7)>,<\sigma,(0.3,0.6)>,<\tau,(0.2,0.7)>\},\{<\delta,(0.4,0.5)>,<\sigma,(0.1,0.4)>,<\tau,(0.2,0.3)>\}\}$ is a $\mathcal{MVT}$ and $(\mathcal{V},\mu_{\mathcal{R}}(\mathcal{S}),\zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Define a mapping $\hbar:(\mathcal{U},\vartheta_{\mathcal{R}}(\mathcal{S}),\eta_{\mathcal{R}}(\mathcal{S}))\longrightarrow(\mathcal{V},\mu_{\mathcal{R}}(\mathcal{S}),\zeta_{\mathcal{R}}(\mathcal{S}))$ by $\hbar(\alpha)=\delta$, $\hbar(\beta)=\sigma$ and $\hbar(\gamma)=\tau$. Then $\hbar$ is $\mathcal{MVGS}$ *Continuous*.

Theorem 4.3

1. Every $\mathcal{MVCM}$ is $\mathcal{MVGCM}$.
2. Every $\mathcal{MVCM}$ is $\mathcal{MV}\alpha\mathcal{CM}$.
3. Every $\mathcal{MVCM}$ is $\mathcal{MVPCM}$.
4. Every $\mathcal{MV}\alpha\mathcal{CM}$ is $\mathcal{MVPCM}$.
5. Every $\mathcal{MVRCM}$ is $\mathcal{MVCM}$.
6. Every $\mathcal{MV}\alpha\mathcal{CM}$ is $\mathcal{MVSCM}$.
7. Every $\mathcal{MVSCM}$ is $\mathcal{MVSPCM}$.
8. Every $\mathcal{MVCM}$ is $\mathcal{MVGSCM}$.
9. Every $\mathcal{MVGCM}$ is $\mathcal{MVGSCM}$.
10. Every $\mathcal{MVSCM}$ is $\mathcal{MVGSCM}$.

11. Every $\mathcal{MV\alpha CM}$ is $\mathcal{MVGSCM}$.

12. Every $\mathcal{MVRCM}$ is $\mathcal{MVGSCM}$.

13. Every $\mathcal{MV\alpha GCM}$ is $\mathcal{MVGSCM}$.

14. Every $\mathcal{MVGSCM}$ is $\mathcal{MVSPCM}$.

15. Every $\mathcal{MVGSCM}$ is $\mathcal{MVGSPCM}$.

16. $\mathcal{MVPCM}$ and $\mathcal{MVGSCM}$ are independent to each other.

17. $\mathcal{MVGPCM}$ and $\mathcal{MVGSCM}$ are independent to each other.

Proof:

1. Let $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ be a $\mathcal{MVCM}$. Let $\mathcal{F}$ be a $\mathcal{MVCS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$. Then $\hbar^{-1}(\mathcal{F})$ is $\mathcal{MVCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. Since every $\mathcal{MVCS}$ is $\mathcal{MVGCS}$, $\hbar^{-1}(\mathcal{F})$ is $\mathcal{MVGCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. Hence $\hbar$ is $\mathcal{MVGCM}$.
2. Let $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ be a $\mathcal{MVCM}$. Let $\mathcal{F}$ be a $\mathcal{MVCS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$. Then $\hbar^{-1}(\mathcal{F})$ is $\mathcal{MVCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. Since every $\mathcal{MVCS}$ is $\mathcal{MV\alpha CS}$, $\hbar^{-1}(\mathcal{F})$ is $\mathcal{MV\alpha CS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. Hence $\hbar$ is $\mathcal{MV\alpha CM}$.
3. Let $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ be a $\mathcal{MVCM}$. Let $\mathcal{F}$ be a $\mathcal{MVCS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$. Then $\hbar^{-1}(\mathcal{F})$ is $\mathcal{MVCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. Since every $\mathcal{MVCS}$ is $\mathcal{MVPCS}$, $\hbar^{-1}(\mathcal{F})$ is $\mathcal{MVPCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. Hence $\hbar$ is $\mathcal{MVPCM}$.

