Free Vibration of Visco-elastic Orthotropic Skew Plate with Thickness and Thermal Effect

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Abstract

A simple model presented here is to study the effect of bi-linear thickness variation on vibration of visco-elastic orthotropic skew plate having simply supported boundary condition on all the four edges. Using the separation of variables method, the governing differential equation has been solved for vibration of visco-elastic orthotropic skew plate. Anapproximate but quite convenient frequency equation is derived by using Rayleigh-Ritz technique with a two-term deflection function. The frequencies corresponding to the first two modes of vibration has been calculated for a simple supported visco-elastic skew plate for various values of taper constant and thermal gradient with the help of MAPPLE software. (today's computational software).

Keywords: Vibration, skew plate, thickness, taper constant, thermal gradient, non-homogeneity constant.

1. INTRODUCTION

Vibration is the mechanical oscillations of an item about an equilibrium factor. The oscillations may be normal which includes the motion of a pendulum or random including the movement of a tire on agravel street. Vibration of Plates offers a comprehensive, self-contained advent to vibration theory andevaluation of two-dimensional plates. Vibrations are encountered in many mechanical and structural applications, for example, mechanisms and machines, homes, bridges, motors, and plane. The effectsof thermally induced vibrations in large machines have always been a most important concern in the field of science and technology. It is desirable to design such large machines for smooth operation with organized vibrations. This analysis is also beneficial for civil and architectural engineers to build earthquake resistant constructions. Also, the tapered plates i.e. the plates with varying thickness variations are frequently used in many engineering and industrial applications.

In recent years, owing to the diversification of engineering materials and for operations in several thermal environments i.e. nuclear weapons, missiles, defense weapons, laser weapons etc. thermal problems have become very important for modern designers and researchers. The classical theory of vibration (which deals with the effect of thermal gradient on vibration) has attracted the attention of many researchers because of its extensive use in diverse fields. The plates of variable thickness are frequently used as structural components and their vibration characteristics are important for practical

design. The structural components are used in many applications involving aerospace, submarine structural, civil engineering structures etc. Depending upon the requirement, durability and reliabilitymaterials are being developed so that these may provide better strength, efficiency and economy. Therefore a study of character and

Behavior of these plates is required so that the full potential of these plates may be used. These platesmay be of any type for example-rectangular plate, square plate, triangular plate. Also, the thickness ofthese plates affects the behavior of plates.

In last few years, a lot of research has been performed in the field of vibration of plate structures of various shapes and sizes. An up-date literature survey is as follows: An extensive review on linear vibration of plates has been given by Leissa [11] in his monograph. The Ritz method is employed forthe all the results. Conwey and Farnham [6] study the free vibration of triangular, rhombic and parallelogram plates. The different skew angles of simply supported and clamped boundary conditionsfor frequencies were calculated. Leissa [27], [31] presented plate vibration research, classical theoryand Plate vibration research, complicating effects. Jain and Soni [14] analyzed Free Vibration of rectangular plates of parabolic ally varying thickness .Gupta and Khanna [131] analyzed vibration ofa visco-elastic rectangular plate under the effect of linearly varying thickness in both directions. Sharma Amit [150]. The present study analyzes the natural vibration of non homogeneous visco elasticskew plate (parallelogram plate) with non uniform thickness under temperature field. Here non homogeneity in the plate's material arises due to circular variation in Poisson's ratio. Gupta, Kumar[143] study the vibration of visco-elastic parallelogram plate whose thickness varies parabolic ally. Itis assumed that the plate is clamped on all the four edges and that the thickness varies parabolic ally in one direction i.e. along length of the plate.

2. ANALYSIS

The parallelogram (skew) plate is assumed to be non-uniform, thin and orthotropic and the plate R be defined by the three number a, b and θ.

