Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 1 (2024)

Methodical Fullness of Hybrid Intellectual System with Cognitive Modeling of Desing-Research Activity on the Basis of Thinking Profiles

Irina Karpacheva[1], Elena Igonina[2], Galina Simonovskaya[3]

[1]*PhD in Pedagogy Sciences, associate professor, Bunin Yelets State University, Yelets, Russia*

[2]*PhD in Physical and Mathematical Sciences, associate professor, Bunin Yelets State University, Yelets, Russia*

[3]*PhD in Pedagogy Sciences, associate professor, Bunin Yelets State University, Yelets, Russia*

Abstract:- Development of methodical fullnesst of hybrid intellectual system with cognitive modeling of desing and research activity of schoolchildren in mathematics. Construction of this system is carried out on the basis of integration of thinking profiles, modified with the development of learning technologies in the context of digitalization of education, and typology of levels of project-research activities of schoolchildren in intellectual knowledge environment (search and reproductive, empirical, theoretical). When forming the content of the system were taken into account the personalized profiles such as: symbolic, geometric, specific, computational, historical and genetic. The filling of the system was determined by research tasks on the main sections of the topic «Complex numbers». Tasks were selected in the context of cognitive characteristics and principles of personalized profiles, their accuracy and ability to reflect the student's preferences and needs.

Keywords: *hybrid intellectual system, design and research activities, thinking profiles, research tasks.*

1. Introduction

Prospects of solving problems of project-research activity management of students in modern conditions are connected with creation of automated learning systems based on artificial intelligence techniques [1, 2]. Intellectual management of project-research activity of students in mathematical education is designed to provide achieving high quality education through individualization. The effectiveness of intelligent learning systems depends on principles of personalized profiles, their accuracy and ability to reflect user's preferences and needs. Currently developed intelligent systems in which personalization is achieved, are based on cognitive modeling of personal characteristics, affective states, learning dynamics, learning styles, creativity [3]. Cognitive modeling allows a priori to predict the results of possible managerial decisions and to choose the most effective solution, to develop an adaptive learning quality management mechanism [4].

The idea of designing intelligent learning systems based on learning styles is recognized by the academic community as dominant. Productivity and performance of adaptive systems, based on learning styles, quality assurance and optimization of learning are noted in a number of studies. One of the most common style-based models is the Felder-Silverman model. This model is based on information acquisition and processing methods and takes into account 4 factors with opposite values: sensory/intuitive (perception mode); visual/verbal (presentation mode); active/reflexive (processing method); sequential /holistic (organization method) [5].

However, the problem of creating a holistic concept of personalized adaptive learning in an electronic environment based on the principles of hybridity, interactivity, intelligence management remains relevant.

The concept of intellectual management of design and research activities of schoolchildren in the hybrid learning environment of mathematics proposed in this paper is based on the cognitive profiles of the learner and typologies of levels of activity of schoolchildren in the intellectual environment of knowledge [3]. Profiles are concretized taking into account the modern tendencies of digitalization of education for students, motivated to study mathematics. Diagnostics of thinking profiles is carried out with the help of the technique «Thinking Profile» (V.A. Ganzen, K.B. Malyshev, L.V. Oginetz) [6].

2. Research methods and materials

The study of the phenomenon of individual thinking is carried out in psychology since the middle of the twentieth century. Thinking skills, along with learning and complex problem solving based on understanding, refer to the basic skills of the XXI century [7, 8]. Cognitive psychology is currently researching thinking styles, types, and profiles. The style of thinking is defined as the functional organization of the emergence and development of cognitive neologisms, their stable manifestations in humans [9]. Type of thinking means individual way of analytic-synthetic transformation of this information [10]. Thinking profile characterizes the dominant ways of intellectual processing of information, which is based on the inherent types of thinking and level of creativity» [6, p.159]. In classical psychology such basic types of thinking are defined as: Subject, figurative, iconic and symbolic [11]. Based on D. Bruner's thesis on thinking as a translation from one language to another, researchers expand the set of types of thinking: subject-form (practical), object-symbolic (humanitarian), object-symbolic (operator), figurative (artistic), figurative-symbolic (technical), iconic-symbolic (theoretical).