Proof of (4) – (17) is same as (1) – (3).

Remark 4.4: The invert of the preceding theorem may not be true as seen in the succeeding examples.

Example 4.5: Let $\mathcal{U} = \{\rho, \lambda, \theta\}$, $\mathcal{U}/\mathcal{R} = \{\{\rho, \theta\}, \{\lambda\}\}$. Let $\mathcal{S} = \{< \rho, (0.1,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.7) >\}$ be a subset of $\mathcal{U}$. Then, $\vartheta_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \rho, (0.1,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.1,0.4) >\}, \{< \rho, (0.2,0.7) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.7) >\}\}$ is a $\mathcal{NVT}$ on $\mathcal{U}$. Let η $=\{< \rho, (0.2,0.3) >, < \lambda, (0.2,0.5) >, < \theta, (0.3,0.4) >\}$. Then, $\eta_R(A) = \{0_{\mathcal{MV}}, 1_{\mathcal{MV}}, \{< \rho\ (0.2,0.3) >, < \lambda, (0.2,0.5) >, < \theta, (0.3,0.4) >\}, \{< \rho, (0.1,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.1,0.4) >\}, \{\rho, (0.2,0.7) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.7) >\}, \{< \rho, (0.1,0.3) >, < \lambda, (0.2,0.5) >, < \theta, (0.1,0.4) >\}, \{< \rho, (0.2,0.3) >, < \lambda, (0.2,0.5) >, < \theta, (0.2,0.4) >\}, \{< \rho, (0.2,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.3,0.4) >\}, \{< \rho, (0.2,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.4) >\}, \{< \rho, (0.2,0.7) >, < \lambda, (0.3,0.5) >, < \theta, (0.3,0.7) >\}\}$ is a $\mathcal{MVT}$ on $\mathcal{U}$ and $(\mathcal{U}, \tau_R(\mathcal{S}), \eta_R(\mathcal{S}))$ is called as the $\mathcal{MVTS}$.

Let $\mathcal{V} = \{\delta, \sigma, \tau\}$, $\mathcal{V}/\mathcal{R} = \{\{\delta\}\{\sigma, \tau\}\}$. Let $\mathcal{S} = \{< \delta, (0.2,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}$ be a subset of $\mathcal{V}$. Then, $\mu_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \delta, (0.2,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}\}$ is a Nano Vague Topology on $\mathcal{U}$. Let ζ $=\{< \delta, (0.2,0.3) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}$. Then $\zeta_{\mathcal{R}}(\mathcal{S}) = \{0_{MV}, 1_{MV}, \{< \delta, (0.2,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}, \{< \delta, (0.2,0.3) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}\}$ is a $\mathcal{MVT}$ and $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Here, $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ by $\hbar(\rho) = \delta$, $\hbar(\lambda) = \sigma$ and $\hbar(\theta) = \tau$ is $\mathcal{MV\alpha CM}$, $\mathcal{MVSCM}$ and $\mathcal{MVPCM}$ but not $\mathcal{MVCM}$ since, $\mathcal{F} = \{< \delta, (0.6,0.8) >, < \sigma, (0.5,0.7) >, < \tau, (0.5,0.7) >\}$ is $\mathcal{MVC}$ set in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ but $\hbar^{-1}(\mathcal{F})$ is not $\mathcal{MVC}$ set in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

