Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 1 (2024)

Divided Square Difference Cordial Labeling of Some Spider Graphs

T. Christy[1], G. Palani[* 2]
[1]Assistant Professor, Department of Mathematics,
Patrician College of Arts and Science,
Affiliated to University of Madras, Chennai
[2]Assistant Professor, Department of Mathematics,
Dr. Ambedkar Government Arts College, Chennai
Corresponding Author
Email: gpalani32@yahoo.co.in

Abstract
Let G be a graph with its vertices and edges. On defining bijective function ρ:V→ $\{0,1,\dots,p\}$. For each edge assign the label with 1 if $\rho^* = \left|\frac{\rho(a)^2-\rho(b)^2}{\rho(a)-\rho(b)}\right|$ is odd or 0 otherwise such that $\left|e_\rho(1)-e_\rho(0)\right| \leq 1$ then the labeling is called as Divided Square difference cordial labeling graph. We prove in this paper for relatively possible set of Spider graphs with atmost one legs greater than one namely SP($1^m,2^n$), SP($1^m,2^n,3^1$), SP($1^m,2^n,3^2$), SP($1^m,2^n,4^1$), SP($1^m,2^n,5^1$).

AMS Mathematics Subject Classification:05C78
Keywords: Spider Graphs

1.Introduction

Graph theory plays a significant role in computers and research area in graph labeling finds a suitable study on constructing algorithm. We consider in our discussion finite graph which constitutes the vertices and edges. Different labeling models have been given in Gallian .J.A [2] 'A dynamic survey of graph labeling'. Most of graph labeling techniques results from the paper by A.Rosa[4]. A graph with difference labeling is summarized by assigning integer values being difference among its vertices where edges are associated with its absolute difference which are associated with each pair of distinct vertices. The Square difference labeling was introduced by Shiama[3] , Alfred Leo [1] identified divided square difference labeling and labeled the graphs and its classes with the divided square difference labeling. Further study on the labeling its behavior is extensively found in many papers by various authors [3] and [5].

2. Preliminaries

Definition2.1: A Tree is spider with center vertex C having degree and other vertices are either degree 2 or a leaf.

Definition2.2: A bijective function ρ:V→ $\{0,1,\dots,p\}$. For each edge assign the label with 1 if $\rho^* = \left|\frac{\rho(a)^2-\rho(b)^2}{\rho(a)-\rho(b)}\right|$ is odd or 0 otherwise such that $\left|e_\rho(1)-e_\rho(0)\right| \leq 1$ then the labeling is called as Divided Square difference cordial labeling graph.

Definition2.3: Spider graph **SP ($1^m,2^n$)** is defined as a tree with m legs of length n.

Definition 2.4: Spider graph **SP ($1^m,2^n,3^1$)** is defined as a tree with 2n legs of length n and 3^1.

Definition 2.5: Spider graph **SP ($1^m,2^n,3^2$)** is defined as a tree with 2n legs of length n and 3^2.

Definition 2.6: Spider graph **SP ($1^m,2^n,4^1$)** is defined as a tree with 2n legs of length n and 4^1.

Definition 2.4: Spider graph **SP ($1^m,2^n,5^1$)** is defined as a tree with 2n legs of length n and 5^1.

3 Main Results

Theorem 3.1: Spider graph **SP (1^m,2^n)** is Divided Square Difference Cordial Labeling.

Proof: Let the graph G=SP (1^m,2^n) with vertices and edges where

V= {a,a_i,b_j,c_j ; $1\leq i,j \leq n\}$ and E={(aa_i) ; $1\leq i \leq n\} \cup \{(ab_j);(b_jc_j); 1 \leq j \leq n\}$

The vertices are computed as below

$\rho(a) = 1; \quad \rho(a_{2i-1}) = 6i-4; \quad \rho(a_{2i}) = 6i; \quad 1 \leq i \leq n$

$\rho(b_{2j-1}) = 6j-4; \quad \rho(b_{2j}) = 6j-1; \quad 1 \leq j \leq n$

$\rho(c_{2j-1}) = 6j-2; \quad \rho(c_{2j}) = 6j+1; \quad 1 \leq j \leq n$

The edges are computed as below

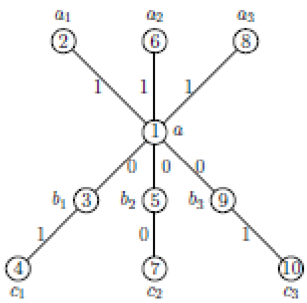
$$\rho^*(aa_i) = 1; \quad 1 \leq i \leq n \qquad \rho^*(ab_j) = 0; \quad 1 \leq j \leq n$$

$$\rho^*(b_jc_j) = 1; \quad j \equiv 1(mod2)\ \rho^*(b_jc_j) = 0; \quad j \equiv 0(mod2)$$

We find that the induced edge labeling satisfies the condition$|e_\rho(1) - e_\rho(0)| \leq 1$.