The thinking profile is the most important personal characteristic of the student. It displays not only the dominant ways of processing information children and the level of their creativity, but also determines the style of activity, inclinations, interests and professional orientation of the person. This paper provides an example of a hybrid intellectual system with cognitive modeling of design and research activities of schoolchildren who are motivated to study mathematics, in this discipline. Filling of the system is carried out on the basis of the following profiles of thinking, modified by E.I. Smirnov taking into account the development of learning technologies in the context of the digitalization of education [12]:

1. *Iconic-symbolic profile*. The dominant type of thinking is symbolic, abstract-theoretical thinking. In the process of solving abstract theoretical problems, the student reveals the regularities of the problem or phenomenon in abstract concepts, resorts to operations performed in the mind (not directly dealing with the experience gained through the senses). Iconic-symbolic profile is typical for students who are engaged in scientific theoretical research.

2. Figure-geometric profile. Dominant thinking type is figurative-symbolic. Figure-geometric profile is connected with the learner's ability to understand the drawings, to understand the schemes of technical devices and their work, solve practical physical and technical problems. It is typical for students inclined to choose a profession such as «man-technique».

3. Concrete activity profile. The dominant type of thinking is subject-form. The thought process is carried out directly in the perception of the surrounding reality and the practical transformation of real objects in real time. Necessary images are presented in short-term and operative memory. This profile is noted among students engaged in productive work, which results in the creation of a material product.

4. Information-computing profile. The dominant type of thinking is object-symbolic. The profile is based on reproductive thinking, aimed at the execution of selected short-term operations according to a given scheme with the use of short-term and operative memory. It is typical for students who like to do counting or computing, accounting work.

5. Historical-genetic profile. Dominant types of thinking is object-symbolic and figurative-symbolic [13]. Object-symbolic type occurs in children of humanitarian orientation, figurative-symbolic type is more prevalent among schoolchildren engaged in some kind of art. The profile is connected with the discovery of new knowledge, with the generation of original ideas.

It is worth noting that the profile of thinking is determined not only by the dominant type of thinking, but also by the creativity of the student. Task facets used to fill the knowledge base of a hybrid intellectual system, are needed to be built, including on the basis of cognitive modelling of the design and research activities of schoolchildren. Mathematics tasks should be offered taking into account the following levels of design and research activity of students in the intellectual environment of knowledge: 1) search and reproductive; 2) empirical; 3) theoretical [13].

3. Results and discussion

Let us illustrate the richness of the hybrid intellectual system taking into account thinking profiles and specified levels on the example of realization of project-research tasks by students on the topic «Complex numbers». The choice of this topic is due to the fact that complex numbers are an interesting subject for study, which provides the key to solving many applied problems that cannot be considered within the narrow limits of elementary mathematics. When solving problems using complex numbers, new interesting facts and generalizations are sometimes found [14].

Logically it would be right to finish the study of the concept of number in the school with the topic «Complex numbers». In the history of domestic mathematical education there were periods, when it was part of the high school math program. Beginning in 1917, Soviet schools studied purely imaginary numbers, since 1932, the topic «Complex numbers» has been included in the program of the 8th grade, the subject continued until the end of the mathematics course in school. However, since 1968, as a result of modernization of the general education course of mathematics, the topic «Complex numbers» in general education classes was not considered, and its study was kept only in major classes. It should be noted that sometimes this topic was also reflected in the school course of mathematics in LH. It is also worth noting that complex numbers are important as a separate field of mathematics, in which the knowledge and skills acquired by students of algebra [15] and trigonometry work perfectly.

