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Complementary 3-Domination Number In Transformation Of Graphs

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Abstract

In Graph theory, a dominating set of a graph G is a subset S of its vertices such that every vertex in $V - S$ is adjacent to atleast one vertex in S. The minimum cardinality of a dominating set is called the domination number $\gamma(G)$. A dominating set of a graph G is called a complementary 3-dominating set of G if for every vertex in S has atleast three neighbors in $V - S$. The complementary 3-domination number $\gamma_3'(G)$ is the minimum cardinality taken over all complementary 3-dominating sets. The transformation graph of G is a simple graph with vertex set $V(G) \cup E(G)$ in which adjacency is defined as follows, (a) Two elements in $V(G)$ are adjacent iff they are non adjacent in G. (b) Two elements in $E(G)$ are adjacent iff they are non adjacent. (c) One element in $V(G)$ and one element in $E(G)$ are adjacent iff they are non adjacent in G. It is denoted by G^{---}. Let $G = (V, E)$ be a simple undirected graph with order n and size m. The transformation graph of G is a simple graph with vertex set $V(G) \cup E(G)$ in which adjacency is defined as follows:(a) Two elements in V(G) are adjacent iff they are adjacent in G. (b) Two elements in E(G) are adjacent iff they are non-adjacent in G. (c) One element in V(G) and one element in E(G) are adjacent iff they are incident in G. It is denoted by G^{+-+}. A transformation graph G^{-+-} of a simple graph G with vertex set $V(G) \cup E(G)$ in which adjacency is defined by, (a) Two elements in $V(G)$ are adjacent iff they are non adjacet in G.(b) Two elements in $E(G)$ are adjacent iff they are adjacent in G.(c) One element in $V(G)$ and one element in $E(G)$ are adjacent iff they are non adjacent in G. In this paper we investigate some results in transformation graph $G^{---}, G^{+-+}, G^{-+-}$ and obtain exact values for some graphs.

Keywords: *Domination number, complementary 3-domination number*

AMS Subject Classification: 05C69

1. Introduction:

A graph $G = (V, E)$ we mean a finite, simple, connected and undirected graph consists of a set V of vertices and a set E of edges, Two vertices that are joined by an edge are called adjacent vertices. A pendant vertex is a vertex that is connected to exactly one other vertex by a single edge. A degree of a vertex is the number of edges incident to the vertex and is denoted by deg(v). A graph G is said to be connected if any two vertices of G are connected by a path. A maximal connected subgraph of G is called a component of G. A walk of a graph G is an alternating sequence of vertices and edges $v_0, e_1, v_1, e_2, \ldots, v_{n-1}, e_n, v_n$ beginning and ending with vertices such that each edge e_i is incident with $v_{i-1}$and v_i. We say that the walk joins v_0 and v_n and it is called a $v_0 - v_n$ walk. v_0 is called a terminal point of the walk. A walk is called a trail if all its vertices are distinct. A $v_0 - v_n$ walk is called closed if $v_0 - v_n$. A closed walk $v_0, v_1, v_2, \ldots, v_n = v_0$ in which $n \geq 3$ and $v_0, v_1, \ldots, v_{n-1}$ are distinct is called a cycle of length n. A cycle of n vertices is denoted by C_n and a path of m vertices is denoted by P_n. The complement $\bar{G}$ of G is the graph with vertex set V in which two vertices are adjacent if and only if they are non adjacent in G.

In graph theory, a dominating set of a graph G is a subset S of its vertices such that every vertex in $V - S$ is adjacent to atleast one vertex in S. The minimum cardinality of a dominating set is called the domination number $\gamma(G)$. A dominating set of a graph G is called a complementary 3-dominating set of G if for every vertex in S has atleast three neighbors in $V - S$. The complementary 3-domination number $\gamma_3'(G)$ is the minimum cardinality taken over all complementary 3-dominating sets. The transformation graph of G is a simple graph with vertex set $V(G) \cup E(G)$ in which adjacency is defined as follows,(a) Two elements in $V(G)$ are adjacent iff they are non adjacent in G. (b) Two elements in $E(G)$ are adjacent iff they are non adjacent. (c) One element in $V(G)$ and one element in $E(G)$ are adjacent iff they are non adjacent in G. It is denoted by G^{---}. The transformation graph of G is a simple graph with vertex set $V(G) \cup E(G)$ in which adjacency is defined as follows:(a) Two elements in V(G) are adjacent iff they are adjacent in G. (b) Two elements in E(G) are adjacent iff they are non-adjacent in G. (c) One element in V(G) and one element in E(G) are adjacent iff they are incident in G. It is denoted by G^{+-+}. A transformation graph G^{-+-} of a simple graph G with vertex set $V(G) \cup E(G)$ in which adjacency is defined by, (a) Two elements in $V(G)$ are adjacent iff they are non adjacet in G.(b) Two elements in $E(G)$ are adjacent iff they are adjacent in G.(c) One element in $V(G)$ and one element in $E(G)$ are adjacent iff they are non adjacent in G. In this paper we investigate some results in transformation graph $G^{---}, G^{+-+}, G^{-+-}$ and obtain exact values for some graphs.

