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Strong and Weak 2-Vertex Duplication Self Switching of a Disconnected Graphs

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Abstract: A vertex v' in G' is said to be a duplication of v in G, if all the vertices which are adjacent to v are also adjacent to v'. $2-$vertex duplication of a graph G is the duplication of any two vertices $u, v \in V(G)$ is u', v' such that u', v' are adjacent to all the vertices that are adjacent to u, v. The $2-$vertex duplication switching of G by $\sigma = \{x, y\}$ is the graph obtained by duplicating any two vertices u, v then by removing all existing edges between σ and its complement $V - \sigma$ in $D((u, v)G)$ and also by adding edges between σ and $V - \sigma$ which are not in G, without affecting the adjacency and non-adjacency of vertices in σ.

Keywords: *$2-$vertex duplication, $2-$vertex duplication switching, $D\big((u, v)G\big)$, $D((u, v)G)^{\{u,v\}}$, $D((u, v)G)^{\{x,y\}}$, $dss_2(G)$*

1. Introduction

For a finite undirected simple graph $G(V, E)$ with $|V(G)| = p$ and a non-empty set $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^{\sigma}(V, E')$ which is obtained from G by removing all edges between σ and its complement, $V - \sigma$ and adding as edges all non-edges between σ and $V - \sigma$. Switching has been defined by Seidel [6] and is also referred to as Seidel switching. The concept of duplication self vertex switching was introduced by C. Jayasekharan and V. Prabhavathy [1,2] . Duplication of a vertex v of a graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. It is denoted by $D(vG)$. A vertex v is called a duplication self vertex switching of a graph G if the resultant graph obtained after duplication of v has v as a self vertex switching. The concept of $2-$vertex duplication self switching is introduced by G. Sumathy and K.S Shruthi[4]. **$2-$vertex duplication** of a connected graph G is the duplication of any two vertices $u, v \in V(G)$ is u', v' such that u', v' are adjacent to all the vertices that are adjacent to u, v.It is denoted by $D\big((u, v)G\big)$. Let $\sigma = \{x, y\} \subseteq V(G)$ is called a **$2-$vertex duplication self switching** of a connected graph G if $D((u, v)G) \cong D((u, v)G)^{\sigma}$. If $\sigma = \{u, v\} \subseteq V(G)$, then σ is called **strong $2-$vertex duplication self switching** of connected graph G. It is denoted by $D((u, v)G)^{\sigma}$. If $\lambda = \big\{\{x, y\}/\{x, y\} \neq \{u, v\}\big\} \subseteq V(G)$, then λ is called the **weak $2-$vertex duplication self switching** of G. It is denoted by $D((u, v)G)^{\lambda}$. The number of $2-$vertex duplication self switching is denoted by $\boldsymbol{dss_2(G)}$.

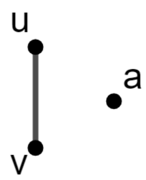
A graph G is called **disconnected** if it is not connected. In this paper we analysed strong and weak $2-$vertex duplication self switching of a disconnected graphs with examples.

Theorem[1.1]: If $\sigma = \{u, v\} \subseteq V(G)$ is a strong $2-$vertex duplication self switching of a graph G, then $d_G(u) + d_G(v) = p$.

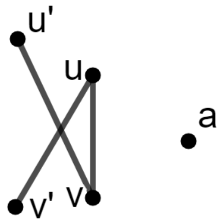
2. Characterization of strong and weak 2 −vertex duplication self switching of a disconnected graphs.

Definition 2.1: The **strong 2 −vertex duplication switching of a disconnected graph G** by σ is the graph obtained by duplicating any two vertices u, v and by removing all edges between σ and its complement $V-\sigma$ also by adding all edges between σ and $V-\sigma$ which are not in G, without affecting the adjacency and non-adjacency of vertices in σ.

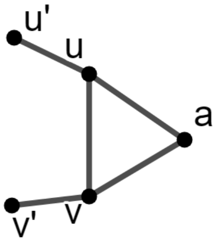
Example 2.2: A disconnected graph G, the duplicated graph $D((u,v)G)$ of G and the strong 2 −vertex duplication switching are given in figures 2.2.1 to 2.2.3



$Fig\ 2.2.1$
G



$Fig\ 2.2.2$
$D((u,v)G)$



$Fig\ 2.2.3$
$D((u,v)G)^{\{u,v\}}$

Definition 2.3: If $D((u,v)G) \cong D((u,v)G)^{\{u,v\}}$, then $\sigma = \{u,v\}$ is a **strong 2-vertex duplication self switching of a disconnected graph G**.