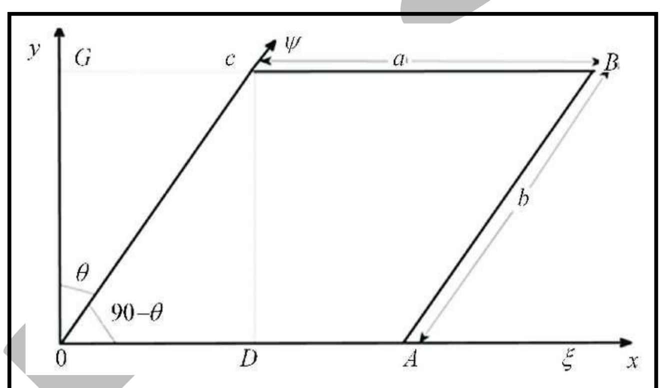


Figure 2.1. The parallelogram plate with skew angle θ

The differential equation of motion and time function for visco elastic plate with thickness variation is given by

$[D_1 (w,_{xxxx} +2 w,_{xxyy} + w,_{yyyy}) +2 D_{1,x} (w,_{xxx} + w,_{xyy}) + 2 w,_{xxxx} +2 D_{1,y}(w,_{yyy} + w,_{yxx}) + 2 D_{1,xx}(w,_{yyy} + w,_{yxx}) + D_{1,xx} (w,_{xx} + \nu w,_{yy}) + D_{1,yy} (w,_{yy} + \nu w,_{xx}) +2 (1-\nu)D_{1,xy} w,_{xy}]- \rho k^2 lw = 0$

(1)

$$\ddot{T} + k^2 D\widetilde{T = 0} \tag{2}$$

Here, comma followed by suffix is known as partial derivative of W with respect to independent variable and double do represent the second derivative with respect to t. Also $D_1 = \frac{yl^3}{12(1-\nu)^2}$ is called flexural rigidity of the plate.

Now the expression for the kinetic energy (M_E) and the strain energy (N_E) is given by:

$$M_E = \frac{1}{2}\omega^2 \rho \iint l W^2 \, dydx \tag{3}$$

and

$$N_E = \frac{1}{2}\iint D_1 \{(W_{,xx})^2+(W_{,yy})^2+2\nu W_{,xx} W_{,yy}+2(1-\nu)(W_{,xy})^2\}\, dydx \qquad (4)$$

The parallelogram (skew) plate is assumed to be non-uniform, thin and orthotropic and the plate R be defined by the three number a, b and θ.

The skew coordinates of the plate are:

$$\xi = x - y\tan\theta,\ \varphi = y\sec\theta \qquad (5)$$

The boundary condition of the plate in skew coordinates is:

$$\xi = 0\ ,\xi = a\ \ and\ \ \varphi = 0\ ,\varphi = b \qquad (6)$$

Using eqn. (5), the equation of K.E. (3) and Strain energy (4) will become:

$$M_E = \frac{1}{2}\, k^2\ \rho\cos\theta \int_0^b \int_0^a l\, W^2\ d\xi d\varphi \qquad (7)$$

$$N_E = \frac{1}{2}\int_0^b \int_0^a D_1\, [\ (W_{,\xi\xi})^2 - 4\sin\theta\,(W_{,\xi\xi})\left(W_{,\xi\varphi}\right) + 2\,(\sin^2\theta + \nu\cos^2\theta)(W_{,\xi\xi})\left(W_{,\varphi\varphi}\right) + 2\,(1+\sin^2\theta - \nu\cos^2\theta)\left(W_{,\xi\varphi}\right)^2 - 4\sin\theta\left(W_{,\xi\varphi}\right)\left(W_{,\varphi\varphi}\right) + \left(W_{,\varphi\varphi}\right)^2]\, d\xi d\varphi \qquad (8)$$

2. ASSUMPTIONS

1. The thickness of the plate is assumed to be bi-linear in two dimensions.

$$g = g_0\ [1 + \beta_1\,(1-\sqrt{1-\frac{\xi}{a}})]\ [1 + \beta_2\,(1-\sqrt{1-\frac{\varphi}{b}})] \qquad (9)$$

Where β_1, β_2 is tapering constant. Thickness of the plate becomes constant at $\xi = 0\,, \varphi = 0$.

2. We consider plate's material to be non-homogeneous. Therefore, either density or Poisson's ratio varies circularly in one dimensions as :

$$\nu = \nu_0\ [1 - m\,(1-\sqrt{1-\frac{\xi}{a}})] \qquad (10)$$

Where m is known as non-homogeneity constant. Poisson's ratio becomes constant i.e. $\nu = \nu_0$ at $\xi = 0\,, \varphi = 0$.