2. Transformation Graph G^{---}

Defintion:2.1 The transformation graph of G is a simple graph with vertex set $V(G) \cup E(G)$ in which adjacency is defined as follows.

(a) Two elements in V(G) are adjacent iff they are non adjacent in G.

(b) Two elements in E(G) are addjacent iff they are non adjacent in G.

(c) One element in V(G) and one element in E(G) are adjacent iff they are non adjacent in G. It is denoted by G^{---}.

Example: 2.2

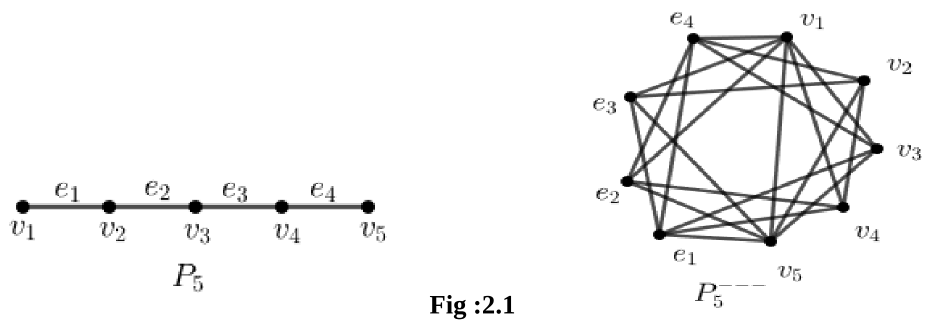


Fig :2.1

In the above example $\{v_1, v_5\}$ is a complementary 3-dominating set and hence $\gamma_3'(P_5^{---}) = 2$.

Theorem:2.3 Let T be any tree of order $n \geq 4$ and $diam(T) > 2$ then $\gamma_3'(G^{---}) = 2$.

Proof: Let T be any tree of order $n \geq 4$ and $diam(T) > 2$, then T is not isomorhic to $K_{1,n}$ and T has atleast two pendant vertices. Without loss of generality let us assume that x and y are the two vertices of T such that $d(x, y) > 2$. Choose x' and y' as the neighbors adjacent with x and y respectively. Then the distance between xy' and $x'y$ are atleast two. Therefore x is adjacent to all vertices in G^{---} other than x' and xx'. Since y is adjacent to all vertices including x' and yx' in G^{---} implies all the vertices in G^{---} are adjacent with x, y in G^{---}. Hence $\{x, y\}$ is a complementary 3-dominating set of G^{---}. Therefore $\gamma_3'(G^{---}) = 2$.

Theorem: 2.4 Let $G_1 = C_n$ and $G_2 = P_n$ of order $n \geq 4$ then $\gamma_3'(G_1^{---} + G_2^{---}) = 2$.

Proof: Let $G_1 = C_n$ and $G_2 = P_n$. Then $G_1^{---} + G_2^{---}$ are connected graphs with $2n$ and $2n - 1$ vertices respectively. Let u and w be the arbitrary vertices of $G_1^{---} + G_2^{---}$ respectively. Then all vertices of $G_1^{---} + G_2^{---}$ are adjacent with w and v respectively in $G_1^{---} + G_2^{---}$. Since v and w are arbitatry any pair of (v,w) is a complementary 3-dominating set of $G_1^{---} + G_2^{---}$. Therefore $\gamma_3'(G_1^{---} + G_2^{---}) = 2$.

3.Transformation Graph G^{+-+}

Definition: 3.1 Let $G = (V, E)$ be a simple undirected graph with order n and size m. The transformation graph of G is a simple graph with vertex set $V(G) \cup E(G)$ in which adjacency is defined as follows:

(a) Two elements in V(G) are adjacent iff they are adjacent in G.

(b) Two elements in E(G) are adjacent iff they are non-adjacent in G.

(c) One element in V(G) and one element in E(G) are adjacent iff they are incident in G. It is denoted by G^{+-+}.