Theorem 2.4: If G is a disconnected graph, then $dss_2(\mathrm{G}) = 0$, for a strong 2 −vertex duplication self switching.

Proof: Let G be a disconnected graph. Then G has atleast two components. Let the k components of G be $G_1, G_2, \ldots., G_k, k \geq 2$. Consider any two vertices u, v of G. Let u', v' be the duplication of (u, v). Since G has k components, $D((u,v)G)$ also has k components.Without loss of generality let us assume that,

Case (i): $u, v \in V(G_i)$,$1 \leq i \leq k$

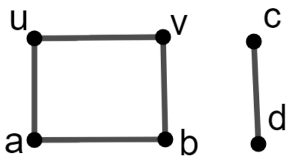
Let $G_1, G_2, \ldots, D((u,v)G_i), \ldots.., G_k$ be the duplication of k components of G. Clearly $u, u', v, v' \in V(D(u,v)G_i)$. Since $u, v \in V(D(u,v)G_i)$, u *and* v are non-adjacent to all the vertices in $G_j, 1 \leq j \leq k, j \neq i$.Therefore, u, v is non-adjacent to atleast one vertex in each and every of component of $D\big((u,v)G\big)$ other than G_i. This implies that $D((u,v)G)^{\{u,v\}}$ is connected. That is, $D((u,v)G) \ncong D((u,v)G)^{\sigma}$. Hence $dss_2(G) = 0$.

Case (ii): $u \in V(G_i)$ & $v \in V\big(G_j\big), 1 \leq i < j \leq k$.

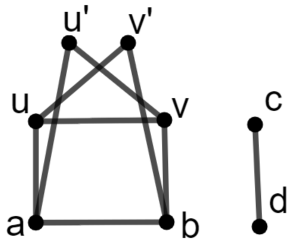
Let $G_1, G_2, \ldots, D((u)G_i) \ldots, D((v)G_j), \ldots.., G_k$ be the duplication of k components of G. Clearly $u, u' \in V(D(u)G_i)$ & $v, v' \in V\big(D(v)G_j\big)$. Therefore, u and v is non-adjacent to atleast one vertex in each component of $D\big((u,v)G\big)$ other than $D(u)G_i$ and $D(v)G_j$ This implies that $D((u,v)G)^{\{u,v\}}$ is connected. That is, $D((u,v)G) \ncong D((u,v)G)^{\sigma}$. Hence $dss_2(G) = 0$.

Definition 2.5: If $\lambda = \big\{\{x,y\}/\{x,y\} \neq \{u,v\}\big\} \subseteq V(G)$. Then λ is called a **weak 2 −vertex duplication self switching of a disconnected graph *G***.(ie) $D((u,v)G) \cong D((u,v)G)^{\{x,y\}}$.

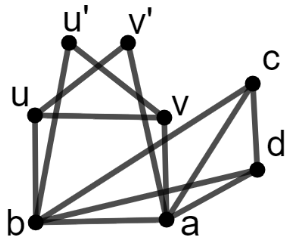
Example 2.6: The graph given in the figures 2.6.1 to 2.6.3 has weak 2 −vertex duplication self switching of a disconnected graph G.



$Fig\ 2.6.1$
G



$Fig\ 2.6.2$
$D((u,v)G)$



$Fig\ 2.6.3$
$D((u,v)G)^{\{a,b\}}$

Theorem 2.7: If $\lambda = \{x, y\} \subseteq V(G)$ is a weak $2-$vertex duplication self switching of a disconnected graph G, then $d_G(x) + d_G(y) = p - 2$, where x and y lies in two different components of G.

Proof: Let $\lambda = \{x, y\} \subseteq V(G)$ be a disconnected weak $2-$vertex duplication self switching of a graph G. Then G has atleast two components, let it be $G_1, G_2, \ldots, G_k, k \geq 2$. Consider any two vertices u, v of G_i. Let u', v' be the duplication of u, v. Since G has k components, duplication of G also has k components.Without loss of generality let us we assume that,

Case (i): $x, y \in V(G_i)$

Let $G_1, G_2, \ldots, D((u, v)G_i) \ldots.., G_k$ be the duplication of k components of G. Clearly $x, u', y, v' \in V(D(u, v)G_1)$. Since $x, y \in V(D(u, v)G_i)$,Clearly $xu', yu' \in V(D(u, v)G_i)$ or $xu', yv' \in V\big(D(u, v)G_j\big)$. Here x and y is non-adjacent to the vertices in G_j for all $i \neq j$. Therefore $D((u, v)G) \ncong D((u, v)G)^{\{x,y\}}$.

