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Impact of Connected Spaces of Neutrosophic Generalized Topology Across Various Disciplines

[1]S. Vijaya, [2]P. Santhi and [3]A. Yuvarani

[1]PG and Research Department of Mathematics, Thiagarajar College, Madurai, Tamil Nadu, India

[2]PG and Research Department of Mathematics, The Standard Fireworks Rajaratnam College for Women, Sivakasi, Tamil Nadu, India.

[3]PG and Research Department of Mathematics, The American College, Madurai, Tamil Nadu, India.

Abstract: - The vital cynosure of this article is to outline the applications of connectedness in Neutrosophic Generalized Topology across various disciplines. For that, initially we delve into the characteristics and properties of ^{N}g-connected and ^{N}g-hyperconnected spaces along with appropriate exemplifications. Subsequently, we instigate ^{N}g-α(β, σ, π)-disconnected spaces, ^{N}g-α(β, σ, π)-hyperconnected spaces and examine its relationships. Along with that, extremally disconnectedness for ^{N}g-TS is studied with essential paradigms.

Keywords*: ^{N}g-separated, ^{N}g-connected, ^{N}g-hyperconnected, ^{N}g-dense, ^{N}g-nowhere dense, ^{N}g-strong, ^{N}g-extremally disconnected.*

2010 Mathematics Subject Classification: 54A05, 54B05, 54D10

1. Introduction

Historically, the interpretation of connectedness notion made its initial advent in the early 20th century over the effort of N.J. Lennes, Frigyes Riesz, and Felix Hausdorff [14]. Connectedness is indeed a fundamental topological property corresponding to the instinctive idea to understand the structure of numerous geometric substances. Here it's why we opted to employ the concept of connectedness, hyperconnectedness and extremally disconnectedness in neutrosophic generalized topology.

Recently in research, the concept of Neutrosophic has risen as an operative tool, due to the intrinsic existence of indeterminacy in various fields of day-to-day life. Researchers can deftly address the challenges modelled by these uncertainties using neutrosophic concepts. During the year 1965, Zadeh [16] proposed the fuzzy set theory as a mode to share out uncertainty in practical situations. Membership elements, non-membership elements in intuitionistic fuzzy sets are considered by Attanassov [1] in 1983. Smarandache captivated his explanations moving towards the degree of uncertainty that directed to Neutrosophic Sets in [5], [6], [7] & [13]. Later, Salama and Albowi [12] familiarized Neutrosophic Topological Spaces.

Csaszar [2] & [3] introduced the concept of Generalized topological spaces and investigated about extremally disconnected spaces. After that, Ekici [4] studied about connectedness and hyperconnectedness in generalized topological spaces and the same is discussed by Karthika M, Smarandache & et al. [8] in neutrosophic complex topological spaces.

By deriving inspiration from their works, in 2020, Murad Arar, Saeid Jafari [9] and Raksha Ben, Hari Siva Annam [10] & [11] presented neutrosophic generalized topological spaces. A short time ago, some Contra Continuous Functions (CCF) in Neutrosophic Generalized Topological Spaces is promoted by Yuvarani, Vijaya and Santhi [15].

The key chore of this article is to delineate the features of ^{N}g-connected, ^{N}g-hyperconnected and ^{N}g- extremally disconnectedness in ^{N}g-TS.

2. Preliminaries

Definition 2.1 [12]

Consider a fixed non-empty set P. Define L = {<ρ, $T_L(\rho)$, $U_L(\rho)$, $F_L(\rho)$) >: ρ ∈ P} is a ^{N}g-subset of P, where $T_L(\rho)$, $U_L(\rho)$ and $F_L(\rho)$ represents the degree of membership, uncertainty and non-membership functions respectively for every ρ ∈ P.

Remark 2.2 [12]

Let L = {<ρ, $T_L(\rho)$, $U_L(\rho)$, $F_L(\rho)$>: ρ ∈ P} be a structured triple ^{N}g-subset on P in $]^-0, 1^+[$.

Remark 2.3 [12]

0_N and 1_N is the ^{N}g-sets in P is considered as below

- 0_N = {<ρ, 0, 0, 1 >: ρ ∈ P}
- 0_N = {<ρ, 0, 1, 1 >: ρ ∈ P}
- 0_N = {<ρ, 0, 1, 0 >: ρ ∈ P}
- 0_N = {<ρ, 0, 0, 0 >: ρ ∈ P}
- 1_N = {<ρ, 1, 0, 0 >: ρ ∈ P}
- 1_N = {<ρ, 1, 0, 1 >: ρ ∈ P}
- 1_N = {<ρ, 1, 1, 0 >: ρ ∈ P}
- 1_N = {<ρ, 1, 1, 1 >: ρ ∈ P}

Definition 2.4 [12]

The complement of L = {< $T_L(\rho)$, $U_L(\rho)$, $F_L(\rho)$>} on P is

- L^c = {<ρ, 1 − $T_L(\rho)$, 1 - $U_L(\rho)$ and 1 - $F_L(\rho)$>: ρ ∈ P}
- L^c = {<ρ, $F_L(\rho)$, $U_L(\rho)$ and $T_L(\rho)$>: ρ ∈ P}
- L^c = {<ρ, $F_L(\rho)$, 1 - $U_L(\rho)$ and $T_L(\rho)$ >: ρ ∈ P}

Definition 2.5 [12]

Let K = {<ρ, $T_K(\rho)$, $U_K(\rho)$, $F_K(\rho)$>: ρ ∈ P} and L = {<ρ, $T_L(\rho)$, $U_L(\rho)$, $F_L(\rho)$>: ρ ∈ P}, P ≠ 0_N. Then

