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Split Geodetic Dominating Sets in Path Graphs

P. Arul Paul Sudhahar[1] and A. Merin Sherly[2]

[1]*Research Supervisor, Department of Mathematics,*
Rani Anna Government College for Women,
Tirunelveli - 627 008, Tamilnadu, India.
[2]*Research Scholar (Register No. 20211172092004),*
Department of Mathematics,
Rani Anna Government College for Women,
Tirunelveli - 627 008,
Affiliated to Manonmaniam Sundaranar University,
Abishekapatti, Tirunelveli 627 012,
Tamilnadu, India.

Abstract
Let $Dsg(P_n, i)$ be the family of split geodetic dominating sets of the path graph P_n. with cardinality i and let $dsg(P_n, i) = | Dsg(P_n, i) |$. Then the split geodetic polynomial $Dsg(P_n, x)$ of Pn is defined as $Dsg(Pn, x)$ $=\sum_{i=\gamma_{sg}(P_n)}^{n} dsg(P_n, i)x^i$, where $\gamma_{sg}(P_n)$ is the split geodetic domination number of Pn.In this paper we have determined the family of split geodetic dominating sets of the path graph Pn with cardinality i.Also , we have obtained the recursive formula to derive the split geodetic domination polynomials of paths and also obtain some properties of this polynomial.

Keywords: *Path, Split geodetic dominating sets , Split geodetic domination Polynomial.*
AMS Subject Classification : 05C12 , 05C69

1 Introduction

Let $G = (V, E)$ be a simple graph of order $| V | = n$. A dominating set for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number $\gamma (G)$ is the number of vertices in a smallest dominating set for G[1]. We call a set of vertices S in a graph G a geodetic dominating set if S is both a geodetic set and a dominating set. The minimum cardinality of a geodetic dominating set of G is its geodetic domination number, and is denoted by $\gamma_g(G)$[2] [3]. Split geodetic number of a graph was studied by in [4]. A geodetic set S of a graph $G = (V, E)$ is the split geodetic set if the induced subgraph $\langle V - S\rangle$ is disconnected.The split geodetic number $g_s(G)$ of G is the minimum cardinality of a split geodetic set. A set $S \subseteq V (G)$ is said to be a split geodetic dominating set of G if S is both a split geodetic set and a dominating set of G.The minimum cardinality of the split geodetic dominating set of G is called the split geodetic domination number of G and is denoted by $\gamma_{gs}(G)$.The concept of split geodetic domination number was introduced by P.Arul Paul Sudhahar and J.Jeba Lisa in [5].A domination polynomial can be studied in [6][7][8][9] and the geodetic domination polynomial was studied in [10]. A path is a connected graph in which two vertices have degree 1 and the remaining vertices have degree 2.Let $P_n$be a path with n vertices. Let $Dsg(Pn, i)$ be the family of split geodetic dominating sets of the graph Pn with cardinality i and let $dsg(P_n, i) = | Dsg(P_n, i) ||$. Then the split geodetic polynomial $Dsg(P_n, x)$ of Pn is defined as $Dsg(P_n, x)$ $=\sum_{i=\gamma_{sg}(Pn)}^{n} dsg(P_n, i)x^i$, where $\gamma_{sg}(P_n)$ is the split geodetic domination number of P_n.

In the next section we construct the families of the split geodetic dominating sets of paths by a recursive method. In section 3, we use the results obtained in section 2 to study the split geodetic domination polynomial of paths.

Lemma 2.1. $\gamma_{sg}(P_n) = \left\lceil \frac{n+2}{3} \right\rceil$

Lemma 2.2. $Dsg(P_n, i) = \Phi$ if and only if $i > n$ or $i < \left\lceil \frac{n+2}{3} \right\rceil$ and $Dsg(Pn, i) > 0$ if $\left\lceil \frac{n+2}{3} \right\rceil \leq i \leq n$.

Lemma 2.3. If $Y \in Dsg(P_{n-1}, i-1)$ or $Dsg(P_{n-2}, i-1)$ or $Dsg(P_{n-3}, i-1)$ then $Y \cup \{x\} \in Dsg(P_n, i)$.

Lemma 2.4. (i) If $Dsg(P_{n-1}, i-1) = Dsg(P_{n-3}, i-1) = \Phi$ then $Dsg(P_{n-2}, i-1) = \Phi$

(ii) If $Dsg(P_{n-1}, i-1) \neq \Phi, Dsg(P_{n-3}, i-1) \neq \Phi$ then $Dsg(P_{n-2}, i-1) \neq \Phi$

(iii) If $Dsg(P_{n-1}, i-1) = Dsg(P_{n-2}, i-1) = Dsg(P_{n-3}, i-1) = \Phi$ then $Dsg(P_n, i) = \Phi$

Proof.