In this paper, the learning of the following sections has been taken into account to determine the base content of the hybrid intellectual system: problems leading to the concept of complex number, definition and forms of writing complex numbers, operations on complex numbers represented by different forms of notation, geometric interpretation of complex number. For the development of methodical support of hybrid intellectual system the above profiles of thinking were used (by E.I. Smirnov) taking into account their specific characteristics and levels of project-research activity of students in the intellectual environment of knowledge. The number of database tasks is represented by double numbering: the first digit is the profile number, the second is the level of knowledge. Depending on the level, volume and stage of students' learning of the studied material facets of tasks can be correlated.

P*rofile 1 (iconic-symbolic)* can be presented with the following tasks:

Task 1.1. Find the quotient of complex numbers in two ways $\frac{1+3i}{1+i}$: by definition and using complex conjugate number.

In the course of the solution the first way learner using the proportion $\frac{1+3i}{1+i} = a + bi$, goes on to solve the equation system $\begin{cases} a - b = 1, \\ a + b = 3. \end{cases}$ To find the quotient of complex numbers by the second way the student uses a complex conjugate number of the denominator of the given fraction. The comparison concludes that second method is more rational and requires no special formulas to be memorized.

Task 1.2. Solve the quadratic equation with the unknown z: $z^2+(-5+6i)z-(1+9i)=0$.

When solving a square equation, the student first calculates the square root of the discriminant, which is a complex number. Since root extraction from a complex number results in a complex number, then root of discriminant is sought as $D=a+bi$. In our case it is $D = -7-24i=a+bi$; $7-24i=(a+bi)^2$, then $7-24i=a^2+2abi-b^2$. Using the knowledge that two complex numbers will be equal if their actual and imaginary parts are equal, respectively, the learner must solve the system to find unknown values a and b. Considering any of the pairs received $a_1=3$, $b_1=-4$ or $a_2=-3$, $b_2=4$, for example, the first, it turns out that $\sqrt{D} = 3 - 4i$, and then $z_1 = 4 - 5i$, $z_2 = 1 - i$.

Task 1.3. Argand Française symbols can be used to write complex numbers in trigonometric form more briefly. These symbols were first used in 1813 by Française, a professor of artillery at Metz. Finally this designation was accepted by Cauchy. The complex number *z*, which amplitude is *r*, *r>0*, and the main value of the amplitude is φ ($-\pi<\varphi\leq\pi$) is denoted by r_φ, that is $r(\cos\varphi+i\sin\varphi)=r_\varphi$. Complex number with $r=0$ doesn't have an amplitude. Positive real number x, will be written as x_0, $x>0$, and minus number will be written as $(-x)=x_\pi$, $x>0$, unit imaginary number will be written as $i=1_{\frac{\pi}{2}}$. Write down the known rules (addition, multiplication, division) of the actions on complex numbers $z_1=r_\alpha$ and $z_2=\rho_\beta$.

It is also possible to invite students to present formulas for the expression of sine and cosine through Argand formulas. Solving this task, the student analytically works with complex numbers written in different forms. Symbolic record, its interpretation allows on the one hand to ignore the essence of the tasks performed, but the resulting formulas can then be used to consider trigonometric functions.

Profile 2 (figure-geometric) can be filled with the following tasks.

Task 2.1. Find the loci Re $z>c$, where *z* is a complex number, *c* is real number.

When working with this task, the student first determines the real part of the complex number $z=x+iy$. The generalization of the gained knowledge on the topic «Solution of inequalities» is made further and then moving to the structure $x>c$. The resulting inequality is the simplest, however, you need to consider the feature that c can run all real numbers, but the response structure is not affected (see Figure 1): $x\in(c;\ +\infty)$.

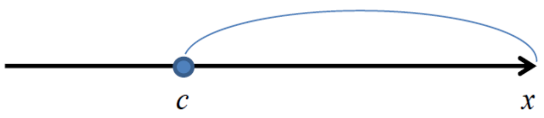

Figure 1: Real number values *c*

Task 2.2. Find the loci $|z-z_0|<c$, where z, z_0 are complex numbers, *c* is real number.

When solving this problem, the student needs to go to the algebraic form of complex numbers: $z=x+iy$, $z=a+ib$. When inequality is transformed, inequality becomes the inequality, which defines the inner region of the circle with the center at the point $(a;b)$ and radius *c* (Figure 2). At this stage, a study should be carried out: impact of values z_0 and *c* on the result of solution.