- $K \subseteq L \Rightarrow T_K(\rho) \leq T_L(\rho), U_K(\rho) \leq U_L(\rho), F_K(\rho) \geq F_L(\rho), \forall \rho \in P$
- $K \subseteq L \Rightarrow T_K(\rho) \leq T_L(\rho), U_K(\rho) \geq U_L(\rho), F_K(\rho) \geq F_L(\rho), \forall \rho \in P$

Definition 2.6 [12]

Let K = {<ρ, $T_K(\rho)$, $U_K(\rho)$, $F_K(\rho)$>: ρ ∈ P} and L = {<ρ, $T_L(\rho)$, $U_L(\rho)$, $F_L(\rho)$>: ρ ∈ P} are ^{N}g-subsets of P ≠ 0_N. Then,

- $K \wedge L = <\rho, T_K(\rho) \wedge T_L(\rho), U_K(\rho) \vee U_L(\rho), F_K(\rho) \vee F_L(\rho)>$
- $K \wedge L = <\rho, T_K(\rho) \wedge T_L(\rho), U_K(\rho) \wedge U_L(\rho), F_K(\rho) \vee F_L(\rho)>$
- $K \vee L = <\rho, T_K(\rho) \vee T_L(\rho), U_K(\rho) \wedge U_L(\rho), F_K(\rho) \wedge F_L(\rho)>$
- $K \vee L = <\rho, T_K(\rho) \vee T_L(\rho), U_K(\rho) \vee U_L(\rho), F_K(\rho) \wedge F_L(\rho)>$

Definition 2.7 [11]

A family of ^{N}g-subsets of $P \neq 0_N$ is called as ^{N}g-topology if the following are satisfied.

- $0_N \in \Psi$
- For any $L_1, L_2 \in \Psi$, $L_1 \vee L_2 \in \Psi$.

Remark 2.8 [11]

- Elements of ^{N}g-TS are ^{N}g-open sets (^{N}g-os).
- The complements of ^{N}g-os are ^{N}g-closed sets (^{N}g-cs).
- Both ^{N}g-open and ^{N}g-closed set is said to be a ^{N}g-clopen set.

Definition 2.9 [11]

Let (P, Ψ) be a ^{N}g-TS and $L = \{<\rho, T_L(\rho), U_L(\rho), F_L(\rho)>\}$ be a ^{N}g-subset in P. Then

- ^{N}g-Closure (L) = $\wedge\{W : L \subseteq W$, W is ^{N}g-cs$\}$.
- ^{N}g-Interior (L) = $\vee\{T : T \subseteq L$, T is ^{N}g-os$\}$.

Definition 2.10 [10]

In a ^{N}g-TS, a ^{N}g-subset T is

- ^{N}g-α-open set (^{N}g-αos) if $T \subseteq {}^N g\text{-}i({}^N g\text{-}c({}^N g\text{-}i(T)))$,
- ^{N}g-σ-open set (^{N}g-σos) if $T \subseteq {}^N g\text{-}c({}^N g\text{-}i(T))$
- ^{N}g-π-open set (^{N}g-πos) if $T \subseteq {}^N g\text{-}i({}^N g\text{-}c(T))$,
- ^{N}g-β-open set (^{N}g-βos) if $T \subseteq {}^N g\text{-}c({}^N g\text{-}i({}^N g\text{-}c(T)))$,
- ^{N}g-regular open set (^{N}g-ros) if $T = {}^N g\text{-}i({}^N g\text{-}c(T))$.

Lemma 2.11 [10]

Each ^{N}g-α-open set is ^{N}g-σ-open set and ^{N}g-π-open set.

Definition 2.12 [10]

The function $\xi : (P_1,\Psi_1) \rightarrow (P_2,\Psi_2)$ is ^{N}g-Cts (^{N}g-SCts, ^{N}g-PCts, ^{N}g-αCts, ^{N}g-βCts) if ξ^{-1} of ^{N}g-cs in (P_2,Ψ_2) is a ^{N}g-cs (^{N}g-σcs, ^{N}g-πcs, ^{N}g-αcs, ^{N}g-βcs) in (P_1,Ψ_1).

Definition2.13 [14]

Let (P_1,Ψ_1) and (P_2,Ψ_2) be ^{N}g-TSs. Then ξ: $P_1 \rightarrow P_2$is said to be contra ^{N}g-continuous function (^{N}g-CCF) [contra ^{N}g-α-continuous function (^{N}g-αCCF), contra ^{N}g-σ-continuous function (^{N}g-σCCF), contra ^{N}g-π-continuous function (^{N}g-πCCF), contra ^{N}g-β-continuous function (^{N}g-βCCF)] if for each ^{N}g-os M in P_2, $\xi^{-1}(M)$ is a ^{N}g-cs [^{N}g-αcs, ^{N}g-σcs, ^{N}g-πcs, ^{N}g-βcs] in P_1.

3. ^{N}g-Connected and ^{N}g-Hyperconnected Spaces

Definition 3.1

Let $K \neq 0_N$ and $L \neq 0_N$ are disjoint ^{N}g-subsets of a ^{N}g-TS (P, Ψ). Then they are said to be ^{N}g-separated if $K \wedge {}^N g\text{-}c(L) = 0_N = {}^N g\text{-}c(K) \wedge L$.

Definition 3.2

In ^{N}g-TS (P, Ψ), a ^{N}g-subset M of is called ^{N}g-disconnected if ∃ ^{N}g-os K & L of (P, Ψ) ∋ $(M \wedge K) \vee (M \wedge L) = 1_N$ and $(M \wedge K) \wedge (M \wedge L) = 0_N$.