3. The temperature variation on the plate is considered to be to bi-linear in ξ direction and bi-parabolic in φ direction as :

$$\eta = \eta_0\,[(\sqrt{1-\frac{\xi^2}{a^2}})\,(\sqrt{1-\frac{\varphi^2}{b^2}})] \qquad (11)$$

Where η and η_0 denotes the temperature excess above the reference temperature on the plate at any point and at the origin the temperature dependence modulus of elasticity for engineering structures is given by:

$$Y = Y_0\,(1-\gamma\,\eta) \qquad (12)$$

Where Y_0 is the Young's Modulus at mentioned temperature (i.e. η = 0) and γ is called slope of variation.
Using equation (11) in equation (12), we get:

$$Y = Y_0\,[1-\gamma\,(\eta_0\,(\sqrt{1-\frac{\xi^2}{a^2}})\,(\sqrt{1-\frac{\varphi^2}{b^2}}))]$$

$$Y = Y_0\,[1-\gamma\,\eta_0\,(\sqrt{1-\frac{\xi^2}{a^2}})\,(\sqrt{1-\frac{\varphi^2}{b^2}}))]$$

Or

$$Y = Y_0\,[1-\alpha(\sqrt{1-\frac{\xi^2}{a^2}})\,(\sqrt{1-\frac{\varphi^2}{b^2}}))] \qquad (13)$$

Where α, (0 ≤ α < 1) is called temperature, which is the product of temperature at origin and γ slope of variation i.e. gradient $\alpha = \gamma\eta_0$

Using equation (9), (10) and (13), flexural rigidity i.e. $D_1 = \frac{yl^3}{(1-\nu)^2}$ of the plate becomes:

$$D_1 = \frac{Y0\ [1-\alpha(\sqrt{1-\frac{\xi^2}{a^2}})(\sqrt{1-\frac{\varphi^2}{b^2}})]l0\ [[1+\beta1(1-\sqrt{1-\frac{\xi}{a}})][1+\beta2(1-\sqrt{1-\frac{\varphi}{b}})]]^3}{12(1-\nu_0{}^2[1-m(1-\sqrt{1-\frac{\xi}{a}})]^2)} \qquad (14)$$

Using (9), (10) and (14), the eqn. of K.E. and Strain Energy becomes:

$$M_E = \frac{1}{2}k^2\rho l_0\int_0^b\int_0^a(1+\beta_1 C1)(1+\beta_2 C2)\ W^2 d\xi d\varphi \qquad (15)$$

$$N_E = \frac{y_0 l_0}{24\cos^4\theta}\int_0^b\int_0^a\left[\frac{\left[1-\alpha(\sqrt{1-\frac{\xi^2}{a^2}})(\sqrt{1-\frac{\varphi^2}{b^2}})\right][(1+\beta_1 C1)(1+\beta_2 C2)]^3}{(1-\nu_0{}^2([1-mC1]^2)}\right][(W_{,\xi\xi})^2-4\left(\frac{a}{b}\right)\sin\theta(W_{,\xi\xi})(W_{,\xi\varphi})+$$
$$2\left(\frac{a}{b}\right)(\sin^2\theta+\nu_0 t[1-mC1]\cos^2\theta)t(W_{,\xi\xi})(W_{,\varphi\varphi})+2\left(\frac{a}{b}\right)^2(1+\sin^2\theta-\nu_0[1-$$
$$mC1]\cos^2\theta)(W_{,\xi\varphi})^2-4\left(\frac{a}{b}\right)^3\sin\theta(W_{,\xi\varphi})(W_{,\varphi\varphi})+\left(\frac{a}{b}\right)^4(W_{,\varphi\varphi})^2]]d\xi d\varphi \qquad (16)$$

Where,

$$C1 = (1-\sqrt{1-\frac{\xi}{a}}),\ C2 = (1-\sqrt{1-\frac{\varphi}{b}})$$

In this paper, we are calculating first two mode of vibration on simply supported boundary condition, therefore we have:

$$W=W_{,\xi}=0 \text{ at } \xi=0,a$$
$$W=W_{,\varphi}=0 \text{ at } \varphi=0,b \qquad (17)$$