Example 4.6: Let $\mathcal{U} = \{\alpha, \beta, \gamma\}$, $\mathcal{U}/\mathcal{R} = \{\{\alpha\}, \{\beta, \gamma\}\}$. Let $\mathcal{S} = \{< \alpha, (0.2,0.5) >, < \beta, (0.2,0.7) >, < \gamma, (0.2,0.4) >\}$. Then, $\vartheta_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \alpha, (0.2,0.5) >, < \beta, (0.2,0.4) >, < \gamma, (0.2,0.4) >\}, \{< \alpha, (0.2,0.5) >, < \beta, (0.2,0.7) >, < \gamma, (0.2,0.7) >\}\}$. Let η $=\{< \alpha, (0.3,0.8) >, < \beta, (0.5,0.8) >, < \gamma, (0.3,0.8) >\}$. Then $\eta_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \alpha, (0.2,0.5) >, < \beta, (0.2,0.4) >, < \gamma, (0.2,0.4) >\}, \{< \alpha, (0.2,0.5) >, < \beta, (0.2,0.7) >, < \gamma, (0.2,0.7) >\}, \{< \alpha, (0.3,0.8) >, < \beta, (0.5,0.8) >, < \gamma, (0.3,0.8) >\}\}$ is a $\mathcal{MVT}$ and $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Let $\mathcal{V} = \{\delta, \sigma, \tau\}$, $\mathcal{V}/\mathcal{R} = \{\{\delta, \tau\}\{\sigma\}\}$. Let $\mathcal{S} = \{< \delta, (0.1,0.7) >, < \sigma, (0.3,0.6) >, < \tau, (0.2,0.3) >\}$**.** Then, $\mu_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \delta, (0.1,0.3) >, < \sigma, (0.3,0.6) >, < \tau, (0.1,0.3) >\}, \{< \delta, (0.2,0.7) >, < \sigma, (0.3,0.6) >, < \tau, (0.2,0.7) >\}\}$. Let ζ =$\{< \delta, (0.1,0.3) >, < \sigma, (0.2,0.5) >, < \tau, (0.1,0.3) >\}$. Then $\zeta_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \delta, (0.1,0.3) >, < \sigma, (0.3,0.6) >, < \tau, (0.1,0.3) >\}, \{< \delta, (0.2,0.7) >, < \sigma, (0.3,0.6) >, < \tau, (0.2,0.7) >\}, \{< \delta, (0.1,0.3) >, < \sigma, (0.2,0.5) >, < \tau, (0.1,0.3) >\}\}$ is $\mathcal{MVT}$ and $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Here, $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ by $\hbar(\alpha) = \delta$, $\hbar(\beta) = \sigma$ and $\hbar(\gamma) = \tau$ is $\mathcal{MVGSCM}$ but not $\mathcal{MVCM}$, $\mathcal{MVSCM}$, $\mathcal{MVGCM}$, $\mathcal{MV\alpha CM}$, $\mathcal{MVRCM}$, $\mathcal{MV\alpha GCM}$ since, $\mathcal{F} = \{< \delta, (0.7,0.9) >, < \sigma, (0.5,0.8) >, < \tau, (0.7,0.9) >\}$ is $\mathcal{MVC}$ set in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ but $\hbar^{-1}(\mathcal{F})$ is not $\mathcal{MVC}$, $\mathcal{MVSC}$, $\mathcal{MVGC}$, $\mathcal{MV\alpha C}$, $\mathcal{MVRC}$, $\mathcal{MV\alpha GC}$ set in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