Definition 3.3

A ^{N}g-TS (P, Ψ) is said to be ^{N}g-disconnected [^{N}g-α-disconnected, ^{N}g-σ-disconnected, ^{N}g-π-disconnected, ^{N}g-β-disconnected] if there exists ^{N}g-os [^{N}g-αos, ^{N}g-σos, ^{N}g-πos, ^{N}g-βos] $K \neq 0_N$, $L \neq 0_N$ and $K \wedge L = 0_N$ of P such that $K \vee L = 1_N$.

P is said to be ^{N}g-connected if it is not ^{N}g-disconnected.

Definition 3.4

If a ^{N}g-TS (P, Ψ) has no proper ^{N}g-clopen subsets, then it is called as ^{N}g-connected space.

Theorem 3.5

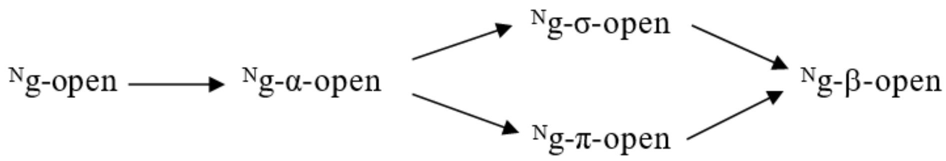
The function ξ: $(P_1,\Psi_1) \rightarrow (P_2,\Psi_2)$ be a contra ^{N}g-α (^{N}g-σ, ^{N}g-π,^{N}g-β) continuous surjection and let P_1 be ^{N}g-α (^{N}g-σ, ^{N}g-π, ^{N}g-β) connected. Then P_2 is ^{N}g-connected.

Proof :

Let ξ: $(P_1,\Psi_1) \rightarrow (P_2,\Psi_2)$ be a contra ^{N}g-α (^{N}g-σ, ^{N}g-π, ^{N}g-β) continuous surjection and let P_1 be ^{N}g-α (^{N}g-σ, ^{N}g-π, ^{N}g-β) connected. Presume that P_2 is not ^{N}g-connected. Then ∃ ^{N}g-os, $Q_1 \neq 0_N$, $Q_2 \neq 0_N$, $Q_1 \wedge Q_2 = 0_N$ of P_2 ∋ $Q_1 \vee Q_2 = 1_N \Rightarrow Q_1, Q_2 \in$ ^{N}g-cs(P_2). As ξ is contra ^{N}g-α (^{N}g-σ, ^{N}g-π, ^{N}g-β) continuous, then $\xi^{-1}(Q_1)$, $\xi^{-1}(Q_2)$ are ^{N}g-α (^{N}g-σ, ^{N}g-π, ^{N}g-β) open. It is known that $\xi^{-1}(Q_1) \wedge \xi^{-1}(Q_2) = 0_N$ and $\xi^{-1}(Q_1) \vee \xi^{-1}(Q_2) = 1_N$. As a result, P_1 is not ^{N}g-α (^{N}g-σ, ^{N}g-π, ^{N}g-β) connected, which contradicts the hypothesis. Consequently, P_2 is ^{N}g-connected.

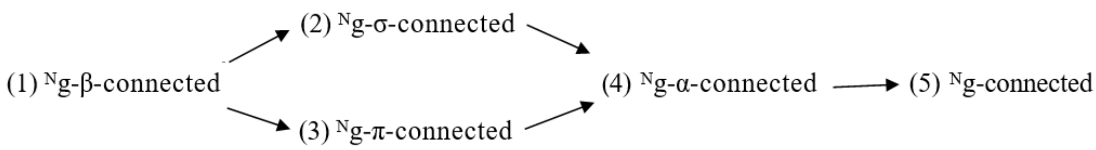
Remarks 3.6

Consider $A \subseteq P$ in a ^{N}g-TS (P,Ψ). The implications given below holds validity.



Theorem 3.7

Consider (P,Ψ) be a ^{N}g-TS. The implications given below holds validity.



Proof :

(1) ⇒ (2). Consider P be ^{N}g-β-connected. Assume P is not ^{N}g-σ-connected. Then ∃ ^{N}g-σos $V_1 \neq 0_N$, $V_2 \neq 0_N$, $V_1 \wedge V_2 = 0_N$ of P ∋ $V_1 \vee V_2 = P$. From Remark 3.6, ^{N}g-σ-open ⇒ ^{N}g-β-open, ∃ ^{N}g-βos $V_1 \neq 0_N$, $V_2 \neq 0_N$, $V_1 \wedge V_2 = 0_N$ of P such that $V_1 \vee V_2 = P$. Then P is not ^{N}g-β-connected. This contradiction proves that P is ^{N}g-σ-connected.

The proof of the remaining implications is as above.

Theorem 3.8

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In a ^{N}g-TS (P, Ψ), the union of two nonempty ^{N}g-separated subsets is ^{N}g-disconnected.

Proof :

In a ^{N}g-TS (P, Ψ), consider two ^{N}g-separated subsets $K \neq 0_N$ and $L \neq 0_N$ such that $K \wedge {}^{N}g\text{-}c(L) = 0_N = {}^{N}g\text{-}c(K) \wedge L$. Let $R = ({}^{N}g\text{-}c(L))^c$ and $S = ({}^{N}g\text{-}c(K))^c$. Then R & S are ^{N}g-os. Also $(K \vee L) \wedge R = 1_N$, $(K \vee L) \wedge S = 1_N$ are disjoint nonempty ^{N}g-subsets with union as $K \vee L$. Accordingly, R & S form a ^{N}g-disconnection of $K \vee L$. Henceforth $K \vee L$ is ^{N}g-disconnected.