(i) If $Dsg(P_{n-1}, i-1) = \Phi$ and $Dsg(P_{n-3}, i-1) = \Phi$ then $i-1 > n-1$ or $i-1 < \left\lceil \frac{n+1}{3} \right\rceil$ and $i-1 > n-3$ or $i-1 > n-1$ or $i-1 < \left\lceil \frac{n-1}{3} \right\rceil \Rightarrow i-1 < \left\lceil \frac{n}{3} \right\rceil$ or $i-1 > n-2$ holds .Therefore $Dsg(P_{n-2}, i-1) = \Phi$.

(ii) If $Dsg(P_{n-1}, i-1) \neq \Phi$ $Dsg(P_{n-3}, i-1) \neq \Phi$ then $\left\lceil \frac{n+1}{3} \right\rceil \leq i-1 \leq n-1$ and $\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq n-3 \Rightarrow \left\lceil \frac{n+1}{3} \right\rceil \leq i-1 \leq n-3$ and $\left\lceil \frac{n}{3} \right\rceil \leq \left\lceil \frac{n+1}{3} \right\rceil \leq i-1 \leq n-3 < n-2 \Rightarrow \left\lceil \frac{n}{3} \right\rceil < i-1 < n-2$. Hence $Dsg(P_{n-2}, i-1) \neq \Phi$.

(iii) If $Dsg(P_{n-1}, i-1) = Dsg(P_{n-2}, i-1) = Dsg(P_{n-3}, i-1) = \Phi$ then $i-1 < \left\lceil \frac{n+1}{3} \right\rceil$ or $i-1 > n-1$; $i-1 < \left\lceil \frac{n}{3} \right\rceil$ or $i-1 > n-2$ and $i-1 < \left\lceil \frac{n-1}{3} \right\rceil$ or $i-1 > n-3 \Rightarrow i-1 < \left\lceil \frac{n-1}{3} \right\rceil$ *or* $i-1 > n-3 \Rightarrow i < \left\lceil \frac{n-1}{3} + 1 \right\rceil$ *or* $i > n \Rightarrow i < \left\lceil \frac{n+2}{3} \right\rceil$ or $i > n$. Therefore $Dsg(P_n, i) = \Phi$.

Lemma 2.5. If $Dsg(P_n, i) \neq \Phi$ then we have

(i) $Dsg(P_{n-1}, i-1) = Dsg(P_{n-2}, i-1) = \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$ if and only if $n = 3k-2, i = k$, for some positive integer k.

(ii) $Dsg(P_{n-2}, i-1) = Dsg(P_{n-3}, i-1) = \Phi$ and $Dsg(P_{n-1}, i-1) \neq \Phi$ if and only if $i = n$.

(iii) $Dsg(P_{n-1}, i-1) \neq \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) = \Phi$ if and only if $i = n-1$.

(iv) $Dsg(P_{n-1}, i-1) = \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$ if and only if $n = 3k$ and $i = \left\lceil \frac{3k+3}{3} \right\rceil$ for some $k \in N$

(v) $Dsg(P_{n-1}, i-1) \neq \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$ if and only if $\left\lceil \frac{n+1}{3} \right\rceil + 1 \leq i \leq n-2$.

Proof

(i)Since $Dsg(P_{n-1}, i-1) = Dsg(P_{n-2}, i-1) = \Phi$ $\Rightarrow i-1 > n-1$ or $i-1 n-2$ or $i-1 < \left\lceil \frac{n}{3} \right\rceil \Rightarrow i-1 < \left\lceil \frac{n}{3} \right\rceil$ or $i-1 > n-1$. If $i-1 > n-1$ then $i > n$ and hence $Dsg(P_n, i) = \Phi$ which is a contradiction.Therefore $i-1 < \left\lceil \frac{n}{3} \right\rceil \Rightarrow i < \left\lceil \frac{n}{3} \right\rceil + 1$. Also since $Dsg(P_{n-3}, i-1) \neq \Phi$, then $\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq n-3$. Hence $\left\lceil \frac{n-1}{3} \right\rceil + 1 \leq i < \left\lceil \frac{n}{3} \right\rceil + 1$. This is true only when $n = 3k-2$ and $i = k$ for some $k \in N$. Conversely assume $n = 3k-2$ and $i = k$ for some $k \in N$ then by lemma 2.2 $Dsg(P_{n-1}, i-1) = \Phi$ $Dsg(P_{n-2}, i-1) = \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$