Example 4.7: Let $\mathcal{U} = \{\rho, \lambda, \theta\}$**,** $\mathcal{U}/\mathcal{R} = \{\{\rho, \theta\}, \{\lambda\}\}$. Let $\mathcal{S} = \{< \rho, (0.1,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.7) >\}$. Then, $\vartheta_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \rho, (0.1,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.1,0.4) >\}, \{< \rho, (0.2,0.7) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.7) >\}\}$. Let η =$\{< \rho, (0.2,0.3) >, < \lambda, (0.2,0.5) >, < \theta, (0.3,0.4) >\}$. Then, $\eta_R(A) = \{0_{\mathcal{MV}}, 1_{\mathcal{MV}}, \{< \rho\ (0.2,0.3) >, < \lambda, (0.2,0.5) >, < \theta, (0.3,0.4) >\}, \{< \rho, (0.1,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.1,0.4) >\}, \{\rho, (0.2,0.7) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.7) >\}, \{< \rho, (0.1,0.3) >, < \lambda, (0.2,0.5) >, < \theta, (0.1,0.4) >\}, \{< \rho, (0.2,0.3) >, < \lambda, (0.2,0.5) >, < \theta, (0.2,0.4) >\}, \{< \rho, (0.2,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.3,0.4) >\}, \{< \rho, (0.2,0.4) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.4) >\}, \{< \rho, (0.2,0.7) >, < \lambda, (0.3,0.5) >, < \theta, (0.3,0.7) >\}\}$ is a $\mathcal{MVT}$ on $\mathcal{U}$ and $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Let $\mathcal{V} = \{\delta, \sigma, \tau\}$, $\mathcal{V}/\mathcal{R} = \{\{\delta, \tau\}\{\sigma\}\}$. Let $\mathcal{S} = \{< \delta, (0.1,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.2,0.4) >\}$. Then, $\mu_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \delta, (0.1,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.1,0.4) >\}, \{< \delta, (0.2,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.2,0.4) >\}\}$. Let ζ =$\{< \delta, (0.1,0.3) >, < \sigma, (0.3,0.5) >, < \tau, (0.1,0.3) >\}$. Then $\zeta_{\mathcal{R}}(\mathcal{S}) = \{0_{MV}, 1_{MV}, \{< \delta, (0.1,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.1,0.4) >\}, \{< \delta, (0.2,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.2,0.4) >\}, \{< \delta, (0.1,0.3) >, < \sigma, (0.3,0.5) >, < \tau, (0.1,0.3) >\}\}$ is a $\mathcal{MVT}$ and $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Here, $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ by $\hbar(\rho) = \delta$, $\hbar(\lambda) = \sigma$ and $\hbar(\theta) = \tau$ is $\mathcal{MVPCM}$, $\mathcal{MVSPCM}$, $\mathcal{MVGPCM}$, $\mathcal{MVGSPCM}$ but not $\mathcal{MV\alpha CM}$ and $\mathcal{MVGSCM}$ since, $\mathcal{F} = \{< \delta, (0.7,0.9) >, < \sigma, (0.5,0.7) >, < \tau, (0.7,0.9) >\}$ is $\mathcal{MVC}$ set in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ but $\hbar^{-1}(\mathcal{F})$ is not $\mathcal{MV\alpha C}$ and $\mathcal{MVGSC}$ set in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

Example 4.8: Let $\mathcal{U} = \{\alpha, \beta, \gamma\}$, $\mathcal{U}/\mathcal{R} = \{\{\alpha, \beta\}, \{\gamma\}\}$. Let $\mathcal{S} = \{< \alpha, (0.5,0.7) >, < \beta, (0.2,0.5) >, < \gamma, (0.3,0.4) >\}$. Then, $\vartheta_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \alpha, (0.2,0.5) >, < \beta, (0.2,0.5) >, < \gamma, (0.3,0.4) >\}, \{< \alpha, (0.5,0.7) >, < \beta, (0.5,0.7) >, < \gamma, (0.3,0.4) >\}\}$. Let η =$\{< \alpha, (0.6,0.7) >, < \beta, (0.6,0.7) >, < \gamma, (0.4,0.8) >\}$. Then $\eta_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \alpha, (0.2,0.5) >, < \beta, (0.2,0.5) >, < \gamma, (0.3,0.4) >\}, \{< \alpha, (0.5,0.7) >, < \beta, (0.5,0.7) >, < \gamma, (0.3,0.4) >\}, \{< \alpha, (0.6,0.7) >, < \beta, (0.6,0.7) >, < \gamma, (0.4,0.8) >\}\}$ is a $\mathcal{MVT}$ and $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Let $\mathcal{V} = \{\delta, \sigma, \tau\}$, $\mathcal{V}/\mathcal{R} = \{\{\delta\}\{\sigma, \tau\}\}$. Let $\mathcal{S} = \{< \delta, (0.4,0.6) >, < \sigma, (0.4,0.4) >, < \tau, (0.2,0.4) >\}$. Then, $\mu_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \delta, (0.4,0.6) >, < \sigma, (0.2,0.4) >, < \tau, (0.2,0.4) >\}, \{< \delta, (0.4,0.6) >, < \sigma, (0.4,0.4) >, < \tau, (0.4,0.4) >\}\}$. Let ζ =$\{< \delta, (0.3,0.4) >, < \sigma, (0.2,0.4) >, < \tau, (0.1,0.3) >\}$. Then $\zeta_{\mathcal{R}}(\mathcal{S}) = \{0_{MV}, 1_{MV}, \{< \delta, (0.4,0.6) >, < \sigma, (0.2,0.4) >, < \tau, (0.2,0.4) >\}, \{< \delta, (0.4,0.6) >, < \sigma, (0.4,0.4) >, < \tau, (0.4,0.4) >\}, \{< \delta, (0.3,0.4) >, < \sigma, (0.2,0.4) >, < \tau, (0.1,0.3) >\}\}$ is a $\mathcal{MVT}$ and $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Here, $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ by $\hbar(\alpha) = \delta$, $\hbar(\beta) = \sigma$ and $\hbar(\gamma) = \tau$ is $\mathcal{MVGCM}$ but not $\mathcal{MVCM}$ since, $\mathcal{F} = \{< \delta, (0.4,0.6) >, < \sigma, (0.6,0.8) >, < \tau, (0.6,0.8) >\}$ is $\mathcal{MVC}$ set in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ but $\hbar^{-1}(\mathcal{F})$ is not $\mathcal{MVC}$ set in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