Theorem 3.9

(P, Ψ) is ^{N}g-disconnected & $\Psi^* \supseteq \Psi \Rightarrow (P, \Psi^*)$ is ^{N}g-disconnected.

Proof :

Consider (P, Ψ) is ^{N}g-disconnected. Take two ^{N}g-os, $K \neq 0_N \in \Psi$ and $L \neq 0_N \in \Psi$, which gives $K \wedge L = 0_N$ & $K \vee L = 1_N$. Given that $\Psi^* \supseteq \Psi$ and $K, L \in \Psi$, then $K, L \in \Psi^*$. Hence $K \wedge L = 0_N$ and $K \vee L = 1_N \Rightarrow (P, \Psi^*)$ is ^{N}g-disconnected.

Theorem 3.10

A ^{N}g-TS (P, Ψ) is ^{N}g-connected iff there exists no nonempty ^{N}g-os K & L of P ∋ $K = L^c$.

Proof :

Necessity : Consider two ^{N}g-os K and L such that $K \neq 0_N$, $L \neq 0_N$ and $K = L^c$. As L is a ^{N}g-os, $L^c = K$ is a ^{N}g-cs and $L \neq 0_N \Rightarrow L^c \neq 1_N$, that is, $K \neq 1_N$. Accordingly, ∃ a proper ^{N}g-os K as $K \neq 0_N$ and $K \neq 1_N$ ∋ K is ^{N}g-clopen, which contradicts that (P, Ψ) is ^{N}g-connected space.

Sufficiency : Consider (P, Ψ) be a ^{N}g-TS and K is ^{N}g-clopen in P ∋ $K \neq 0_N$ & $K \neq 1_N$. Here $K = L^c$. Now, L is ^{N}g-os and $K \neq 1_N \Rightarrow L = K^c \neq 0_N$, ⇒⇐ to the assumption. Hence, ∃ no proper ^{N}g-subset in K which is ^{N}g-clopen. As a result, (P, Ψ) is ^{N}g-connected space.

Definition 3.11

In the ^{N}g-TS (P, Ψ), a ^{N}g-subset K is

(i) ^{N}g-dense if $^{N}g\text{-}c(K) = 1_N$.

(ii) ^{N}g-nowhere dense if $^{N}g\text{-}i(^{N}g\text{-}c(K)) = 0_N$.

Definition 3.12

A ^{N}g-TS (P, Ψ) is called as ^{N}g-hyperconnected (^{N}g-α, ^{N}g-σ, ^{N}g-π, ^{N}g-β)-hyperconnected if every ^{N}g-os (^{N}g-α-os, ^{N}g-σ-os, ^{N}g-π-os, ^{N}g-β-os) $U \neq 0_N$ of P is ^{N}g-dense.

Example 3.13

Let P = {T_1, T_2} and Ψ = {0_N, K_1, K_2, K_3, K_4} where

K_1 ={<T_1,0.2,0.4,0.3>, <T_2,0.5,0.1,0.0>, T_1, T_2 ∈ P},

K_2 ={<T_1,0.1,0.5,0.6>, <T_2,0.4,0.2,0.4>, T_1, T_2 ∈ P},

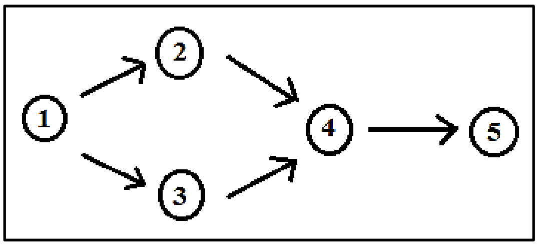
K_3 ={<T_1,0.2,0.5,0.3>, <T_2,0.5,0.2,0.0>, T_1, T_2 ∈ P},

K_4 ={<T_1,0.1,0.4,0.6>, <T_2,0.4,0.1,0.4>, T_1, T_2 ∈ P}.

P is ^{N}g-hyperconnected, since $^{N}g\text{-}c(K_1) = {}^{N}g\text{-}c(K_2) = {}^{N}g\text{-}c(K_3) = {}^{N}g\text{-}c(K_4) = 1_N$.

Theorem 3.14

Consider a ^{N}g-TS (P,Ψ). Then the given below implications holds validity.



where, 1 : ^{N}g-β-hyperconnected space

2 : ^{N}g-σ-hyperconnected space

3 : ^{N}g-π-hyperconnected space

4 : ^{N}g-α-hyperconnected space

5 : ^{N}g-hyperconnected space

Proof :

(1) ⇒ (2). Let P be ^{N}g-β-hyperconnected space and $K \neq 0_N$ be a ^{N}g-σos of P. By Remarks 3.6, K is a ^{N}g-βos. As P is ^{N}g-β-hyperconnected space ⇒ ${}^{N}g\text{-}c(K) = 1_N$ ⇒ P is ^{N}g-σ-hyperconnected space.

The proof of the remaining implications can be done as above.

Theorem 3.15

In a ^{N}g-TS (P, Ψ), the given below properties are equivalent.

(i) P is ^{N}g-hyperconnected space.

(ii) 0_N and 1_N are the only ^{N}g-ros in P.

Proof

(i) ⇒ (ii) If $K \neq 0_N$ is a ^{N}g-ros in a ^{N}g-hyperconnected space (P, Ψ), then $K = {}^{N}g\text{-}i({}^{N}g\text{-}c(K)) \Rightarrow ({}^{N}g\text{-}i({}^{N}g\text{-}c(K)))^{c} = 1_N - ({}^{N}g\text{-}i({}^{N}g\text{-}c(K))) = {}^{N}g\text{-}c[1_N - {}^{N}g\text{-}c(K)] = {}^{N}g\text{-}c(K^{c}) = K^{c} \neq 1_N$, which contradicts the hypothesis. As a result, 0_N and 1_N are the only ^{N}g-ros in P.