Example: 3.2

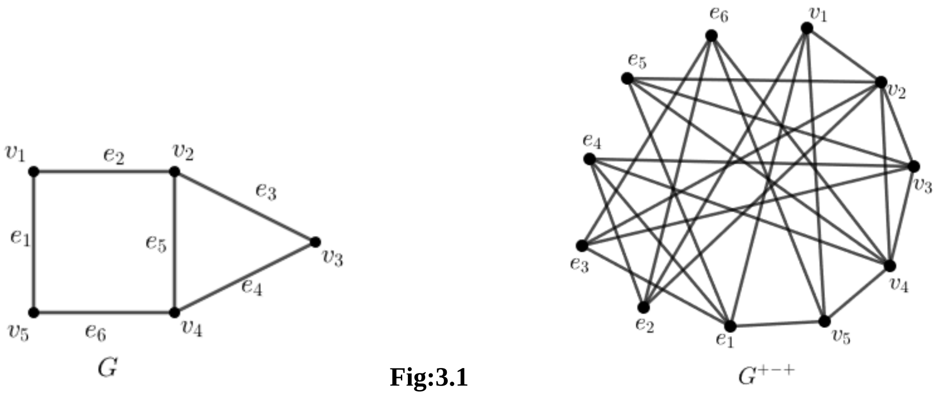


Fig:3.1

In the above example $\{v_3, e_1, e_6\}$ is a complementary 3-dominating set and hence $\gamma_3'(G^{+-+}) = 3$.

Theorem:3.3 Let G$\cong K_{1,n}$ where $n \geq 3$ if and only if $\gamma_3'(K_{1,n}) = 1$

Proof: Suppose that $G \cong K_{1,n}$, $n \geq 3$. Let v be an universal vertex of G. Then v is a neighbor to all vertices and edges of G in G^{+-+}. Therefore $S = \{v\}$ is an complementary 3-dominating set of G^{+-+}. Therefore $\gamma_3'(K_{1,n}) = 1$. Conversely we assume that $\gamma_3'(K_{1,n}) = 1$ and S be a minimum complementary 3-dominating set of G^{+-+}. If $S = \{w\}$ then w occurs in exactly two vertices of G and thus $G \cong K_2 \cong K_{1,1}$. If $S = \{v\}$ then v is adjacent to every vertices and coincides with edges of G. Hence $G \cong K_{1,n}$.

Theorem:3.4 For any complete graph $\gamma_3'(K_n^{+-+}) = 3$

Proof: Let $V(K_n) = \{v_1, v_2, \ldots., v_n\}$. By theorem $\gamma_3'(K_n^{+-+}) \geq 2$. Let S be a subset of $V(K_n^{+-+})$. Suppose $S = \{v_r, v_s\}$. Since $n \geq 4$, there exists two vertices v_t and v_u such that $v_t v_u$ in $E(G)$ which is adjacent to neither $v_r\ nor\ v_s$ in K_n^{+-+}, Suppose $v_r \neq v_s$ then since $n \geq 4$ there exists a vertex v_u such that $v_s v_r, v_t v_u$ in $E(G)$ adjacent to neither v_r nor $v_s v_t$ in K_n^{+-+}. Suppose $S = \{v_s v_r, v_t v_u\}$. If $v_s v_r\ and\ v_t v_u$ are adjacent in K_n, then there is a vertex v_w in $V(K_n)$ which is not adjacent to $v_s v_r\ and\ v_t v_u$ in K_n^{+-+}. If $v_s v_r\ and\ v_t v_u$ are non adjacent in K_n then $v_s v_r, v_t v_u$ are not adjacent to $v_s v_r\ and\ v_t v_u$ in K_n^{+-+}. Thus S is not a $\gamma_3' - set$ of K_n^{+-+}.

Let $v_r\ and\ v_s$ be any two vertices in K_n. Then v_r dominates every vertices of K_n in K_n^{+-+}. All edges which are adjacent to $v_r v_s$ are dominated by $\{v_r, v_s\}$ in K_n^{+-+} and the other edges of K_n which are non adjacent to $v_r v_s$ in K_n^{+-+}. Hence $\{v_r, v_s, v_r v_s\}$ is a complementary 3-dominating set of K_n^{+-+}. Therefore $\gamma_3'(K_n^{+-+}) = 3$.

Theorem: 3.5 For a complete bipartite graph $K_{m,n}$, $\gamma_3'(K_{m,n}^{+-+}) = 3$

Proof: Let (T, U) be the bipartition of $K_{m,n}$ with $|T| = m$ and $|U| = n$. Let S be the subset of $V(K_{m,n}^{+-+})$. Suppose $S = \{t, u\}$. If $t, u \in T$, then there is a vertex $v \in T$ which is adjacent to neither t nor u in $K_{m,n}^{+-+}$. If $t \in T$ and $u \in U$ then there exists an $'q'$ is not adjacent to t and u in $K_{m,n}^{+-+}$. If t and u are edges of $K_{m,n}$ then atmost four vertices of $K_{m,n}$ re dominated by S. Therefore there is a vertex in $K_{m,n}$ which is not adjacent to t and y in $K_{m,n}^{+-+}$. If $t \in T$ and $u \in E(K_{m,n})$ then atmost two vertices of T are dominated by S in $K_{m,n}^{+-+}$. Hence there is a vertex $v \in T$ which is non adjacent to t and u in $K_{m,n}^{+-+}$. Therefore S is not a $\gamma_3' - set$ of $K_{m,n}^{+-+}$.