Result 4.9: The relationship between various types of $\mathcal{MVCMs}$ is given in the following figure.

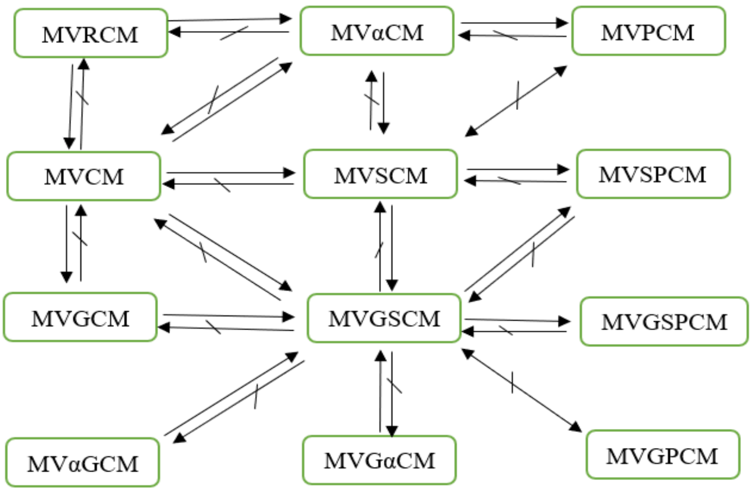

Figure 1

Theorem 4.10: A mapping $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVGSCM}$ if and only if the inverse image of each $\mathcal{MVOS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVGSOS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

Proof: Necessity: Let $\mathcal{P}$ be a $\mathcal{MVOS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$. This implies that $\mathcal{P}^{\mathcal{C}}$ is $\mathcal{MVCS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$. Since $\hbar$ is $\mathcal{MVGSCM}$, $\hbar^{-1}(\mathcal{P}^{\mathcal{C}})$ is $\mathcal{MVGSCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. Since $\hbar^{-1}(\mathcal{P}^{\mathcal{C}}) = (\hbar^{-1}(\mathcal{P}))^{\mathcal{C}}$, $\hbar^{-1}(\mathcal{P})$ is $\mathcal{MVGSOS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

Sufficiency: The proof is obvious from the definition of $\mathcal{MVGSCM}$.

Theorem 4.11: A mapping $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVGSCM}$ if $\mathcal{MV}cl(\mathcal{MV}int(\mathcal{MV}cl(\hbar^{-1}(\mathcal{P})))) \subseteq \hbar^{-1}(\mathcal{MV}cl(\mathcal{P}))$ for every $\mathcal{MV}$ set $\mathcal{P}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$.