(ii) ⇒ (i) The only ^{N}g-ros in P are 0_N and 1_N. Presume, P is not a ^{N}g-hyperconnected space. Then ∃ a ^{N}g-os $K \neq 0_N$ of P ∋ ${}^{N}g\text{-}c(K) \neq 1_N \Rightarrow {}^{N}g\text{-}c({}^{N}g\text{-}i(K)) \neq 1_N$. By hypothesis, ${}^{N}g\text{-}c({}^{N}g\text{-}i(K)) = 0_N$. This gives ${}^{N}g\text{-}c(K) = 0_N$, a contradiction to $K \neq 0_N$. Hence, P is ^{N}g-hyperconnected space.

Theorem 3.16

A ^{N}g-TS (P, Ψ) is ^{N}g-hyperconnected iff every ^{N}g-subsets of P is either ^{N}g-dense or ^{N}g-nowhere dense.

Proof

Let $K \subseteq 1_N$ be any ^{N}g-subsets of a ^{N}g-hyperconnected space P. Assume K is not ^{N}g-nowhere dense. Then ${}^{N}g\text{-}c(1_N - {}^{N}g\text{-}c(K)) = 1_N - ({}^{N}g\text{-}i({}^{N}g\text{-}c(K))) \neq 1_N$. Since ${}^{N}g\text{-}i({}^{N}g\text{-}c(K)) \neq 0_N$, this implies that ${}^{N}g\text{-}c({}^{N}g\text{-}i({}^{N}g\text{-}c(K))) = 1_N \subseteq {}^{N}g\text{-}c(K)$ and so ${}^{N}g\text{-}c(K) = 1_N$. As a result of this, K is ^{N}g-dense set. Conversely, let $K_1 \neq 0_N$ be any ^{N}g-os in P. Then $K_1 \subset {}^{N}g\text{-}i({}^{N}g\text{-}c(K_1)) \Rightarrow K_1$ is not ^{N}g-nowhere dense. Then, K_1 is ^{N}g-dense set, by hypothesis.

Proposition 3.17

If K_1 and K_2 are ^{N}g-σos in a ^{N}g-hyperconnected space (P, Ψ), then $K_1 \wedge K_2$ is ^{N}g-σos.

Proof

Let $K_1 \neq 0_N$ and $K_2 \neq 0_N$ be any two ^{N}g-σos in a ^{N}g-hyperconnected space P. Then, $K_1 \subseteq {}^N g\text{-}c[{}^N g\text{-}i[K_1]]$ and $K_2 \subseteq {}^N g\text{-}c[{}^N g\text{-}i[K_2]]$. It follows that, ${}^N g\text{-}c(K_1) = {}^N g\text{-}c[{}^N g\text{-}i[K_1]] = 1_N$ and ${}^N g\text{-}c(K_2) = {}^N g\text{-}c[{}^N g\text{-}i[K_2]] = 1_N$. Also, $K_1 \wedge K_2 \neq 0_N$. Consequently, ${}^N g\text{-}c[{}^N g\text{-}i[K_1 \wedge K_2]] = {}^N g\text{-}c[{}^N g\text{-}i[K_1]] \wedge {}^N g\text{-}c[{}^N g\text{-}i[K_2]] = 1_N \Rightarrow K_1 \wedge K_2 \subseteq {}^N g\text{-}c[{}^N g\text{-}i[K_1]] \wedge {}^N g\text{-}c[{}^N g\text{-}i[K_2]] = {}^N g\text{-}c[{}^N g\text{-}i[K_1 \wedge K_2]]$. Hence $K_1 \wedge K_2$ is ^{N}g-σos.

Theorem 3.18

Consider a ^{N}g-TS (P, Ψ), where ${}^N g\text{-}c(0_N) = 0_N$. Then the given below properties are equivalent.

1) (P, Ψ) is ^{N}g-hyperconnected,
2) For every ^{N}g-πos $0_N \neq H \subset 1_N$, H is ^{N}g-dense,
3) For every ^{N}g-πos $0_N \neq H \subset 1_N$, ${}^N g\text{-}\sigma c(H) = 1_N$,
4) For every ^{N}g-σos $0_N \neq H \subset 1_N$, ${}^N g\text{-}\pi c(H) = 1_N$.

Proof

(1) ⇒ (2). Let $H \neq 0_N$ be a ^{N}g-πos of ^{N}g-hyperconnected space (P, Ψ), then $0_N \neq H \subset {}^N g\text{-}i[{}^N g\text{-}c[H]]$. Consequently, ${}^N g\text{-}c(H) = {}^N g\text{-}c({}^N g\text{-}i({}^N g\text{-}c(H))) = 1_N$.

(2) ⇒ (3). Suppose ∃ a ^{N}g-πos $H \neq 0_N \ni {}^N g\text{-}\sigma c(H) \neq 1_N$, then ∃ ^{N}g-σos $A \neq 0_N \ni A \wedge H = 0_N$. Thus ${}^N g\text{-}i(A) \wedge H = 0_N$. By (ii), $0_N = {}^N g\text{-}i(A) \wedge {}^N g\text{-}c(H) = {}^N g\text{-}i(A)$, which contradicts the assumption.

(3) ⇒ (4). Presume that ∃ a nonempty ^{N}g-σos $H \ni {}^N g\text{-}\pi c(H) \neq 1_N$. Then ∃ a ^{N}g-πos $A \neq 0_N \ni A \wedge H = 0_N$. Consequently, $A \wedge {}^N g\text{-}i(H) = 0_N$. It follows that $0_N = {}^N g\text{-}c(A) \wedge {}^N g\text{-}i(H) \supset {}^N g\text{-}\sigma c(A) \wedge {}^N g\text{-}i(H) = {}^N g\text{-}i(H)$ by (iii), which contradicts the hypothesis.