Suppose $x \in T$ and $y \in U$. Then every vertices of $K_{m,n}$ and every edges which are adjacent to xy are dominated by $\{x, y\}$ in $K_{m,n}^{+-+}$. Also, the other edges which are non adjacent to xy are dominated by xy. Therefore $\{x, y, xy\}$ is a complementary 3-dominating set of $K_{m,n}^{+-+}$. Hence $\gamma_3'(K_{m,n}^{+-+}) = 3$.

Theorem:3.6 For any wheel graph W_n of order $n \geq 4, \gamma_3'(W_n^{+-+}) = 3$.

Proof: Let $u_1, u_2, \dots, u_{n-1}$ be the vertex of degree 3 and u be the vertex of degree $n-1$ in W_n. Let $e_i = uu_i$ where $1 \leq i \leq n-1, e_{i(i+1)} = u_i u_{i+1}$ where $1 \leq i \leq n-2$ and $e_{1(n-1)} = u_1 u_{n-1}$. If $n = 4$ then by theorem $\gamma_3'(W_n^{+-+}) = 3$. Let us consider the cases for $n > 4$. Suppose $\{u, u_i\}$ is not a complementary 3-dominating set of W_n^{+-+}. Since there is an edge e_{rs} which is non adjacent to u and u_i in W_n^{+-+}, $\{u_i, u_r\}$ is not a complementary 3-dominating set of W_n^{+-+}. Since there exists an edge e_s which is adjacent to neither u_i nor u_r in W_n^{+-+}. Since any two edges of W_n are adjacent to atmost four vertices of W_n in W_n^{+-+}, no two elements of $E(W_n)$ is a complementary 3-dominating set of W_n^{+-+} since adjacent edges of e_i in C_{n-1} are adjacent to neither u nor e_i in W_n^{+-+}. Since adjacent edges of e_{ir} in C_{n-1} are adjacent to neither u nor e_{ir} in W_n^{+-+}, $\{u, e_{ir}\}$ is not a complementary 3-dominating set of W_n^{+-+}. $\{u_i, e_r\}$ is not a complementary 3-dominating set of W_n^{+-+}since there is an edge e_t which is neither u_i nor e_r. $\{u_i, e_{rs}\}$ is not a complementary 3-dominating set of W_n^{+-+} since atleast one of $\{e_r, e_s\}$ is adjacent to neither u_i nor e_{rs}. Hence no two element of $V(W_n^{+-+})$ is a complementary 3-dominating set of W_n^{+-+}.

Suppose $S' = \{u, e_{ir}, e_{rs}\}$ then u is adjacent to every vertices of W_n and every spokes of W_n in W_n^{+-+}, e_{ir} is adjacent to every non adjacent edges of e_{ir}, e_{rs} is adjacent to every adjacent edges of e_{ir} in C_{n-1}. Hence S' is a complementary 3-dominating set of W_n^{+-+}. Therefore $\gamma_3'(W_n^{+-+}) = 3$.

Theorem:3.7 For any connected graph with maximum degree $n-2$, $\gamma_3'(G^{+-+}) \leq 3$.

Proof: Let u be the vertex of maximum degree and $V - N[u] = \{v\}$. Let v be adjacent to u_i in $N[u]$. Then u dominates all the vertices except v of G and every edges incident with u in G^{+-+}. Also u_i dominates v and every edges incident with u_i in G^{+-+}. Furthermore all the other edges are dominated by uu_i in G^{+-+}. Thus $\gamma_3'(G^{+-+}) \leq 3$.

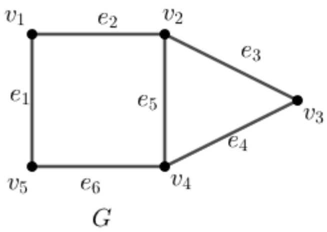
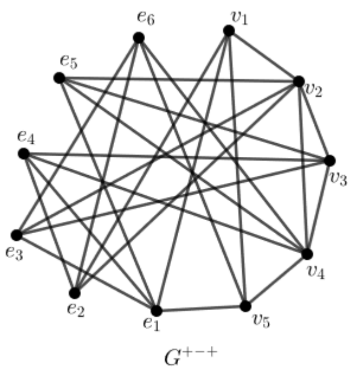


Fig :3.2



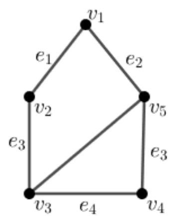
In the above example $\{e_1, e_6, v_3\}$ is a complementary 3-dominating set for G^{+-+}. Hence $\gamma_3'(G^{+-+}) = 3$ and the sharpness is attained.