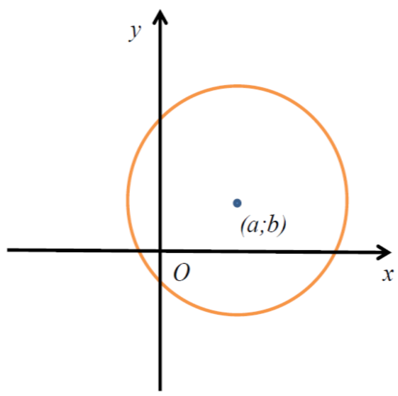

Figure 2: Loci

Task 2.3. Consider vector interpretation of addition operations and subtraction of complex numbers.

During the task the student uses the knowledge that each point M, matching the complex number *z*, on the plane of complex numbers is uniquely matched by the vector $\overline{OM}$ starting at zero point O. The addition and subtraction of complex numbers written in algebraic form clearly corresponds to the addition and subtraction of corresponding vectors. That is, if a and b are complex coordinates of points A and B, then number $c=a+b$ is the coordinate of point C according to Figure 3. The complex number $d = - b$ is corresponded by point D.

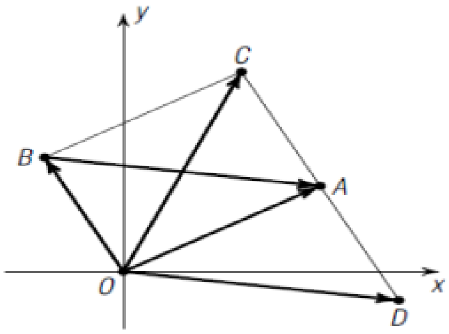

Figure 3: Vector interpretation of addition and subtraction operations of complex numbers

Students can also be asked to consider the geometric interpretation of the multiplication operation of complex numbers represented in trigonometric form. Geometric imaging tasks are examined with the support of an interactive environment that allows to perform dynamic images.

Profile 3 (*Concrete activity)* can be represented by practical objectives.

Task 3.1. For example, one of the most visible interpretations, geometric is points in the plane, whose coordinates correspond to the actual and imaginary part of the complex number. Considering some curve as a population of such points, one can describe it as a complex function of the real argument.

For example, when considering a function f(t)= i^t , by explanating t as temporal value, it results the degree of rotation of unit vector. If you change the value of the argument from 0 to 4, the vector describes the complete circle. It is possible conversion to function and canonical, more familiar: $i^t = e^{\frac{i\pi t}{2}}$.

A graphical dynamic representation of functions of this kind is also possible.

Another practical use of complex numbers is the use of the numbers in solving planimetric problems. It should be noted that further consideration of linkages will be required.

Task 3.2. The triangle ABC is refined in a circle centered at the origin. The triangle A'B'C' is symmetrical with respect to the actual axis. Prove that their orthocenters (height intersection points) are symmetric with respect to the actual axis.

A solution using complex numbers can be presented as follows.

Let the points of ABC have complex coordinates $A(z_1)$, $B(z_2)$, $C(z_3)$. Then the complex coordinate of its orthocenter is $H(z_1+z_2+z_3)$. The triangle A'B'C' will be refined in the same circle (by definition of symmetry and by property of diameter of circle, perpendicular to chord), complex coordinates of its points are respectively $\overline{z_1}$, $\overline{z_2}$, $\overline{z_3}$. Complex coordinate then H' will be $\overline{z_1}+\overline{z_2}+\overline{z_3}$, but for complex numbers the $\overline{z_1}+\overline{z_2}+\overline{z_3}=\overline{z_1+z_2+z_3}$. As was to be proved.

In the next level tasks it is advisable to reflect the application of complex numbers, for example, application in the economy (in particular in financial transactions when considering capital formation).