(4) ⇒ (1). Let $H \neq 0_N$ be a ^{N}g-os of (P, Ψ). Since H is ^{N}g-σos, by (iv), ${}^N g\text{-}c(H) \supset {}^N g\text{-}\pi c(H) = 1_N$. As a result, (P, Ψ) is ^{N}g-hyperconnected.

Theorem 3.19

Consider a ^{N}g-strong space (P, Ψ). Then the properties given below are equivalent.

(1) P is ^{N}g-hyperconnected space.

(2) For each ^{N}g-σos $K \neq 0_N$ and ^{N}g-πos $L \neq 0_N$, $K \wedge L \neq 0_N$.

Proof

(1) ⇒ (2). Assume P is ^{N}g-hyperconnected space. Consider $L \neq 0_N$ is a ^{N}g-πos and $K \neq 0_N$ be a ^{N}g-σos $\ni K \wedge L = 0_N$. By theorem 3.18, ${}^N g\text{-}\sigma c(L) = 1_N$. Now, $1_N = {}^N g\text{-}\sigma c(L) \subseteq {}^N g\text{-}\sigma c(1_N - K) = 1_N - K \Rightarrow K = 0_N$, which contradicts the hypothesis.

(2) ⇒ (1). Presume, $K \neq 0_N$ and $L \neq 0_N$ are ^{N}g-os of P. By (2), $K \wedge L \neq 0_N \Rightarrow$ P is a ^{N}g-hyperconnected space.

Theorem 3.20

The following are equivalent in a strong ^{N}g-TS (P, Ψ).

(i) P is ^{N}g-hyperconnected space.

(ii) For every nonempty $L \in {}^N$g-σos, ${}^N g\text{-}\beta c(L) = 1_N$.

(iii) For every nonempty $L \in {}^N$g-σos, ${}^N g\text{-}\pi c(L) = 1_N$.

Proof

(i) ⇒ (ii). Let $L \neq 0_N$ be a ^{N}g-σos of ^{N}g-hyperconnected space (P, Ψ). Then ${}^N g\text{-}i(L) \neq 0_N \Rightarrow {}^N g\text{-}c({}^N g\text{-}i(L)) = 1_N$. Hence ${}^N g\text{-}\beta c(L) \supset L \vee {}^N g\text{-}i({}^N g\text{-}c({}^N g\text{-}i(L))) = L \vee {}^N g\text{-}i(1_N) = 1_N \Rightarrow {}^N g\text{-}\beta c(L) = 1_N$.

(ii) ⇒ (iii). This result can be proved obviously.

(iii) ⇒ (i). If $L \neq 0_N$ is a ^{N}g-os of (P, Ψ), then L is ^{N}g-σos and so by assumption, ^{N}g-πc(L) = 1_N. Since ^{N}g-os ⊂ ^{N}g-πos, ^{N}g-πc(L) ⊂ ^{N}g-c(L) and so ^{N}g-c(L) = 1_N ⇒ P is ^{N}g-hyperconnected space.

Theorem 3.21

The following are equivalent in a ^{N}g-TS (P, Ψ) where ^{N}g-c(0_N) = 0_N.

(i) (P, Ψ) is ^{N}g-hyperconnected space.

(ii) For every ^{N}g-βos $0_N \neq K \subset 1_N$, K is ^{N}g-dense.

(iii) For every ^{N}g-βos $0_N \neq K \subset 1_N$, ^{N}g-σc(K) = 1_N

Proof

(i) ⇒ (ii). Assume $K \neq 0_N$ is a ^{N}g-βos of ^{N}g-hyperconnected space (P, Ψ). This follows that ^{N}g-i[^{N}g-c(K)] ≠ 0_N. Hence, 1_N = ^{N}g-c(^{N}g-i(^{N}g-c(K))) = ^{N}g-c(K).

(ii) ⇒ (iii). Let $K \neq 0_N$ be any ^{N}g-βos of (P, Ψ). Then, ^{N}g-σc(K) = K ∨ ^{N}g-i[^{N}g-c(K)] = K ∨ ^{N}g-i(1_N) = 1_N.

(iii) ⇒ (i). Let $K \neq 0_N$ be any ^{N}g-os of (P, Ψ). From (iii), ^{N}g-σc(K) = K ∨ ^{N}g-i[^{N}g-c(K)] = 1_N. Thus ^{N}g-c(K) = 1_N. Accordingly, (P, Ψ) is ^{N}g-hyperconnected.

Corollary 3.22

The following properties are equivalent in a ^{N}g-TS (P, Ψ) where ^{N}g-c(0_N) = 0_N.

(i) (P, Ψ) is ^{N}g-hyperconnected space,

(ii) $K \wedge L \neq 0_N$ for each ^{N}g-σos $K \neq 0_N$ & ^{N}g-βos $L \neq 0_N$ of (P, Ψ),

(iii) $K \wedge L \neq 0_N$ for each ^{N}g-os $K \neq 0_N$ & ^{N}g-βos $L \neq 0_N$ of (P, Ψ).

Theorem 3.23

The following properties are equivalent in a ^{N}g-strong TS (P, Ψ).

(i) P is ^{N}g-hyperconnected space.

(ii) For each ^{N}g-σos $K \neq 0_N$ & ^{N}g-βos $L \neq 0_N$, $K \wedge L \neq 0_N$

(iii) For each ^{N}g-σos $K \neq 0_N$ & $L \neq 0_N$, $K \wedge L \neq 0_N$

Proof

Using Corollary 3.22, the results can be proved apparently.

