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Centralizing and Commuting Jordan Generalized Reverse Derivations on Prime Rings

M. Sri Kameswara Rao[1], C. Jaya Subba Reddy[2], A. Sivakameshwara Kumar[3], Chennupalle Divya[4]

[1] *Research Scholar, Department of Mathematics, S.V.University, Tirupati, Andhra Pradesh, India*

[2] *Department of Mathematics, S.V.University, Tirupati, Andhra Pradesh, India*

[3] *Department of Mathematics, Santhiram Engineering College, Nandyal, Andhra Pradesh, India*

[4] *Department of CSE, Amrita School of Engineering, Bangalore, India*

Abstract:- Let R be a prime ring and d a derivation on R. If f is a Jordan generalized reverse derivation on R such that f is centralizing on left ideal U of R, then R is commutative.

Keywords*: Derivations, Jordan generalized derivation, Jordan generalized reverse derivations, Prime ring, centralizing and commuting.*

1. Introduction

H. E. Bell and W. S. Martindale III, [2] proved that a semiprime ring R must have a nontrivial central ideal, if it admits an appropriate endomorphism or derivation which is centralizing on U. Then R is commutative. M.Bresar [3] proved that some concrete additive mappings cannot be centralizing on certain subsets of non-commutative prime rings. He also described the structure of an arbitrary additive mapping which is centralizing on a prime ring. J. H. Mayne [8] had discussed about the existence of a non-trivial automorphism or derivation which is centralizing on a non-zero ideal in a prime ring and then shown that the ring is commutative. Asif Ali and Tariq Shah [1] has considered f as a generalized derivation on such that f is centralizing on a left ideal U of R. Jaya Subba Reddy et.al. [5-7], studied some results on centralizing and commuting left generalized derivations, $(\alpha, 1)$-reverse and generalized $(\alpha, 1)$-reverse derivations in rings. In this paper we extended some results on Jordan generalized reverse derivation on prime rings.

2. Preliminaries

Throughout this paper R will denote a prime ring and Z its center. A ring R is said to be a prime if $aRb = 0$ implies that either $a = 0$ or $b = 0$. An additive mapping $d: R \to R$ is said to be derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. A mapping f is said to be Jordan generalized reverse derivation if there exists a derivation d from R to R such that $f(x^2) = f(x)x + xd(x)$, for all x in R. A mapping f is said to be commuting on a left ideal U of R if $[f(x), x] = 0$, for all $x \in R$ and f is said to be centralizing if $[f(x), x] \in Z(R)$, for all $x \in R$.

Remark 2.1: For a nonzero element $a \in Z(R)$, $ab \in Z(R)$, then $b \in Z(R)$.

To prove the main results, we find it necessary to establish the following lemma.

Lemma 2.1: If f is an additive mapping from R to R such that f is centralizing on a left ideal U of R then for all $x \in U \cap Z(R)$, $f(x) \in Z(R)$.

Proof: Since f is centralizing on U, we have $x, y \in U \Rightarrow y + x \in U$.

For all $y + x \in U \Longrightarrow [f(y + x), y + x] \in Z(R)$.

f is additive then $[f(y) + f(x), y + x] \in Z(R)$, for all $x, y \in U$.

$[f(y), y] + [f(y), x] + [f(x), y] + [f(x), x] \in Z(R)$, for all $x, y \in U$.

Since f is centralizing, we have $[f(x), y] + [f(y), x] \in Z(R)$. (2.1)

Now if $x \in Z(R)$, then equation (2.1) becomes $[f(x), y] \in Z(R)$.

Replacing y by $f(x)y$ in above equation, we get $[f(x), f(x)y] \in Z(R)$, for all $x, y \in U$.

$\Rightarrow f(x)[f(x), y] \in Z(R)$, for all $x, y \in U$.

Case 1: $[f(x), y] = 0$ for all $x, y \in U$ we have $(x)y - yf(x) = 0 \Longrightarrow f(x) \in C_R(U)$.

The centralizer of U in R and hence {[1, identity IV]} $f(x) \in Z(R)$.

Case 2: $[f(x), y] \neq 0$. It again follows from remark 1, we have

$x \in Z(R), f(x)[f(x), y] \in Z(R) \Longrightarrow f(x) \in Z(R)$. Hence the lemma.

3. Main Results

Theorem 3.1: Let R be a prime ring. Let $d: R \to R$ be a non zero derivation and f be a Jordan generalized reverse derivation on a left ideal U of R. If f is commuting on U then R is commutative.

Proof: Since f is commuting on U, we have $[f(x), x] = 0$ for all $x \in U$.

Replacing x by $y + x$ in above equation, we get $[f(y + x), y + x] = 0$, for all $x, y \in U$.

$[f(y), x] + [f(x), y] = 0$, for all $x, y \in U$. (3.1)

We replace x by x^2 in (3.1), we get $[f(y), x^2] + [f(x^2), y] = 0$, for all $x, y \in U$.

$x[f(y), x] + [f(y), x]x + [f(x)x + xd(x), y] = 0$, for all $x, y \in U$.

$x[f(y), x] + [f(y), x]x + f(x)[x, y] + [f(x), y]x + [x, y]d(x) + x[d(x), y] = 0$, for all $x, y \in U$.

Using equation (3.1) and replacing x by y in the above relation, we get $y[d(y), y] = 0$, for all $y \in U$.

Replace $d(y)$ by $d(y)xr$, for all $r \in R$ in the above relation, we get $yd(y)x[r, y] = 0$, for all $x \in U$. Finally, we generalized $d(y)U[r, y] = 0$.

By the primeness of R, we have either $d(y) = 0$ or $[r, y] = 0$, for all $y \in U$ and $r \in R$.

So for any element $y \in U$, either $y \in Z(R)$ or $d(y) = 0$, for all $y \in U$. Since d is nonzero on R, by [4, Lemma 2] d is nonzero on U.

Suppose $d(y) \neq 0$, for $y \in U, y \in Z(R)$.

Suppose $z \in U$ is such that $z \notin Z(R)$, then $d(z) = 0$ and $y + z \notin Z(R)$.

This implies that $d(y + z) = 0$ and so $d(y) = 0$, a contradiction.

This implies that $z \in Z(R)$ for all $z \in U$.

Thus U is commutative and hence by [6, Lemma 3], R is commutative.

Theorem 3.2: Let U be a left ideal of a prime ring R such that $U \cap Z(R) \neq 0$. Let d be a nonzero derivation and f be a Jordan generalized reverse derivation on R such that f is centralizing on U. Then R is commutative.

Proof: We assume that $Z(R) \neq 0$, otherwise f is a commuting on U and there is nothing to prove. We replace y by z^2 in equation (2.1), we get

$z[f(x), z] + [f(x), z]z + [f(z)z + zd(z), x] \in Z(R)$.

$z[f(x), z] + [f(x), z]z + f(z)[z, x] + [f(z), x]z + z[d(z), x] + [z, x]d(z) \in Z(R).$ (3.2)

Since $z \in Z(R)$ and f is centralizer, we have $[z, x] = 0$ and $[f(z), x] = 0 = [f(x), z]$, for all $x \in R$.

Equation (3.2) becomes,$[d(z), x] \in Z(R)$, since z is non zero and replacing z by x, we get $[d(x), x] \in Z(R)$.

This implies that d is centralizing on U and hence by [1, Theorem 4] we conclude that R is commutative.

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