Task 3.3. Let's look at the rise at the complex interest rate given by the formula:

$$F\,V = P\,V\,(1 + r)^t, \quad (1)$$

where the initial amount $P\,V$ is extended over a period of t at the rate of r – complex number of type $a+bi$, $F\,V$ *is* final amount. Knowing the interest rate $r = a + 1{,}09i$, it is necessary to construct a schedule of growth and determine the period of capital accumulation and the rate of capital growth for this period.

To solve the problem, the student must first consider the trigonometric form of representation of the complex number z , which participates in the formula of accretion (1):

$$z = 1 + r = (1 + a) + bi = mod\ (\cos\alpha + i \cdot sin\alpha)\ , \qquad (2)$$

$$mod = \ (1 + a)^2 + b^2)^{1/2},\ \alpha = arctg\frac{b}{1+a}.$$

According to the Moivre formula, accretion (1) looks like

$$F\ V = \ P\ V \,\mathrm{mod}^t\ (\cos t\,\alpha + + i \cdot \sin t\,\alpha).$$

Complex component of number FV can be discarded and considered as a real monetary sum only the real part. Note that the schedule of accumulation of the complex sum FV in the complex plane is a spiral which intersects with the real axis in its positive and negative part.. At compound rate $r = a + 1{,}09i$ according to the formulas (2) we will get a gain graph at the Figure 4. The capital accumulation period is 5 years, the capital growth rate for this period is equal to 2.

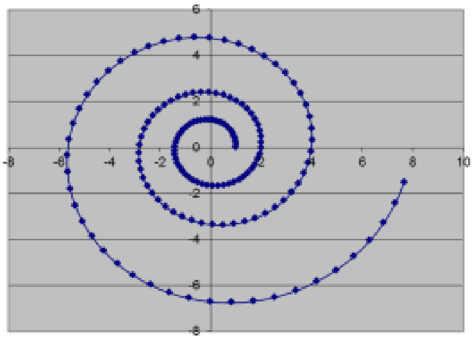

Figure 4: Complex sum *FV* accumulation schedule at the rate of $r = a + 1{,}09i$

As an alternative task, one can consider the use of complex numbers in physics. For example, in electrical engineering [16] real numbers are used to measure resistance. It is the property of the object to prevent electric current from passing through it And imaginary numbers are used to measure inductance (ratio of magnetic flux to current in coil) and capacitances (ratio of electric charge to potential difference between capacitor plates). Application of complex numbers allows to use laws, formulas and methods of calculation, which are used in DC circuits to calculate AC circuits. It is possible to invite students to consider other applications of complex numbers.

For the Profile 4 *(informational-computing)* could be offered the tasks of such a plan as:

Task 4.1. Using the properties of complex numbers

$|z_1 + z_2| \ \leq |z_1| + |z_2|$ and $|z_1 + z_2| \geq ||z_1| - |z_2||$,

show, that $7 \leq |\ z + 6 - 8i\ | \leq 13$, if $|\ z\ | = 3$.

When working with this task, the student tries to see the application of the proposed properties to accomplish the task in a context. Namely, the first property is implemented as follows:

$|z + 6 - 8i\ | \leq |z| + |6 - 8i|$, where $|z| + |6 - 8i| = 3 + \sqrt{6^2+8^2} = 3 + \sqrt{100} = 13$,

second property application:

$|z + 6 - 8i\ | \geq ||z| - |6 - 8i||$, where $||z| - |6 - 8i|| = |3 - \sqrt{6^2+8^2}| = |3 - \sqrt{100}\ | = 7$,

given the received we will have:

$7 \leq |\ z + 6 - 8i\ | \leq 13$.

Task 4.2. Develop a program (or algorithm) to solve the following problem. Complex numbers are given $a=\alpha+\beta i$, $b=\gamma+\delta i$, $c=\sigma+\mu i$. Find complex number $d = \varphi + \omega i$, represented by formula $d=a^2 * \frac{a+b}{a-bc}$.

Task 4.3. Express cos5φ and sin5φ in terms of cosφ and sinφ, using a combination of the Newton binome formula and the Moivre formula.