Proof: Let $\mathcal{P}$ be $\mathcal{MVOS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$, then $\mathcal{P}^{\mathcal{C}}$ is $\mathcal{MVCS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$. By hypothesis, $\mathcal{MV}cl(\mathcal{MV}int(\mathcal{MV}cl(\hbar^{-1}(\mathcal{P}^{\mathcal{C}})))) \subseteq \hbar^{-1}(\mathcal{MV}cl(\mathcal{P}^{\mathcal{C}})) = \hbar^{-1}(\mathcal{P}^{\mathcal{C}})$, since $\mathcal{P}^{\mathcal{C}}$ is $\mathcal{MVCS}$. Now $(\mathcal{MV}int(\mathcal{MV}cl(\mathcal{MV}int(\hbar^{-1}(\mathcal{P})))))^{\mathcal{C}} = \mathcal{MV}cl(\mathcal{MV}int(\mathcal{MV}cl(\hbar^{-1}(\mathcal{P}^{\mathcal{C}})))) \subseteq \hbar^{-1}(\mathcal{P}^{\mathcal{C}}) = \hbar^{-1}(\mathcal{P})^{\mathcal{C}}$. This implies that $\hbar^{-1}(\mathcal{P}) \subseteq \mathcal{MV}int(\mathcal{MV}cl(\mathcal{MV}int(\hbar^{-1}(\mathcal{P}))))$. Hence $\hbar^{-1}(\mathcal{P})$ is $\mathcal{MV}\alpha\mathcal{OS}$ and hence it is $\mathcal{MVGSOS}$. Therefore $\hbar$ is $\mathcal{MVGSCM}$ continuous mapping.

Theorem 4.12: Let $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ be $\mathcal{MVGSCM}$ and $\imath: (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{W}, \varphi_{\mathcal{R}}(\mathcal{S}), \xi_{\mathcal{R}}(\mathcal{S}))$ be continuous mapping, then $\imath \circ \hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{W}, \varphi_{\mathcal{R}}(\mathcal{S}), \xi_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVGSCM}$.

Proof: Let $\mathcal{P}$ be $\mathcal{MVCS}$ in $(\mathcal{W}, \varphi_{\mathcal{R}}(\mathcal{S}), \xi_{\mathcal{R}}(\mathcal{S}))$. Then $\imath^{-1}(\mathcal{P})$ is $\mathcal{MVCS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ by hypothesis. Since $\hbar$ is $\mathcal{MVGSCM}$, $\hbar^{-1}(\imath^{-1}(\mathcal{P}))$ is $\mathcal{MVGSCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. Hence $\imath \circ \hbar$ is $\mathcal{MVGSCM}$.

Remark 4.13: Composition of two $\mathcal{MVGSCM}$ need not to be a $\mathcal{MVGSCM}$ and it is shown in the following example.

Example 4.14: Let $\mathcal{U} = \{\rho, \lambda, \theta\}$, $\mathcal{U}/\mathcal{R} = \{\{\rho, \theta\}, \{\lambda\}\}$. Let $\mathcal{S} = \{<\rho, (0.1,0.4)>, <\lambda, (0.3,0.5)>, <\theta, (0.2,0.7)>\}$. Then, $\vartheta_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{<\rho, (0.1,0.4)>, <\lambda, (0.3,0.5)>, <\theta, (0.1,0.4)>\}, \{<\rho, (0.2,0.7)>, <\lambda, (0.3,0.5)>, <\theta, (0.2,0.7)>\}\}$. Let η $=\{<\rho, (0.2,0.3)>, <\lambda, (0.2,0.5)>, <\theta, (0.3,0.4)>\}$. Then, $\eta_R(A) = \{0_{MV}, 1_{MV}, \{<\rho\, (0.2,0.3)>, <\lambda, (0.2,0.5)>, <\theta, (0.3,0.4)>\}$, $\{<\rho, (0.1,0.4)>, <\lambda, (0.3,0.5)>, <\theta, (0.1,0.4)>\}$, $\{\rho, (0.2,0.7)>, <\lambda, (0.3,0.5)>, <\theta, (0.2,0.7)>\}$, $\{<\rho, (0.1,0.3)>, <\lambda, (0.2,0.5)>, <\theta, (0.1,0.4)>\}$, $\{<\rho, (0.2,0.3)>, <\lambda, (0.2,0.5)>, <\theta, (0.2,0.4)>\}$, $\{<\rho, (0.2,0.4)>, <\lambda, (0.3,0.5)>, <\theta, (0.3,0.4)>\}$, $\{<\rho, (0.2,0.4)>, <\lambda, (0.3,0.5)>, <\theta, (0.2,0.4)>\}$, $\{<\rho, (0.2,0.7)>, <\lambda, (0.3,0.5)>, <\theta, (0.3,0.7)>\}\}$ is a $\mathcal{MVT}$ on $\mathcal{U}$ and $(\mathcal{U}, \tau_R(\mathcal{S}), \eta_R(\mathcal{S}))$ is called as the $\mathcal{MVTS}$.