Theorem:3.8 If G is a graph with $diam(G) = 2$ then $\gamma_3'(G^{+-+}) \leq \delta(G) + 1$ and the bound is sharp.

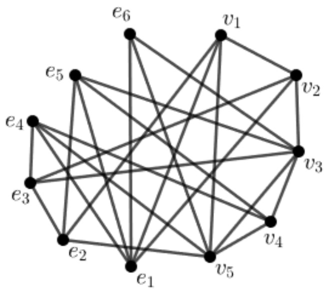
Proof: Let u be a vertex in $V(G)$ such that $d(u) = \delta(G)$ and $N(u) = \{u_1 u_2 u_3 \dots \dots . u_\delta\}$. Then uu_i is adjacent to every non adjacent edges of uu_i in G^{+-+}. Also $N(u)$ dominates all the edges incident with u or u_i and every vertices of G in G^{+-+}.Therefore $N(u) \cup \{uu_i\}$ is a complementary 3-dominating set of G^{+-+}. Hence $\gamma_3'(G^{+-+}) \leq \delta(G) + 1$. The sharpness is attained for C_n where $n > 4$ and P_n where $n > 3$.

Theorem:3.9 If G is any connected graph with $\Delta(G) < n - 1$ then $\gamma_3'(G^{+-+}) \leq n - \Delta(G) + 1$ and the bound is sharp.

Proof: Suppose $d(u) = \Delta(G)$ and $N(u) = \{u_1 u_2 u_3 \dots \dots . u_\Delta\}$. Since G is connected there is a vertex v in $V(G) - N[u]$ such that v is adjacent to atleast one vertex u_i in $N(u)$. Then $[V(G) - N(u)] - \{u\}] \cup \{u_i\}$ dominates every vertices of G. Now u dominates every edges incident with u in G^{+-+}. The vertex u_i dominates every edges incident with u_i and the edges uu_i dominates every non adjacent edges of uu_i in G^{+-+}. Hence $[V(G) - N(u)] - \{v\}] \cup \{u_i, uu_i\}$ is a complementary 3-dominating set of G^{+-+}. Thus $\gamma_3'(G^{+-+}) \leq n - \Delta(G) + 1$.



G



G^{-+-}

Fig:3.3

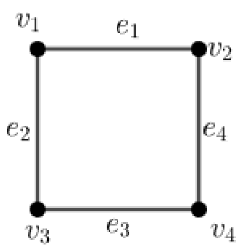
For the above graph the complementary 3-dominating set is $\{v_2, v_5, e_2\}$. Hence $\gamma_3'(G^{+-+}) = 3$. Therefore $\gamma_3'(G^{+-+}) \leq n - \Delta(G) + 1 = 5 - 3 + 1 = 3$, the bound is sharp.

Theorem:3.10 For any connected graph G, $\gamma(G) \leq \gamma_3'(G^{+-+}) \leq \gamma(G) + 2$ and the bound is sharp.

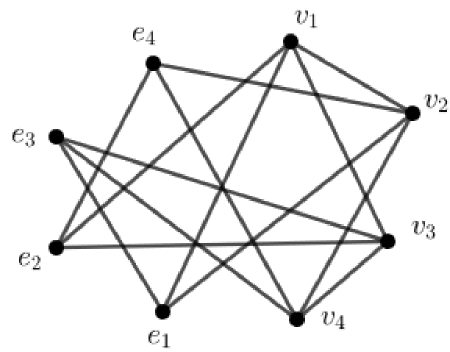
Proof: To prove this theorem we consider two cases. Let S be the minimum dominating set of G and S' be the minimum complementary 3-dominating set of G^{+-+}.

***Case*(1)**: $\gamma(G) \leq \gamma_3'(G^{+-+})$

Assume that $\gamma(G) > \gamma_3'(G^{+-+})$. Then $|S| > |S'|$. If S' has no dges of G then S is a complementary 3-dominating set of G with $|S'| < |S|$, which is a contradiction. If S' contains $q \geq 1$ edges of G say $u_1 v_1, u_2 v_2, \dots \dots, u_q v_q$ then this q edges dominate $q_1 \leq 2q$ vertices of G in G^{+-+}. Therefore $[S'^{-} u_1 v_1, u_2 v_2, \dots \dots, u_q v_q\}] \cup \{u_1, u_2, \dots \dots, u_q\}$ is a complementary 3-dominating set with cardinality $|S'| < |S|$ which is a contradiction. Therefore $\gamma(G) \leq \gamma_3'(G^{+-+})$.