Case (ii): $x \in V(G_i)$ & $y \in V\big(G_j\big), i \neq j$.

Let $G_1, G_2, \ldots D\big((u)G_i\big), D((v)G_j), \ldots.., G_k$ be the duplication of k components of G and $xu \in E(G_i)$ and $yv \in E\big(G_j\big)$. Clearly xu' and $yv' \in D\big((u)G_i\big)$ and $D((v)G_j)$ respectively. In $D((u, v)G)^{\{x,y\}}$ x is adjacent to v, v' and y is adjacent to u, u'. Therefore $D((u, v)G) \cong D((u, v)G)^{\lambda}$. Hence $|E(D(u, v)G)| = \big|E(D(u, v)G)^{\lambda}\big|$.

This implies that,

$$q + d_G(x) + d_G(y) = \quad q + d_G(x) + d_G(y) + \big[p + 2 - 1 - d_{D((u,v)G)}(x)\big] - d_{D((u,v)G)}(x) + \big[p + 2 - 1 - d_{D((u,v)G)}(y)\big] - d_{D((u,v)G)}(y) - 2.$$

$0 \ = p + 1 - [d_G(x) + 1] - (d_G(x) + 1) + p + 1 - [d_G(y) + 1] - (d_G(y) + 1) - 2.$

$0 \ = 2p + 2 - 2d_G(x) - 2 - 2d_G(y) - 2 - 2.$

$0 \ = 2p - 2d_G(x) - 2d_G(y) - 4$

$p - 2 = d_G(x) + d_G(y).$

Theorem 2.8: If G is a weak $2-$vertex duplication self switching disconnected graph, then for $2 \leq n \leq 3$,

(1) $dss_2(P_2 \cup P_2) = \begin{cases} 1, & if \ \ xy \notin E(P_2 \cup P_2) \\ 0 & otherwise. \end{cases}$

(2) $dss_2(P_3 \cup P_3) = \begin{cases} 1, & if \ \ xy \notin E(P_3 \cup P_3) \\ 0 & otherwise. \end{cases}$

(3) $dss_2(P_2 \cup P_3) = \begin{cases} 1 & if \ \ xy \notin E(P_2 \cup P_3) \\ 0 & otherwise. \end{cases}$

Proof: Let G be the disconnected graph consists of two components G_1 and G_2. Let $u, v \in V(G)$ be the duplication of G. Let $D((u,v)G)$ be the duplicated components of G and $\lambda = \{x, y\}$ be a weak $2-$ vertex duplication self switching of G. There arises three cases.

Case(1): $G_1 = G_2 = P_2$.

There arises two subcases.

Subcase(1(i)): If $xy \in E(G)$

Clearly $x, y \in V(G_1)$. Here the graph gets connected after switching x and y. Then $D((u,v)G) \ncong D((u,v)G)^{\lambda}$. Therefore $\lambda = \{x, y\}$ is not a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 0$.

Subcase(1(ii)): If $xy \notin E(G)$

Then $x \in V(G_1)$ and $y \in V(G_2)$. Also u and v lies in the two different components of G. Clearly $xu \in E(G_1)$ and $yv \in E(G_2)$. The duplicated vertices u' and v' lies in G_1 and G_2 respectively. This implies that $D((u,v)G) \cong D((u,v)G)^{\lambda}$. Therefore, $\lambda = \{x, y\}$ be a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 1$.

Case(2): $G_1 = P_3$ and $G_2 = P_3$

There arises two subcases.

Subcase(2(i)): If $xy \in E(G)$

Clearly $x \in V(G_1)$ and $x \in V(G_2)$. Here the graph gets connected after switching x and y. Then $D((u,v)G) \ncong D((u,v)G)^{\lambda}$. Therefore $\lambda = \{x, y\}$ is not a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 0$.