Let $\mathcal{V} = \{\delta, \sigma, \tau\}$, $\mathcal{V}/\mathcal{R} = \{\{\delta\}\{\sigma, \tau\}\}$. Let $\mathcal{S} = \{< \delta, (0.2,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}$. Then, $\mu_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \delta, (0.2,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}\}$. Let ζ $=\{< \delta, (0.2,0.3) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}$. Then $\zeta_{\mathcal{R}}(\mathcal{S}) = \{0_{MV}, 1_{MV}, \{< \delta, (0.2,0.4) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}, \{< \delta, (0.2,0.3) >, < \sigma, (0.3,0.5) >, < \tau, (0.3,0.5) >\}\}$ is a $\mathcal{MVT}$ and $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Let $\mathcal{W} = \{\alpha, \beta, \gamma\}$, $\mathcal{W}/\mathcal{R} = \{\{\alpha, \gamma\}\{\beta\}\}$. Let $\mathcal{S} = \{< \alpha, (0.3,0.4) >, < \beta, (0.3,0.5) >, < \gamma, (0.3,0.6) >\}$. Then, $\varphi_{\mathcal{R}}(\mathcal{S}) = \{0_{NV}, 1_{NV}, \{< \alpha, (0.3,0.4) >, < \beta, (0.3,0.5) >, < \gamma, (0.3,0.4) >\}, \{< \alpha, (0.3,0.6) >, < \beta, (0.3,0.5) >, < \gamma, (0.3,0.6) >\}\}$. Let ξ $=\{< \alpha, (0.5,0.8) >, < \beta, (0.5,0.8) >, < \gamma, (0.4,0.8) >\}$. Then $\xi_{\mathcal{R}}(\mathcal{S}) = \{0_{MV}, 1_{MV}, \{< \alpha, (0.3,0.4) >, < \beta, (0.3,0.5) >, < \gamma, (0.3,0.4) >\}, \{< \alpha, (0.3,0.6) >, < \beta, (0.3,0.5) >, < \gamma, (0.3,0.6) >\}, \{< \alpha, (0.5,0.8) >, < \beta, (0.5,0.8) >, < \gamma, (0.4,0.8) >\}\}$ is a $\mathcal{MVT}$ and $(\mathcal{W}, \varphi_{\mathcal{R}}(\mathcal{S}), \xi_{\mathcal{R}}(\mathcal{S}))$ is $\mathcal{MVTS}$.

Define mappings $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ by $\hbar(\rho) = \delta$, $\hbar(\lambda) = \sigma$, $\hbar(\theta) = \tau$ and $\imath: (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{W}, \varphi_{\mathcal{R}}(\mathcal{S}), \xi_{\mathcal{R}}(\mathcal{S}))$ by $(\delta) = \alpha$, $\hbar(\sigma) = \beta$, $\hbar(\tau) = \gamma$ where $\hbar$ and $\imath$ are $\mathcal{MVGSCM}$. Then the composite mapping $\imath \circ \hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{W}, \varphi_{\mathcal{R}}(\mathcal{S}), \xi_{\mathcal{R}}(\mathcal{S}))$ is not $\mathcal{MVGSCM}$ since $\mathcal{F} = \{< \alpha, (0.2,0.5) >, < \beta, (0.2,0.5) >, < \gamma, (0.2,0.6) >\}$ is $\mathcal{MVC}$ set in $(\mathcal{W}, \varphi_{\mathcal{R}}(\mathcal{S}), \xi_{\mathcal{R}}(\mathcal{S}))$ but $\hbar^{-1}(\mathcal{F})$ is not $\mathcal{MVGSC}$ set in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