C_4



C_4^{+-+}

Fig :3.4

Illustration for which $\gamma(G) = \gamma_3'(G^{+-+})$

***Case*(2)**: $\gamma'_3(G^{+-+}) \leq \gamma(G) + 2$

Every vertices of G are dominated by S in G^{+-+}. If G is disconnected then $\gamma'_3(G^{+-+}) = \gamma(G) = n$. If G is connected then G has an edge $e = uv$ whose one end is in S. Assume that $u \in S$, then all the edges adjacent to e adjacent to u or v in G^{+-+} and every edges which is not adjacent to uv in G are adjacent to uv in G^{+-+}. Hence $S \cup \{uv, v\}$ is a complementray 3-dominating set of G^{+-+}. Therefore $\gamma'_3(G^{+-+}) \leq \gamma(G) + 2$.

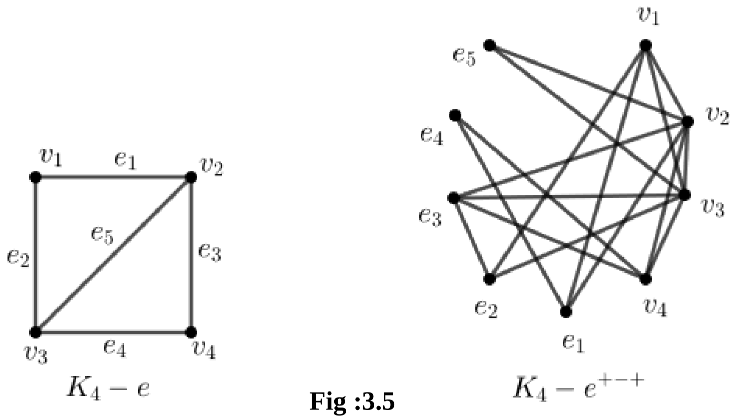


Fig :3.5

Illustration for which $\gamma'_3(G^{+-+}) = \gamma(G) + 2$

Theorem:3.11 If G is any connected graph of order $n = 6$ then $\gamma'_3(G^{+-+}) \leq 3$ and the sharpness is attained.

Proof: Suppose $\gamma(G) = 1$ then by theorem 3.10 $\gamma'_3(G^{+-+}) \leq 3$. If $\gamma(G) = 2$ and $\delta(G) = 1$ then $\gamma'_3(G^{+-+}) \leq 3$. Suppose $\gamma(G) = 3$ then $G \cong H^\circ K_1$ where H is isomorphic to P_3 or C_3. Then $V(H)$ is a minimum complementary 3-dominating set of G^{+-+}. Hence $\gamma'_3(G^{+-+}) = 3$. Let us assume that $\delta(G) \geq 3$. Suppose if $\Delta(G) = 5$ *or* 4, then by theorem 3.7 $\gamma'_3(G^{+-+}) \leq 3$. If $\Delta(G) = 2$ then G is isomorphic to C_6 and hence $\gamma'_3(G^{+-+}) = 3$. Now let us asume that $\Delta(G) = 3$. Let u be a vertex of degree 3. Let $N(u) = \{u_1, u_2, u_3\}$ and $V - N[u] = \{v_1, v_2\}$. If $N(v_1) \cap N(v_2) \neq \emptyset$ in $N(u)$, then let $u_i \in N(v_1) \cap N(v_2)$. Now u dominates every vertices of $N[u]$ and every edges which are incident with u in G^{+-+}. Also, uu_i dominates all the other edges which are non adjacent to uu_i. Hence $\{u, u_i, uu_i\}$ is a complementary 3-dominating set of G^{+-+}. Suppose $N(v_1) \cap N(v_2) = \emptyset$ in $N(u)$, then let v_1 be adjacent to u_1 and v_2 be adjacent to u_2. Then u_1v_1 dominates v_1, u_1 and every edges which are non adjacent to v_1u_1 in G^{+-+}, u_2v_2 dominates v_2, u_2 and all the other edges which are adjacent to v_1u_1 except v_1v_2, u_1u_2 in G^{+-+}, uu_3 dominates u, u_3 and the edges v_1v_2, u_1u_2 in G^{+-+}. Hence $\{v_1u_1, v_2u_2, uu_3\}$ is a complementary 3-dominating set of G^{+-+}. Therefore $\gamma'_3(G^{+-+}) \leq 3$.