Hence, the two term deflection function, which satisfies eqn. (17), is:

$$W(\xi,\varphi) = [B1\left(\frac{\xi}{a}\right)^2\left(\frac{\varphi}{b}\right)^2\left(1-\frac{\xi}{a}\right)^2\left(1-\frac{\varphi}{b}\right)^2+B_2\left(\frac{\xi}{a}\right)^3\left(\frac{\varphi}{b}\right)^3\left(1-\frac{\xi}{a}\right)^3\left(1-\frac{\varphi}{b}\right)^3]$$
$$= \left(\frac{\xi}{a}\right)^2\left(\frac{\varphi}{b}\right)^2\left(1-\frac{\xi}{a}\right)^2\left(1-\frac{\varphi}{b}\right)^2[B1+B_2\left(\frac{\xi}{a}\right)\left(\frac{\varphi}{b}\right)\left(1-\frac{\xi}{a}\right)(1-\frac{\varphi}{b})] \qquad (18)$$

Where B_1 and B_2 are arbitrary constant.

4. Solution for frequency equation by Rayleigh-Ritz method

We used Rayleigh-Ritz method to solve frequency equation and frequency mode i.e. in Rayleigh-Ritz method maximum kinetic energy must be equal to maximum strain energy.

Hence we have:

$$\delta(N_E - M_E) = 0 \qquad (19)$$

Using equation (15) and (16), we get:

$$\delta(N_E{}^* - \lambda^2 M_E{}^*) = 0 \qquad (20)$$

Where,

$$M_E{}^* = \int_0^b\int_0^a(1+\beta_1 C1)(1+\beta_2 C2)\ W^2 d\xi d\varphi \qquad (21)$$

And

$$N_E^* = \frac{1}{cos^4\theta}\int_0^b\int_0^a\{\left[\frac{\left[1-\alpha\left(1-\frac{\xi}{a}\right)\left(1-\frac{\varphi}{b}\right)\right][(1+\beta_1 C1)(1+\beta_2 C2)]^3}{(1-\nu_0{}^2([1-mC1])^2}\right][(W_{,\xi\xi})^2 - 4\left(\frac{a}{b}\right)\sin\theta\,(W_{,\xi\xi})(W_{,\xi\varphi}) + 2\left(\frac{a}{b}\right)(\sin^2\theta+\nu_0 t[1-mC1]\cos^2\theta)t(W_{,\xi\xi})(W_{,\varphi\varphi}) + 2\left(\frac{a}{b}\right)^2(1+\sin^2\theta-\nu_0[1-mC1]\cos^2\theta)(W_{,\xi\varphi})^2 - 4\left(\frac{a}{b}\right)^3\sin\theta(W_{,\xi\varphi})(W_{,\varphi\varphi}) + \left(\frac{a}{b}\right)^4(W_{,\varphi\varphi})^2]\}d\xi d\varphi \quad (22)$$

And $\lambda 2 = \frac{12\omega^2 a^4 \rho}{Y0\ell_0{}^2}$ is known as frequency parameter.

Equation (20) consists of two unknown constants which are obtained by the substitution of W and these constant can be evaluated by the following formula:

$$\frac{\partial}{\partial B_1}(N_E{}^* - \lambda^2 M_E{}^*) = 0\,,\ \frac{\partial}{\partial B_2}(N_E{}^* - \lambda^2 M_E{}^*) = 0 \quad (23)$$

after solving equation (23) , we get,

$$d_{11}B + d_{12}B_2 = 0 \quad (24)$$

$$d_{21}B_1 + d_{22}B_2 = 0 \quad (25)$$

Where d_{11}, $d_{12} = d_{21}$ and d_{22} involve parametric constant and frequency parameter.
For a non- trivial solution the determinant of the coefficients of Equation (24) & (25) must be zero.
Therefore, we get the frequency equation,

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = 0 \quad (26)$$

With the help of equation (26), we get quadratic equation in λ^2. We can obtain two roots of λ2 from this equation. These roots give the first (λ_1) and second (λ_2) modes of vibration of frequency for various parameters.