4. ^{N}g-Extremally disconnected spaces

Definition 4.1

Let (P, Ψ) be a ^{N}g-TS. P is said to be ^{N}g-extremally disconnected if the ^{N}g-*closure* of every ^{N}g-os is ^{N}g-os in P.

Definition 4.2

A ^{N}g-TS (P, Ψ) is ^{N}g-strong if $1_N \in \Psi$. Undoubtedly, (P, Ψ) is ^{N}g-strong iff ^{N}g-c(0_N) = 0_N, when 0_N is closed.

Theorem 4.3

If a ^{N}g-TS (P, Ψ) is ^{N}g-extremally disconnected, then it is ^{N}g-strong.

Proof

It is known that ^{N}g-c(0_N) = 1_N - ^{N}g-i(1_N) = 1_N - $\mathcal{M}_{g^N}$, where $\mathcal{M}_{g^N}$ = ∨{K / K ∈ ^{N}g-os} ≠ 1_N. Now, ^{N}g-i(^{N}g-c(0_N)) = ^{N}g-i(1_N - $\mathcal{M}_{g^N}$) = 0_N. As (P, Ψ) is a ^{N}g-extremally disconnected space and 0_N is ^{N}g-os, ^{N}g-i(^{N}g-c(0_N)) = ^{N}g-c(0_N). Hence ^{N}g-c(0_N) =0_N.

Theorem 4.4

A ^{N}g-TS (P, Ψ) is ^{N}g-extremally disconnected if it is both ^{N}g-strong and ^{N}g-hyperconnected space.

Proof

Let K ≠ 0_N be a ^{N}g-os of P. Since P is ^{N}g-hyperconnected, ^{N}g-c(K)= 1_N. As P is ^{N}g-strong, 1_N ∈ Ψ and so ^{N}g-c(K) is ^{N}g-os. Therefore, P is ^{N}g-extremally disconnected.

Remark 4.5

The following illustration shows that the converse of Theorem 4.4 may not be true.

Example 4.6

Let P ={T}with Ψ = {0_N, K_1, K_2, K_3, K_4} where

K_1 ={<T,0.5,0.7,0.2>; T ∈ P}, K_2 ={<T,0.2,0.7,0.5>; T ∈ P},

K_3 ={<T,0.2,0.3,0.5>; T ∈ P}, K_4 ={<T,0.5,0.3,0.2>; T ∈ P}

Here ^{N}g-c(K_1) = K_3, ^{N}g-c(K_2) = K_4, ^{N}g-c(K_3) = K_1 and ^{N}g-c(K_4) = K_2 are ^{N}g-os but not ^{N}g-dense. This illustration shows that (P, Ψ) is ^{N}g-extremally disconnected but not ^{N}g-hyperconnected.

Preposition 4.7

A ^{N}g-TS (P, Ψ) is ^{N}g-extremally disconnected iff ^{N}g-c(K) ∧ ^{N}g-c(L) = 0_N, for ^{N}g-os's K and L, with K ∧ L = 0_N.

Theorem 4.8

A ^{N}g-TS (P, Ψ) is ^{N}g-extremally disconnected space iff for each ^{N}g-os K and ^{N}g-cs L with K ⊆ L, ∃ a ^{N}g-os K_1 and ^{N}g-cs L_1 ∋ K ⊆ L_1 ⊆ K_1 ⊆ L.

Proof

Let K be a ^{N}g-os and L be a ^{N}g-cs ∋ K ⊆ L, in the ^{N}g-extremally disconnected space (P, Ψ). Then K ∧ L^c = 0_N. By Preposition 4.8, ^{N}g-c(K) ∧ ^{N}g-c(L^c) = 0_N, i.e. ^{N}g-c(K) ⊆ (^{N}g-c(L^c))^c. Using (^{N}g-c(L^c))^c ⊆ L, ^{N}g-c(K) = L_1 and (^{N}g-c(L^c))^c = K_1, it is obtained that K ⊆ L_1 ⊆ K_1 ⊆ L.

Conversely, let the condition hold. Let K ≠ 0_N and L ≠ 0_N be two ^{N}g-os in P. Then, K ⊆ L^c and L^c is ^{N}g-cs. Consequently, ∃ a ^{N}g-os G and a ^{N}g-cs F ∋ K ⊆ F ⊆ G ⊆ L^c, ⇒ ^{N}g-c(K) ∧ (^{N}g-i(L^c))^c = 0_N. But (^{N}g-i(L^c))^c = ^{N}g-c(L). From Preposition 4.7, ^{N}g-c(K) ∧ ^{N}g-i(L) = 0_N, and so P is ^{N}g-extremally disconnected space.

Theorem 4.9

A ^{N}g-TS (P, Ψ) is ^{N}g-extremally disconnected space ⟺ for each ^{N}g-rcs of P is ^{N}g-os in P.

Proof

Let H be a ^{N}g-rcs in ^{N}g-extremally disconnected space (P, Ψ). Then H = ^{N}g-c(^{N}g-i(H)). Since ^{N}g-i(H) is ^{N}g-os, by hypothesis, ^{N}g-c(^{N}g-i(H)) is ^{N}g-os and so ^{N}g-i(H) = ^{N}g-i(^{N}g-c(^{N}g-i(H))) = ^{N}g-c(^{N}g-i(H)) = H. Hence H is ^{N}g-os.

Conversely, presume the condition holds. Let L be a ^{N}g-os of P. Since ^{N}g-c(L) = ^{N}g-c(^{N}g-i(L)) is ^{N}g-rcs, ^{N}g-i(^{N}g-c(L)) = ^{N}g-c(L). Thus, ^{N}g-c(L) is ^{N}g-os and as a result, (P, Ψ) is ^{N}g-extremally disconnected.