Subcase(2(ii)): If $xy \notin E(G)$

Then $x \in V(G_1)$ and $y \in V(G_2)$. Also u and v lies in the two different components of G. Clearly $xu \in E(G_1)$ and $yv \in E(G_2)$. The duplicated vertices u' and v' lies in G_1 and G_2 respectively. This implies that $D((u,v)G) \cong D((u,v)G)^{\lambda}$. Therefore, $\lambda = \{x, y\}$ be a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 1$.

Case(3): $G_1 = P_2$ and $G_2 = P_3$

There arises two subcases.

Subcase(3(i)): If $xy \in E(G)$

Clearly $x \in V(G_1)$ and $x \in V(G_2)$. Here the graph gets connected after switching x and y. Then $D((u,v)G) \ncong D((u,v)G)^{\lambda}$. Therefore $\lambda = \{x, y\}$ is not a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 0$.

Subcase(3(ii)): If $xy \notin E(G)$

Then $x \in V(G_1)$ and $y \in V(G_2)$. Also u and v lies in the two different components of G. Clearly $xu \in E(G_1)$ and $yv \in E(G_2)$. The duplicated vertices u' and v' lies in G_1 and G_2 respectively. This implies that $D((u,v)G) \cong D((u,v)G)^{\lambda}$. Therefore, $\lambda = \{x, y\}$ be a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 2$.

Theorem 2.9: If G is a weak $2-$vertex duplication self switching disconnected graph, then for $2 \le n \le 3$,

(1) $dss_2(K_2 \cup P_2) = \begin{cases} 1, & if\ \ xy \notin E(K_2 \cup P_2) \\ 0 & otherwise. \end{cases}$

$$(2)\ dss_2(K_3 \cup P_2) = \begin{cases} 2, & if\ \ xy \notin\ E(K_3 \cup P_2) \\ 0 & otherwise. \end{cases}$$

$$(3)\ dss_2(K_3 \cup P_3) = \begin{cases} 2, & if\ \ xy\ \notin E(K_3 \cup P_3) \\ 0 & otherwise. \end{cases}$$

Proof: Let G be the disconnected graph consists of two components G_1 and G_2. Let $u, v \in V(G)$ be the duplication of G. Let $D((u,v)G)$ be the duplicated components of G and $\lambda = \{x, y\}$ be a weak $2-$ vertex duplication self switching of G. There arises three cases.

Case(1): $G_1 = K_2$ and $G_2 = P_2$.

Since $K_2 \cong P_2$. By theorem 2.4 $dss_2(K_2 \cup P_2) = 1$

Case(2): $G_1 = K_3$ and $G_2 = P_2$.

There arises two subcases.

Subcase(2(i)): If $xy \in E(G)$

Either $xy \in G_1$ or G_2. Here the graph gets connected after switching x and y. Then $D\big((u,v)G\big) \ncong D((u,v)G)^{\lambda}$. Therefore $\lambda = \{x, y\}$ is not a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 0$.

Subcase(2(ii)): If $xy \notin E(G)$

Then $x \in V(G_1)$ and $y \in V(G_2)$. Also u and v lies in the two different components of G. Clearly $xu \in E(G_1)$ and $yv \in E(G_2)$. The duplicated vertices u' and v' lies in G_1 and G_2 respectively. This implies that $D\big((u,v)G\big) \cong D((u,v)G)^{\lambda}$. Therefore, $\lambda = \{x, y\}$ be a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 2$.

Case(3): $G_1 = K_3$ and $G_2 = P_3$.

There arises two subcases.

Subcase(3(i)): If $xy \in E(G)$

Either $xy \in G_1$ or G_2. Here the graph gets connected after switching x and y. Then $D\big((u,v)G\big) \ncong D((u,v)G)^{\lambda}$. Therefore $\lambda = \{x, y\}$ is not a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 0$.

Subcase(3(ii)): If $xy \notin E(G)$

Then $x \in V(G_1)$ and $y \in V(G_2)$. Also u and v lies in the two different components of G. Clearly $xu \in E(G_1)$ and $yv \in E(G_2)$. The duplicated vertices u' and v' lies in G_1 and G_2 respectively. This implies that $D\big((u,v)G\big) \cong D((u,v)G)^{\lambda}$. Therefore, $\lambda = \{x, y\}$ be a weak $2-$ vertex duplication self switching of G. Hence $dss_2(G) = 2$.

2. Conclusion

In this paper we have discussed about strong and weak $2-$vertex duplication self switchings of disconnected graphs and further we are analyzing more in disconnecting graphs.

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