5. Result and Discussion

The frequency (λ) for first and second mode of vibration of an orthotropic skew (parallelogram) plate has been determined for different values of thermal constant(α), tapering constant (β1 and β2), aspect ratio (a/b) and non-homogeneity constant (m) and skew angle(θ).Every one of the outcomes are acquired by utilizing MATLAB/MAPLE programming. All the results are shown with the help of Figures.
Following boundaries are utilized for this estimation is: v_0=0.345, a/b=1.5

In Fig I: Thickness (tapering parameter (β1) variation in plate v/s frequency (λ) with fixed value of θ = 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant (β1 =β2= m =α = 0, 0.4, 0.8).From fig.1 that as value of taper constant (β1) increases from 0 to 0.8 correspondingfrequency value (λ) for 1st and 2nd mode of vibration also increases.

In Fig II: Thickness (tapering parameter (β2) variation in plate v/s frequency (λ) for θ = 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant (β1 =β2 = m= α = 0, 0.4, 0.8).From fig.II that as value of taper constant (β2) increases from 0 to 0.8 corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

In Fig III: non-homogeneity (m1) variation in plates material v/s vibrational frequency (λ) for $\theta = 30^0$ and a/b = 1.5 and different values of taper constants and non-homogeneity constant (β1 =β2 = m = α = 0, 0.4, 0.8).From fig.III that as value of non-homogeneity (m) increases from 0 to 0.8 corresponding frequency value (λ) for 1st and 2nd mode of vibration is decreases.

In Fig IV: Thermal gradient (α) variation in plates material v/s frequency (λ) for $\theta = 30^0$ and a/b = 1.5 and different values of taper constants and non-homogeneity constant (β1 =β2 = m = 0, 0.4,0.8).From fig.IV that frequency mode decreases as value of thermal gradient increases from 0 to 0.8 i.e. Corresponding frequency value (λ) for 1st and 2nd mode of vibration decreases.

In Fig V: skew angle (θ) variation in plates material v/s frequency (λ) for a/b = 1.5 and different values of taper constants and non-homogeneity constant (β1 =β2 = α =0.4, m = 0, 0.4, 0.8). From fig.V clear that frequency mode increases sharply as value of skew angle increases from 0 to 75 i.e. corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

Figure -1 Taper Constant (β_1) v/s Frequency (λ)

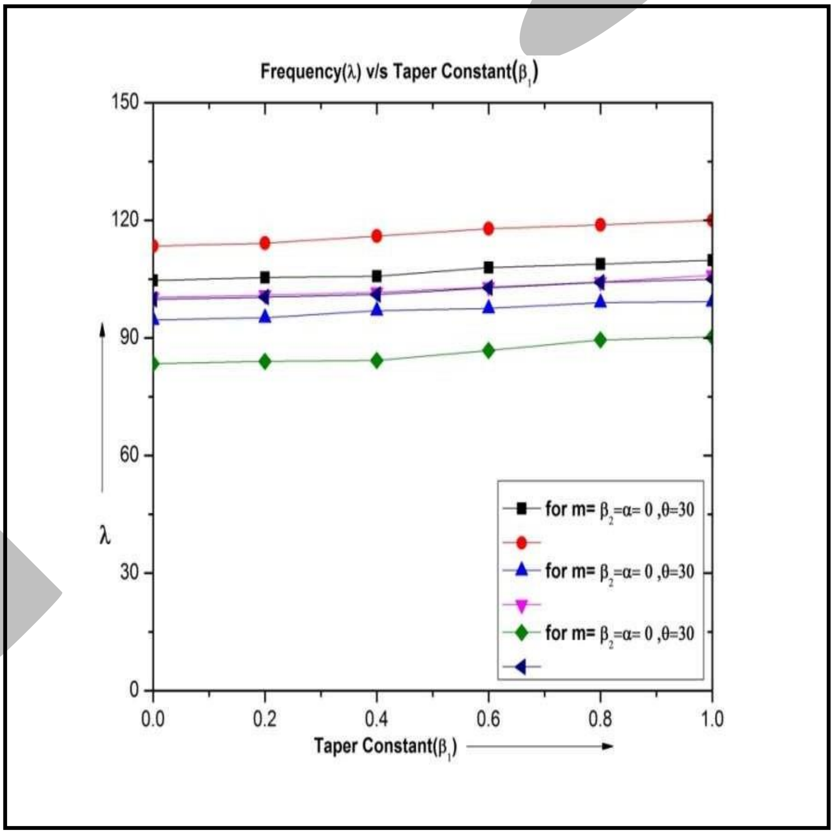


Figure -2 Taper Constant (β_2) v/s Frequency (λ)