The combination of the Newton binome formula and the Moivre formula is inexhaustible source of combinatorial and trigonometric identities.

According to the Newton binome formula we have:

$(\cos\varphi + isin\,\varphi)^5 = cos^5\varphi + 5icos^4\varphi\sin\varphi + 10i^2cos^3\varphi sin^2\varphi + 10i^3cos^2\varphi sin^3\varphi + 5i^4\cos\varphi sin^4\varphi +$
$+i^5sin^5\varphi = cos^5\varphi - 10cos^3\varphi sin^2\varphi + 5\cos\varphi sin^4\varphi + (5cos^4\varphi\sin\varphi - -10cos^2\varphi sin^3\varphi + sin^5\varphi)i$.

According to the Moivre formula at n=5 will have: $(\cos\varphi + isin\,\varphi)^5 = \cos5\varphi + isin\,5\varphi$. Therefore we have ,$\cos5\varphi = cos^5\varphi - 10cos^3\varphi sin^2\varphi + 5\cos\varphi sin^4\varphi$, $sin\,\varphi = 5cos^4\varphi\sin\varphi - 10cos^2\varphi sin^3\varphi + sin^5\varphi$.

To the Profile 5 (historical-genetic) can be attributed tasks in which complex number is «covered» by historical layer. It is necessary to open it and understand the mathematical component, for example:

Task *5.1.* Conduct research on Argand symbols.

During the assignment, students need to learn more than just historical information about the works of the French mathematician Argand (1768-1822), but the history of the introduction of symbols developed by scientists to denote complex numbers. To test knowledge on the studied topic in Task 5.1 you can offer specific test questions.

For the second level, it is possible to use tasks related to the study of complex numbers and their relationship with different types of transformation..

Task 5.2. Let this transformation *T* be operation that each complex number *z*, that is, each point of the plane, aligns the other point *T*(*z*). Let's consider the photo on the plane of any size and show its image, obtained by transformation: each pixel of a photo will pass to the place of the plane, where its transformation carries over. For example, to display *T*: *T*(*z*) = *z*/2 each number is divided by 2. Therefore, the photo will just shrink in half - this is compression. This transformation in geometry is called homothety. To map *T*: *T*(*z*) = *iz* the photo is rotated counterclockwise by 90° by definition of the number *i*.

Students can be asked to perform such transformations as:

1) *T*(*z*) = (1+*i*)*z*. Since the number module 1+*i* is equal to $\sqrt{2}$, and the amplitude is 45°, then the transformation is a composition of turning the preimage by 45° and homothetics with coefficient $\sqrt{2}$ – this transformation is called uniformity. The great advantage of complex numbers is that they can describe uniformity transformations simply as multiplications by numbers.

2) *T*(*z*) = -1/*z*. This transformation is close to the inversion, namely to the origin of the coordinates, which corresponds to the number 0, this transformation cannot be applied, but we will agree to assume that point 0 goes for infinity. The motivation is very simple: when a complex number z approaches 0, that is, its module tends to 0, its image module |-1/z| is attached to the module *z*, which means he's going for infinity. So, this transmutation in zero has a «blast»: the little neighborhood of zero goes very far beyond the screen. Conversely, points that were very far from zero, retract, moving to points very close to zero. The main property of inversion is that it turns circles into circles or lines. Artists often use transformations of this kind, calling them anamorphoses.

The next level of research tasks is devoted to acquaintance with the views of P.A. Florensky on suspicion in geometry. The content of 5.3 tasks can be related to the study of the system of the most common complex numbers (double, dual, ordinary complex numbers, quaternions, octaves, hypercomplex number system). Students can also be asked to continue with the 5.2 task to conduct research work on the topic: «Complex numbers and fractals». The proposed assignments relate to a rather complex profile and imply a high level of learning.