(ii) Since $Dsg(P_{n-2}, i-1) = Dsg(P_{n-3}, i-1) = \Phi$,then $i-1 < \left\lceil \frac{n}{3} \right\rceil$ or $i-1 > n-2$ .If $i-1 < \left\lceil \frac{n}{3} \right\rceil$ then $i-1 < \left\lceil \frac{n+1}{3} \right\rceil$, then $Dsg(P_{n-1}, i-1) = \Phi$,which is a contradiction, so we have $i-1 > n-2 \Rightarrow i > n-1 \Rightarrow i \geq n$.Also since $Dsg(P_{n-1}, i-1) \neq \Phi$ then $\left\lceil \frac{n+1}{3} \right\rceil < i-1 \leq n-1 \Rightarrow i \leq n$.Hence $i = n$.Conversely if $i = n$, then $Dsg(P_{n-2}, i-1) = Dsg(P_{n-2}, n-1) \neq \Phi$, $Dsg(P_{n-3}, i-1) = Dsg(P_{n-3}, n-1) \neq \Phi$ and $Dsg(P_{n-1}, i-1) = Dsg(P_{n-1}, n-1) \neq \Phi$ [Since $Dsg(P_{n-1}, n-1) = 1$].

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(iii)Assume $Dsg(P_{n-1}, i-1) \neq \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) = \Phi$. Since $Dsg(P_{n-3}, i-1) = \Phi$, $i-1 > n-3$ or $i-1 < \left\lceil \frac{n-1}{3} \right\rceil$. Since $Dsg(P_{n-2}, i-1) \neq \Phi$, $\left\lceil \frac{n}{3} \right\rceil < i-1 \leq n-2$. That is, $i-1 < \left\lceil \frac{n-1}{3} \right\rceil$ is not possible. Therefore, $i-1 > n-3 \Rightarrow i-1 \geq n-2$, But $i-1 \leq n-2 \Rightarrow i-1 = n-2 \Rightarrow i = n-1$. Conversely suppose $i = n-1$, then $Dsg(P_{n-1}, i-1) = Dsg(P_{n-1}, n-2) \neq \Phi$, $Dsg(P_{n-2}, i-1) = Dsg(P_{n-2}, n-2) \neq \Phi$, but $Dsg(P_{n-3}, i-1) = Dsg(P_{n-3}, n-2) = \Phi$.

(iv)Assume $Dsg(P_{n-1}, i-1) = \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$. Since $Dsg(P_{n-1}, i-1) = \Phi$, $i-1 > n-1 \Rightarrow i-1 > n-2 \Rightarrow Dsg(P_{n-2}, i-1)$ *and* $Dsg(P_{n-3}, i-1)$are empty, which is a contradiction. Therefore $i-1 < \left\lceil \frac{n+1}{3} \right\rceil \Rightarrow i < \left\lceil \frac{n+1}{3} \right\rceil + 1$. Since $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$,we have $\left\lceil \frac{n}{3} \right\rceil \leq i-1 \leq n-2$ and $\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq n-3$. Therefore $\left\lceil \frac{n}{3} \right\rceil \leq i-1 \leq n-3$. Hence $\left\lceil \frac{n}{3} \right\rceil + 1 \leq i < \left\lceil \frac{n+1}{3} \right\rceil + 1$. This holds only when $n = 3k$ *and* $i = k+1 = \left\lceil \frac{3k+3}{3} \right\rceil$ for some $k \in N$. Conversely, assume $n = 3k$ *and* $i = k+1 = \left\lceil \frac{3k+3}{3} \right\rceil$, then $Dsg(P_{n-1}, i-1) = \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$.

(v) Assume $Dsg(P_{n-1}, i-1) \neq \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$. Then $\left\lceil \frac{n+1}{3} \right\rceil \leq i-1 \leq n-1$; $\left\lceil \frac{n}{3} \right\rceil \leq i-1 \leq n-2$ and $\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq n-3 \Rightarrow \left\lceil \frac{n+1}{3} \right\rceil \leq i-1 \leq n-3 \Rightarrow \left\lceil \frac{n+1}{3} \right\rceil + 1 \leq i \leq n-2$. Conversely, suppose $\left\lceil \frac{n+1}{3} \right\rceil + 1 \leq i \leq n-2$. Therefore $\left\lceil \frac{n+1}{3} \right\rceil \leq i-1 \leq n-1$; $\left\lceil \frac{n}{3} \right\rceil \leq i-1 \leq n-2$ and $\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq n-3$. From these we obtain $Dsg(P_{n-1}, i-1) \neq \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$.