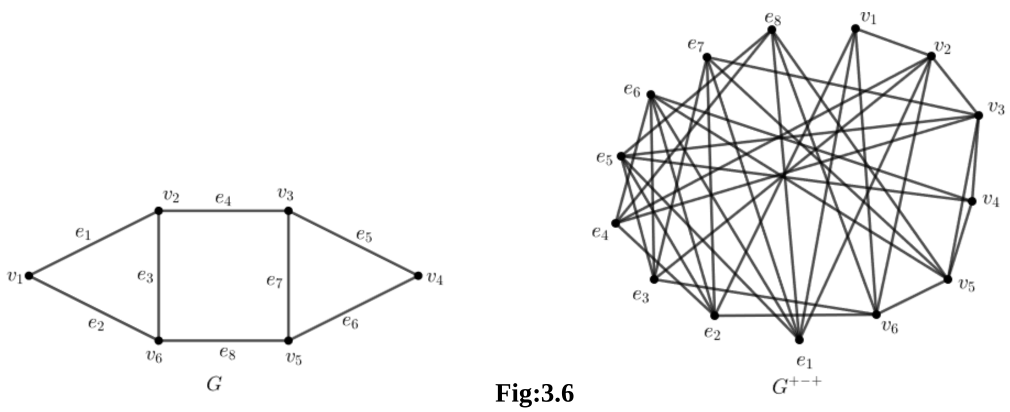


Fig:3.6

Illustration for which $\gamma'_3(G^{+-+}) = 3$

4.Transformation graph G^{-+-}

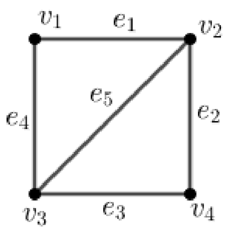
Definition: 4.1 A transformation graph G^{-+-} of a simple graph G with vertex set $V(G) \cup E(G)$ in which adjacency is defined by,

(a) Two elements in $V(G)$ are adjacent iff they are non adjacet in G.

(b) Two elements in $E(G)$ are adjacent iff they are adjacent in G.

(c) One element in $V(G)$ and one element in $E(G)$ are adjacent iff they are non adjacent in G.

Example: 4.2



$C_4 + e$

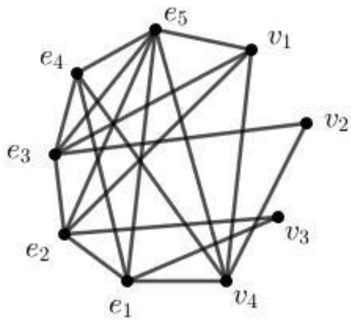


Fig :4.1

$(C_4 + e)^{-+-}$

In the above example $\{v_4, e_2\}$ is a complementary 3-dominating set and hence $\gamma_3'(C_4 + e)^{-+-} = 2$.

Theorem:4.3 Let G be any connected graph of order $n > 4$, $\gamma_3'(G^{-+-}) \leq 3$.

Proof: Let G be any connected graph and $x, y \in V(G), e = xy \in E(G)$. We have to claim that $S = \{x, y, xy\}$ is a complementary 3-dominating set of G^{-+-}. Now e dominates every vertices except x and y every edgs incident with x and y. Also x dominates itself and every edges which are not incident with x and y dominates itself. Hence S is a $\gamma_3' -$ set in G^{-+-} and $|S| = 3$. Therefore $\gamma_3'(G^{-+-}) \leq 3$.

Theorem:4.4 For any connected graph G of order $n \geq 3, \gamma_3'(G^{-+-}) = 1$ if and only if G has an isolated vertex.

Proof: Let G be a connected graph and assume that $\gamma_3'(G^{-+-}) = 1$. Suppose G has no isolated vertex then all the vertices y is adjacent with some e=xy. Now y will non-adjacent to x and xy in G^{-+-} and xy will be non-adjacent to x and y in G^{-+-}. Thus $S \neq \{xy\}$ and $S \neq \{y\}$. Therefore $\gamma_3'(G^{-+-}) \geq 2$ is a contradiction. Hence there exists an isolated vertex in G. Conversely let us assume that y is an isolated vetex of G. By the definition of a transformation graph G^{-+-}, y is adjacent to every vertices in $V(G) \cup E(G)$ in G^{-+-}. Therefore $S = \{y\}$ is a $\gamma_3' -$ set. Hence $\gamma_3'(G^{-+-}) = 1$.

Observation:4.5 Let G be any connected graph with no isolated vertices of order $n > 4, \gamma_3'(G^{-+-}) \geq 2$.

Theorem:4.6 Let G be any connected graph of order $n > 4$ with atleast two pendant edges having no vertex in common $\gamma_3'(G^{-+-}) = 2$.