Definition 4.15

Let A be a $\mathcal{MV}$ set of the $\mathcal{MVTS}$ $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. The Micro Vague Generalized Semi interior of A $(shortly\ \mathcal{MV} - gs - int(A))$ and Micro Vague Generalized Semi closure of A $(shortly\ \mathcal{MV} - gs - cl(A))$ are defined as:

1. $\mathcal{MV} - gs - int(A) = \cup\{H/H\ is\ a\ \mathcal{MVGSOS}\ in\ U\ and\ H \subseteq A\}$.
2. $\mathcal{MV} - gs - cl(A) = \cap\{\mathcal{T}/\mathcal{T}\ is\ a\ \mathcal{MVGSCS}\ in\ U\ and\ A \subseteq \mathcal{T}\}$.

If A is $\mathcal{MVGSCS}$, then $\mathcal{MV}gscl(A) = A$.

Theorem 4.16

Let $\hbar: (\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S})) \longrightarrow (\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ be a $\mathcal{MVGSCM}$. Then the following conditions hold:

i). $\hbar(\mathcal{MV}gscl(\mathcal{P})) \subseteq \mathcal{MV}cl(\hbar(\mathcal{P}))$, for every $\mathcal{MV}$ set $\mathcal{P}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

ii). $\mathcal{MV}gscl(\hbar^{-1}(\mathcal{Q})) \subseteq \hbar^{-1}(\mathcal{MV}cl(\mathcal{Q}))$, for every $\mathcal{MV}$ set $\mathcal{Q}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$.

Proof: i). Since $\mathcal{MV}cl(\hbar(\mathcal{P}))$ is $\mathcal{MVCS}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$ and $\hbar$ is $\mathcal{MVGSCM}$, then $\hbar^{-1}(\mathcal{MV}cl(\hbar(\mathcal{P})))$ is $\mathcal{MVGSCS}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$. That is $\mathcal{MV}gscl(\mathcal{P}) \subseteq \hbar^{-1}(\mathcal{MV}cl(\hbar(\mathcal{P})))$. Therefore, $\hbar(\mathcal{MV}gscl(\mathcal{P})) \subseteq \mathcal{MV}cl(\hbar(\mathcal{P}))$, for every $\mathcal{MV}$ set $\mathcal{P}$ in $(\mathcal{U}, \vartheta_{\mathcal{R}}(\mathcal{S}), \eta_{\mathcal{R}}(\mathcal{S}))$.

ii). Replacing $\mathcal{P}$ by $\hbar^{-1}(\mathcal{Q})$ in (i), we get $\hbar(\mathcal{MV}gscl(\hbar^{-1}(\mathcal{Q}))) \subseteq \mathcal{MV}cl(\hbar(\hbar^{-1}(\mathcal{Q}))) \subseteq \mathcal{MV}cl(\mathcal{Q})$. Hence, $\mathcal{MV}gscl(\hbar^{-1}(\mathcal{Q})) \subseteq \hbar^{-1}(\mathcal{MV}cl(\mathcal{Q}))$, for every $\mathcal{MV}$ set $\mathcal{Q}$ in $(\mathcal{V}, \mu_{\mathcal{R}}(\mathcal{S}), \zeta_{\mathcal{R}}(\mathcal{S}))$.

Statements and Declarations

Conflict of interest: On behalf of all authors, the corresponding author declares that there is no conflict of interest.

Availability of data and materials: No data was used for the research described in the article.

5. Conclusion

Several types of continuous mappings in Micro Vague Topological spaces are introduced and the theorems based on Micro Vague Continuous mappings are presented. Micro Vague Generalized Semi interior and closer are introduced. The inter-relation between Micro Vague Generalized Semi Continuous mappings and the other existing mappings are discussed and proved with the help of solid numerical.

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