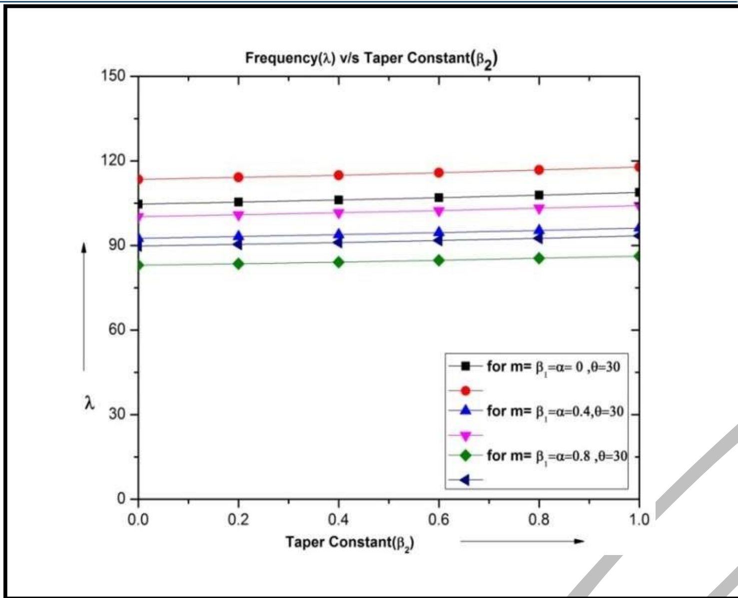


Figure -3 Non-Homogeneity (m) v/s Frequency (λ)

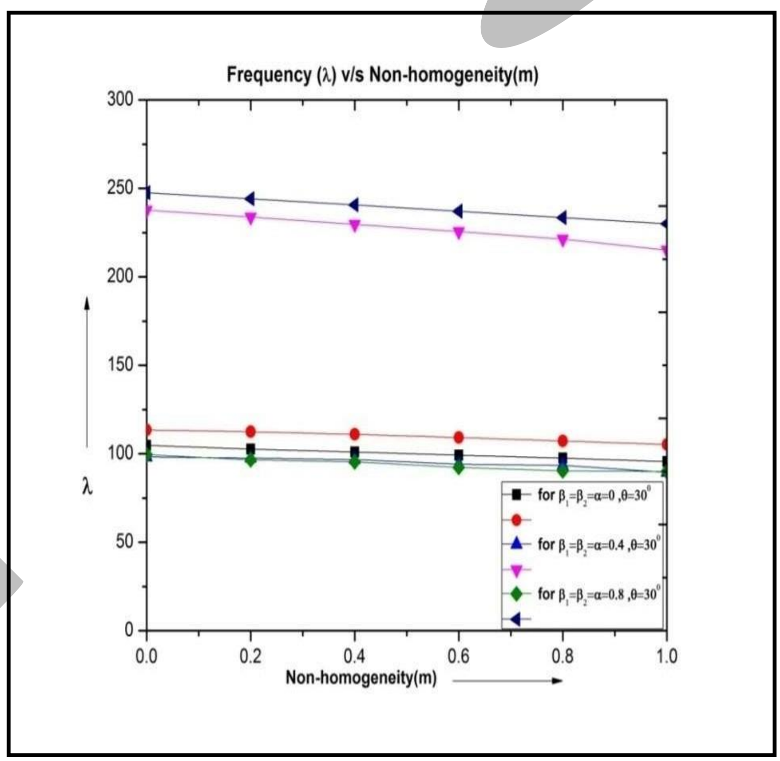


Figure - 4 Thermal Gradients (α) v/s Frequency (λ)