Proof: Let G be any connected graph of order $n > 4$ with no common vertex and having atleast two pendant edges. Let $e_1 = x_1 y_1$ and $e_2 = x_2 y_2$ be the pendant edges of G having no vertex in common and y_1 and y_2 be the pendant vertices. By observation $\gamma_3'(G^{-+-}) \geq 2$. Now, except e_1 and x_1, y_1 is adjacent to every elements of G^{-+-} and y_2 is adjacent with e_1 and x_1. Hence $\{y_1, y_2\}$ is a γ_3' −set of G^{-+-}. Therefore $\gamma_3'(G^{-+-}) = 2$.

Theorem:4.7 Let G be a graph without isolated vertices of order $n = 4$ *or* 5. If G is not isomorphic to $K_{1,n}$ then $\gamma_3'(G^{-+-}) = 2$.

Proof: Let G be a graph without isolated vertices then by theorem 4.4 $\gamma_3'(G^{-+-}) \neq 1$.

Case: (1) G is connected

By hypothesis, there exists two independent edges of G say e_1 and e_2. Then all the edges of G is adjacent to any one of e_1 and e_2 and all the vertices of G is not incident with atleast one of e_1 and e_2. Thus $\{e_1, e_2\}$ is a $\gamma_3' -$ set of G^{-+-}.

Case: (2) G is disconnected

Then G has two components say H_1 and H_2. Now, $x \in V(H_1)$ dominates every vertices and edges of H_2 and $y \in V(H_2)$ each points and egde of H_1. Therefore {x,y} is a $\gamma_3' -$ set of G^{-+-}. Hence $\gamma_3'(G^{-+-}) = 2$.

Theorem:4.8 Let G be any connected graph of order $n > 4$. For the following cases the complementary 3-dominating set in G^{-+-} has no 2-element sets.

(a) $u, v \in V(G)$ where u and v are adjacent.

(b) Two adjacent edges of G.

(c) An edge and one of its end points in G.

Proof: Let $u, v \in V(G^{-+-})$ and $S = \{u, v\}$

Case: (1) If u and v are adjacent in G then the edge uv in G is adjacent to neither u nor v in G^{-+-}.

Case: (2) If u and v are adjacent edges in G then they incident with a common vertex say $w \in V(G)$ but w is adjacent to neither u nor v in G^{-+-}.

Case: (3) Suppose if $u = xy$ in $E(G)$ and $v = x$ in $V(G)$ then y is adjacen tto neither u nor v in G^{-+-}. Therefore in all the above cases S is not a complementary 3-dominating set of G^{-+-}.

Observation:4.9 Let G be any connected graph of order $n > 4$ with $diam(G) = 2$ and $S = \{u, v\}$ in $V(G^{-+-})$. If u,v are non adjacent vertices of G and $v \in V(G), u \in E(G)$ and v is not an end vertes of u, then S is not a $\gamma_3' -$ set of G^{-+-}.

Theorem:4.10 For any connected graph of order $n > 4$ with $diam(G) \geq 2, \gamma_3'(G^{-+-}) = 2$.

Proof: Let G be a connected graph of order $n > 4$ and $diam(G) \geq 2$.

Case: (1) If $diam(G) = 2$

Let us assume that $S' = \{e_1, e_2\}$ be a minimum independent edge dominating set of G. Since S' is an edge dominating set, S' will dominate all edges of G in G^{-+-}. Since e_1 and e_2 are indeendent every points except the terminal points of e_1 will dominated by e_1 and terminal points of e_1 will dominated by e_2. Thus $\{e_1, e_2\}$ is a γ_3' $-$set of G^{-+-}. Therefore $\gamma_3'(G^{-+-}) = 2$.

Case: (2) If $diam(G) \geq 3$.

Let $diam(G) \geq 3$. There exists two vertices x and y such that $d(x, y) = 3$. If suppose $y \in N(y)$ in G^{-+-} is not dominated by x then x is adjacent to y_i in G. Then there exists a x, y path (x, y_i, y) of length two in G which contradics $d(x, y) = 3$. Hence x and y dominates $N[y]$ and x respecively. Suppose x does not dominate an edge e which is coinciding with y in G. Then e coincides with x in G. Then $e = xy$ contadicts the choice of x and y. Therefpre x dominates y on all edges of G. Similarly y dominated all edges accompanying x. All other vertices and edges are dominant for both x and y. Therefore $\{x, y\}$ is a γ_3' $-$set in G^{-+-} . Hence $\gamma_3'(G^{-+-}) = 2$.

Conclusion: In this paper we determined complementary 3-domination number for some graphs and obtained some results in trasnsformation graph concerning this parameter.

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