Theorem 4.10

A ^{N}g-TS (P, Ψ) is ^{N}g-extremally disconnected iff ^{N}g-σos ⊂ ^{N}g-πos ⇔ ^{N}g-αos = ^{N}g-σos.

Proof

Assume that (P, Ψ) is ^{N}g-extremally disconnected. If K ∈ ^{N}g-σos, then K ⊂ ^{N}g-*c*(^{N}g-*i*(K)). Since P is ^{N}g-extremally disconnected, K ⊂ ^{N}g-*c*(^{N}g-*i*(K)) = ^{N}g-*i*(^{N}g-*c*(^{N}g-*i*(K))) ⊂ ^{N}g-*i*(^{N}g-*c*(K)) and so K ∈ ^{N}g-πos ⇒ ^{N}g-σos ⊂ ^{N}g-πos.

Conversely, let L be a ^{N}g-rcs in (P, Ψ). Then L = ^{N}g-*c*(^{N}g-*i*(L)) ⇒ L∈ ^{N}g-σos ⇒ L∈ ^{N}g-πos. Accordingly, L ⊂ ^{N}g-*i*(^{N}g-*c*(L)) = ^{N}g-*i*(^{N}g-*c*(^{N}g-*c*(^{N}g-*i*(L)))) = ^{N}g-*i*(^{N}g-*c*(^{N}g-*i*(L))) = ^{N}g-*i*(L) and so L is ^{N}g-os. As per Theorem 4.9, (P, Ψ) is ^{N}g-extremally disconnected. Using Lemma 2.11, ^{N}g-σos ⊂ ^{N}g-πos ⇔ ^{N}g-αos = ^{N}g-σos.

Theorem 4.11

Consider K as a subset of a ^{N}g-TS (P, Ψ). Then the below given statements hold.

(i) Each nonempty ^{N}g-σos has nonempty ^{N}g-interior, if P is ^{N}g-strong.

(ii) Every set having nonempty ^{N}g- interior is ^{N}g-σos, if P is ^{N}g-hyperconnected.

Proof

(i) Consider a ^{N}g-σos K ≠ 0_N. Then K ⊂ ^{N}g-*c*(^{N}g-*i*(K)). If ^{N}g-*i*(K) = 0_N, then K ⊂ ^{N}g-*c*(0_N) = 0_N, which is a contradiction. Therefore ^{N}g-*i*(K) ≠ 0_N.

(ii) Consider a ^{N}g-set K ≠ 0_N containing a ^{N}g-os U ≠ 0_N. As P is ^{N}g-hyperconnected space and U is ^{N}g-dense, ^{N}g-*c*(U) = 1_N and so U ⊂ K ⊂ ^{N}g-*c*(U) = 1_N. Hence K is ^{N}g-σos.

5. Applications

The following illustrates the essential role of connectedness in Neutrosophic Topological spaces across different fields such as Transportation, Geographical Environmental Spaces, Modeling Physical Occurrences, Knot theory, Epidemiology and Cosmology.

Establishing connectivity is crucial when planning transportation networks, especially in road and rail systems or public transport. The mobility of People and goods can be systematic by a well-connected transportation network. Efficient transportation infrastructure supports in minimizing travel time, soothing traffic congestion and enhancing convenience for all inhabitants. It is essential to create a connected network of pathways and sidewalks while restructuring urban to smart city. Ensuring such proper connected sidewalks throughout a city or neighborhood diminishes the reliance on automobiles, and promotes healthier lifestyles.

Within the realm of smart cities, connectivity extends to include the distribution of data and information. To monitor traffic, air quality, and other aspects of urban life connected sensor-based networks is essential which helps to improve the city management system in a better manner.

Connectedness helps study the continuity and accessibility of spaces in construction, especially the architectural planning of urban/metropolitan/smart cities and geographical or spatial information systems.

While planning for disaster management, connectedness is significant to make sure that urgent situation services can reach affected regions and that people can move out efficiently. Connected flora spaces, greenery parks, and avenues festooned with trees contribute to sustainability of environment reducing the urban heat and supporting biodiversity.

In engineering and physics domains, the idea of connectedness is significant as it plays a crucial role in understanding the properties of physical systems. For instance, in the field of solid-state physics and quantum physics, the notion of connectedness is straightly linked with the electrical conductivity of materials.

Connectedness in topology derived from knot theory holds application in the analysis of physical systems like liquid crystals and plasma physics. We use connected graphs in knot theory to develop Ising models for investigating the way particles interaction. In disease spread modeling, weak connectivity can indicate the possibility for indirect transmission of infections via contact through intermediate carriers.

Connected topology has impact in the learning of the large-scale structure of the space. Cosmic topology pertains to the spatial distribution of galaxies and it can be examined via topological approaches.

In Neutrosophic topology, the concept of connectedness provides a framework for comprehending the unique properties and phenomena that arise in the aforementioned systems.

6. Conclusion

This chapter outlined the vital part of connectedness in Neutrosophic Generalized Topology in a multitude of fields. For that, the features of ^{N}g-connected and ^{N}g-hyperconnected spaces are deliberated along with suitable exemplifications. Afterward, ^{N}g-α(β, σ, π)-disconnected spaces, ^{N}g-α(β, σ, π)-hyperconnected spaces and ^{N}g-extremally disconnected spaces are instigated and studied its associations with other notions. The concept of connectedness in Neutrosophic Generalized Topology offers a mathematical structure and researchers can utilize this framework to provide solutions of the problems concerning indeterminacy.

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