4. Conclusion

The proposed approach to the definition of training profiles allowed to develop a methodically substantiated system of tasks to fill the knowledge base of the hybrid intelligent support system of design and research activities of schoolchildren in mathematics based on cognitive modeling When developing the system of assignments, cognitive abilities of students were taken into account, based on the dominant type of thinking and creativity. The set of tasks for each profile was determined taking into account their implementation by students at three levels of project-research activities in the intellectual environment of knowledge: research, and reproductive, empirical and theoretical. During the course of the work, a knowledge base was developed and presented by five clusters of three levels in each cluster. Informative filling of hybrid intellectual system was carried out on the example of realization of project-research tasks on the topic «Complex numbers», which took into account the mastery of the main sections of the topic. This approach to intellectualization of learning systems based on individual needs and learning styles was proposed by authors for the first time.

Refrences

[1] Khasbulatova B. Principles of building intelligent learning systems // E-learning in continuing education 91). 2016. pp. 957–962.

[2] Alyosheva L. Intelligent learning systems // Bulletin of the University, 2018. Vol. 1. pp. 149–155. doi: https:// doi.org/10.26425/1816-4277-2018-1-149-155.

[3] Karpacheva I., Igonina E., Simonovskaya G. Methodological support of a hybrid intellectual system with cognitive modeling of design and research activities based on thinking profiles // Fundamental problems of teaching mathematics, informatics and informatization of educatio : Collection of abstracts of the international scientific conference, Yelets, September 30 – 02 2022. 2022. pp. 139-142.

[4] Grechko M. Cognitive modeling as a tool of adaptive quality management of education // National interests: priorities and security. 2017. Vol 4 (349). doi: https:// doi.org/10. 24891/ni.13.4.725

[5] Samia D., Badji M. An Adaptive E-Learning System based on Student's Learning Styles: An Empirical Study // International Journal of Distance Education Technologies. 2016. Vol. 14. Issue 3. pp. 34-51. doi: https:// doi.org/10.4018/IJDET.2016070103

[6] Ganzen V., Malyshev K, Oginets L. Profile of thinking // A workshop on the psychology of professional activity. St. Petersburg: Peter. 2001. pp. 159–164.

[7] Hall M., Farkas J. Cognitive skills and attitude behavioral characteristics in adolescence and the level of wages // Questions of education. 2013. Vol.3. pp. 5–57.

[8] Ertmer P., Newby T. Behaviorism, Cognitivism, Constructivism: Comparing Critical Features from an Instructional Design Perspective Research // Update on Key Training and Mentoring Topics. 2013. Vol. 26(2). pp. 43–71. doi: https:// doi.org/10.1111/j.1937-8327.1993.tb00605.x

[9] Belousova A. Development of a Personal Potential in Collaborative Thinking Activity Procedia // Social and Behavioral Sciences. 5th ICEEPSY International Conference on Education & Educational Psychology 2015. Vol. 171. pp. 987–994. doi:10.1016/j.sbspro.2015.01.217

[10] Malyshev K. The study of thinking using a multidimensional typological approach // Yaroslavl Pedagogical Bulletin. 2014. Vol. 2(2), pp. 218–222.

[11] Bruner J. Psychology of cognition. Beyond the immediate information: monograph. Moscow: Direct-Media 2008.

[12] Smirnov E., Dvoryatkina S., Shcherbatykh S. Parameters and structure of neural network databases for assessment of learning outcomes // International Journal of Criminology and Sociology. 2020. Vol. 9, pp. 1638–1648. doi: https://doi.org/10.6000/1929-4409.2020.09.188

[13] Dvoryatkina S., Smirnov E., Shcherbatykh S. Intellectual support of project and research activities of schoolchildren in a hybrid environment of teaching mathematics. Yelets. 2021.

[14] Ouchi M. Entertaining mathematics. Complex numbers; per. with yap. S. L. Plekhanova. M.: DMK Press, 2019.

[15] Voitenko T., Tsygankova A. Methodological features of studying the topic "complex numbers" in specialized classes of high school // Modern problems of science and education. 2019. Vol. 4. P. 134.

[16] Mizinova L., Popova S. Application of complex numbers in electrical engineering // International Student Scientific Bulletin. 2018. Vol 3(1). pp. 104-106.