Lemma 2.6. If $Dsg(P_n, i) \neq \Phi$, then

(i) $Dsg(P_{n-1}, i-1) = Dsg(P_{n-2}, i-1) = \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$, then $Dsg(P_n, i) = \{1, 4, \ldots, 3k-5, 3k-2\}$

(ii) $Dsg(P_{n-2}, i-1) = Dsg(P_{n-3}, i-1) = \Phi$ and $Dsg(P_{n-1}, i-1) \neq \Phi$ then $Dsg(P_n, i) = \{1, 2, \ldots, n\}$

(iii) $Dsg(P_{n-1}, i-1) \neq \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) = \Phi$ then $Dsg(P_n, i) = \{[n] - x / x \in n - \{1, n\}\}$.

(iv) $Dsg(P_{n-1}, i-1) = \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$, then $Dsg(P_n, i) = \{X_1 \cup 3k / X_1 \in P_{3k-2}, k\} \cup \{X_2 \cup 3k / X_2 \in P_{3k-3}, k\}$

(v) $Dsg(P_{n-1}, i-1) \neq \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$ then $Dsg(P_n, i) = \{X_1 \cup n / X_1 \in P_{n-1}, i-1\} \cup \{X_2 \cup n / X_2 \in P_{n-2}, i-1\} \cup \{X_3 \cup n / X_3 \in P_{n-3}, i-1\}$

Proof.

(i)Since $Dsg(P_{n-1}, i-1) = Dsg(P_{n-2}, i-1) = \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$, then by Lemma 2.5 (i) $n = 3k-2$ and $i = k$ for some $k \in N$. Hence $Dsg(P_n, i) = \{1, 4, \ldots, 3k-5, 3k-2\}$.

(ii)Since $Dsg(P_{n-2}, i-1) = Dsg(P_{n-3}, i-1) = \Phi$ and $Dsg(P_{n-1}, i-1) \neq \Phi$, then by lemma 2.5 (ii) $i = n$ .Therefore $Dsg(P_n, i) = \{1, 2, \ldots, n\}$.

(iii)Since $Dsg(P_{n-1}, i-1) \neq \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) = \Phi$, then by lemma 2.5 (iii) $i = n-1$, then $Dsg(P_n, i) = \{[n] - x / x \in [n] - \{1, n\}\}$.

(iv)Since $Dsg(P_{n-1}, i-1) = \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$ by Lemma 2.5 (iv) $n = 3k$ and $i = k+1$ for some $k \in N$.Let $X_1 = \{1, 4, \ldots, 3k-3\} \in P_{3k-3}, k$ then $\{X_1 \cup \{3k\} \in P_{3k}, k+1\}$. Also if $X_2 = \{1, 4, \ldots, 3k-2\} \in P_{3k-2}, k$ then $X_2 \cup \{3k\} \in P_{3k}, k+1$ .Therefore $\{X_1 \cup \{3k\} / X_1 \in P_{3k-3}, k\} \cup \{X_2 \cup \{3k\} / X_2 \in P_{3k-2}, k\} \subseteq P_{3k}, k+1$. Now let $Y \in P_{3k}, k+1$. Then the vertices labelled 1 and 3k must belong to $P_{3k}, k+1$.If the vertex $3k-3$ is in Y ,then $Y = \{X_1 \cup \{3k\} / X_1 \in P_{3k-3}, k\}$. Similarly if the vertex $3k-2$ is in Y ,then $Y = \{X_2 \cup \{3k\} / X_2 \in P_{3k-2}, k\}$. Hence $P_{3k}, k+1 \subseteq = \{X_1 \cup 3k / X_1 \in P_{3k-2}, k\} \cup \{X_2 \cup 3k / X_2 \in P_{3k-3}, k\}$.

(v)Suppose $Dsg(P_{n-1}, i-1) \neq \Phi$; $Dsg(P_{n-2}, i-1) \neq \Phi$ and $Dsg(P_{n-3}, i-1) \neq \Phi$.Let $X_1 \in Dsg(P_{n-1}, i-1)$, then $n-1, n-2$ or $n-3$ is in X_1. If $n-1, n-2$ or $n-3 \in X_1$ then $X_1 \cup \{n\} \in$