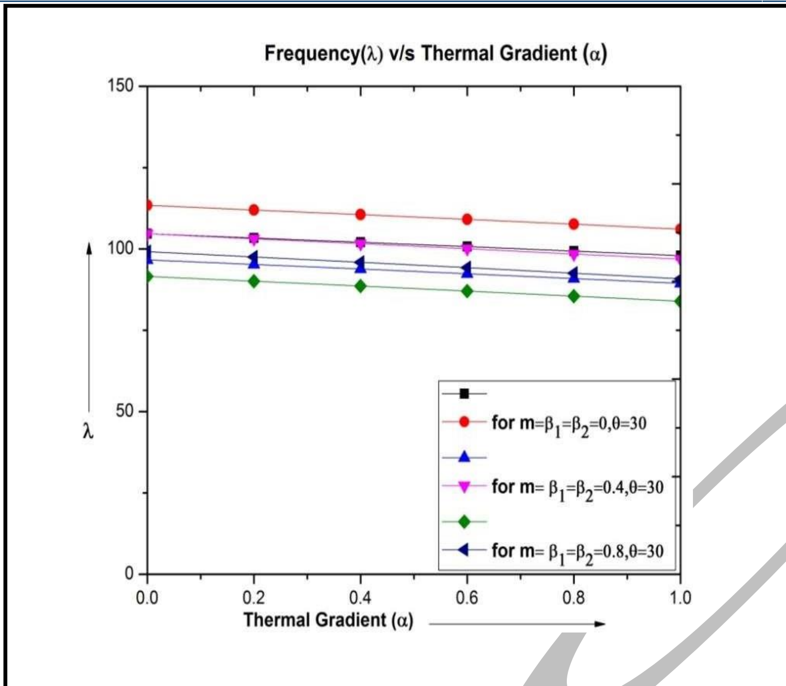
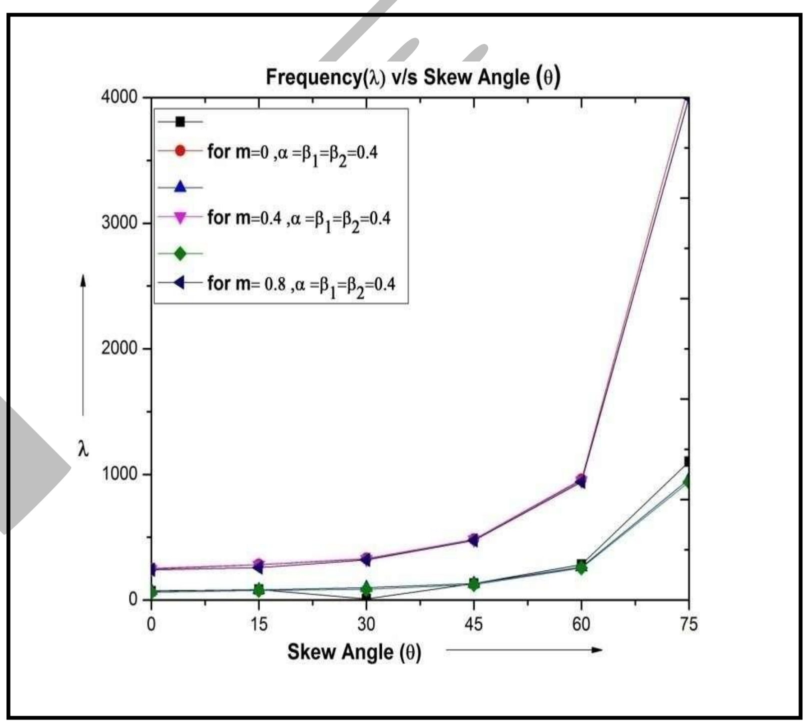


Figure -5 Skew Angle (θ) v/s Frequency (λ)



6. Conclusion

Rayleigh - Ritz technique is applied to study the effect of various parameters (taper constants, thermalconstant, and non-homogeneity constant, skew angle) on vibration of non-homogeneous parallelogram skew plate with circular variation in bi-linear thickness and bi-parabolic temperature variation. From the result discussion author conclude that as tapering constant (β_1 and β_2) and skew angle (θ) increases, frequency increases for both modes of vibration. While it decreases as thermal gradient and Non- Homogeneity increases. This paper gives good appropriate numerical data of frequency modes whichis helpful for researchers and scientists, making good optimal structural designs.

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