$Dsg(P_n, i)$. Let $X_2 \in Dsg(P_{n-2}, i-1)$, then $n-2$ or $n-3$ or $n-4$ is in X_2.If $n-2, n-3$ or $n-4 \in X_2$ then $X_2 \cup \{n\} \in Dsg(P_n, i)$.Now let $X_3 \in Dsg(P_{n-3}, i-1)$, then $n-3, n-4$ or $n-5$ is in X_3.If $n-3, n-4$ or $n-5 \in X_3$ then $X3 \cup \{n\} \in Dsg(P_n, i)$.Thus we have$\{X_1 \cup \{n\} / X_1 \in P_{n-1}, i-1\} \cup \{X_2 \cup \{n\} / X_2 \in P_{n-2}, i-1\} \cup \{X_3 \cup \{n\} / X_3 \in P_{n-3}, i-1\} \subseteq Dsg(P_n, i)$.If $n \in Y$, then $Y = X_1 \cup \{n\}$ for some $X_1 \in Dsg(P_{n-1}, i-1)$.If $n-1 \in Y$ then $Y = X_2 \cup \{n\}$ for some $X_2 \in Dsg(P_{n-2}, i-1)$.If $n-2 \in Y$ then $Y = X_3 \cup \{n\}$, for some $X_3 \in Dsg(P_{n-3}, i-1)$.So $Dsg(P_n, i) = \{X_1 \cup n/X_1 \in P_{n-1}, i-1\} \cup \{X_2 \cup n / X_2 \in P_{n-2}, i-1\} \cup \{X_3 \cup n/X_3 \in P_{n-3}, i-1\}$.

Example 2.7. Consider P_6 with V (P_6) = 6.

We use Lemma2.6, to construct Dsg(P_6, i) for $2 \le i \le 6$. $Dsg(P_6, 2) = \phi$ Since Dsg(P_5, 2) = ϕ , $Dsg(P_4, 2) = \{1, 4\}$, $Dsg(P_3, 2) = \{1, 3\}$ then by Lemma2.6 (iv) $Dsg(P_6, 3) = \{1, 4, 6\}, \{1, 3, 6\}$. Since $Dsg(P_5, 2) = \{1, 3, 5\}, \{1, 2, 5\}, \{1, 4, 5\}$, $Dsg(P_4, 3) = \{1, 3, 4\}, \{1, 2, 4\}$, $Dsg(P_3, 3) = \{1, 2, 3\}$. Therefore by Lemma2.6 (v) $Dsg(P_6, 4) = \{1, 3, 5, 6\}, \{1, 2, 5, 6\}, \{1, 4, 5, 6\}, \{1, 3, 4, 6\}, \{1, 2, 4, 6\}, \{1, 2, 3, 6\}$. By Lemma2.6(iii) Dsg($P_6$, 5) = $\{1, 3, 4, 5, 6\}, \{1, 2, 4, 5, 6\}, \{1, 2, 3, 5, 6\}, \{1, 2, 3, 4, 6\}$. By Lemma2.6 (ii),$Dsg(P_6, 6) = \{1, 2, 3, 4, 5, 6\}$.

References

[1] T. W. Haynes, S. Hedetniemi, and P. Slater, Fundamentals of domination in graphs. CRC press, 1998.

[2] A. Hansberg and L. Volkmann, “On the geodetic and geodetic domination numbers of a graph,” Discrete mathematics, vol. 310, no. 15-16, pp. 2140–2146, 2010.

[3] H. Escuadro, R. Gera, A. Hansberg, N. Jafari Rad, and L. Volkmann, “Geodetic domination in graphs,” JCMCC-Journal of Combinatorial Mathematicsand Combinatorial Computing, vol. 77, p. 89, 2011.

[4] T. Gubbi and S. Shankarghatta, “Split geodetic number of a graph,” 2014.

[5] P. A. P. Sudhahar and J. J. Lisa, “Split geodetic domination number of a graph,” in Don BoscoCollege (Co-Ed), Yelagiri Hills: International Conference on Mathematics and Computer Applications (ICMCA 2020) accepted, 2022.

[6] S. Alikhani and Y.-h. Peng, “Introduction to domination polynomial of a graph,” arXiv preprint arXiv:0905.2251, 2009.

[7] S. Alikhani and Y.-H. Peng, “Introduction to domination polynomial of a graph,” Ars combinatoria, vol. 114, pp. 257–266, 2014.

[8] S. Alikhani, “The domination polynomial of a graph at- 1,” Graphs and Combinatorics, vol. 29, no. 5, pp. 1175–1181, 2013.

[9] S. Alikhani and Y.-H. Peng, “Dominating sets and domination polynomials of paths,” International journal of Mathematics and mathematical sciences, vol. 2009, 2009.

[10] N. J. Beaula and A. Vijayan, “22. geodetic dominating sets and geodetic dominating polynomials of triangular ladder graph ,” Recent Trends and Techniques in, p. 200, 2021.