Thus, Spider graph **SP (1^m,2^n)** is Divided Square Difference Cordial Labeling.

Example 3.1: SP (1^3,2^3)



Theorem 3.2: Spider graph **SP (1^m,2^n,3^2)** is Divided Square Difference Cordial Labeling.

Proof: Let the graph G=SP (1^m,2^n,3^2) with vertices and edges where

V= {a, a_i, b_j, c_j, l_k ; $1\leq i,j \leq n, 1 \leq k \leq 6\}$ and E={(aa_i) ; $1\leq i \leq n\} \cup \{(ab_j);(b_jc_j); 1 \leq j \leq n\} \cup \{al_k; 1 \leq k \leq 6\}$

The vertices are computed as below

$\rho(a) = 1; \quad \rho(a_1) = 2; \ \rho(a_{2i+1}) = 6i+8; \quad \rho(a_{2i}) = 6i+6; \quad 1 \leq i \leq n$

$\rho(b_1) = 3; \ \rho(b_{2j+1}) = 6j+9; \quad \rho(b_{2j}) = 6j+5; \quad 1 \leq j \leq n$

$\rho(c_1) = 4; \ \rho(c_{2j+1}) = 6j+10; \quad \rho(c_{2j}) = 6j+7; \quad 1 \leq j \leq n$

$$\rho(l_k) = k+4; \quad 1 \leq k \leq 3; \ \rho(l_4) = 9; \quad \rho(l_5) = 8; \quad \rho(l_6) = 10$$

The edges are computed as below

$$\rho^*(aa_i) = 1; \quad 1 \leq i \leq n \qquad \rho^*(ab_j) = 0; \quad 1 \leq j \leq n$$

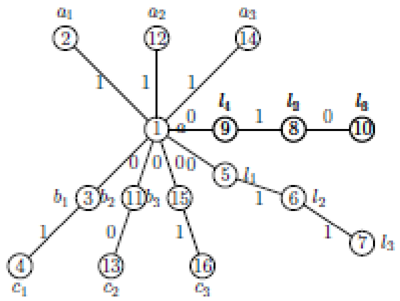
$$\rho^*(b_{2j-1}c_{2j-1}) = 1; \quad j \equiv 1(mod2)\ \rho^*(b_{2j}c_{2j}) = 0; \quad j \equiv 0(mod2)$$

$$\rho^*(al_k) = 0; \quad k = 1,4\ \rho^*(l_kl_k) = 1; k = 1\ to\ 4 \quad \rho^*(l_5l_6) = 0$$

We find that the induced edge labeling satisfies the condition$|e_\rho(1) - e_\rho(0)| \leq 1$.

Thus, Spider graph **SP (1^m,2^n,3^2)** is Divided Square Difference Cordial Labeling.

Example 3.2: SP (1^3,2^3,3^2)



Theorem 3.3: Spider graph **SP (1^m,2^n,3^1)** is Divided Square Difference Cordial Labeling.

Proof: Let the graph G=SP (1^m,2^n,3^1) with vertices and edges where

V= {a, a_i, b_j, c_j, l_k ; $1\leq i,j \leq n, 1 \leq k \leq 3$} and E={($aa_i$) ; $1\leq i \leq n$} $\cup \{(ab_j); (b_jc_j); 1 \leq j \leq n\} \cup \{(al_1) \cup (l_1l_2) \cup (l_2l_3)\}$

The vertices are computed as below

$\rho(a) = 1; \quad \rho(a_1) = 2; \quad \rho(a_{i+1}) = 3i + 5; \quad 1 \leq i \leq n$

$\rho(b_1) = 3; \ \rho(b_{2j+1}) = 6j + 7; \quad \rho(b_{2j}) = 6j + 3; \quad 1 \leq j \leq n$

$\rho(c_1) = 4; \ \rho(c_{2j+1}) = 6j + 6; \quad \rho(c_{2j}) = 6j + 4; \quad 1 \leq j \leq n$

$\rho(l_1) = 5; \ \ \rho(l_2) = 7; \ \ \rho(l_3) = 6$

The edges are computed as below

$$\rho^*(aa_i) = 1; \ \ 1 \leq i \leq 2 \quad \rho^*(aa_{2i+1}) = 0; \quad \rho^*(aa_{2i+2}) = 1; \ \ 1 \leq i \leq n$$

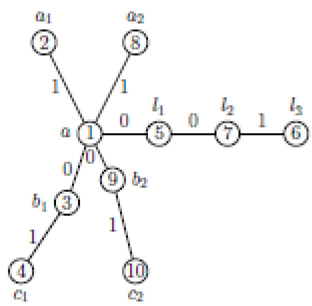
$$\rho^*(ab_j) = 0; \quad \rho^*(b_jc_j) = 1; \quad 1 \leq j \leq n$$

$$\rho^*(al_1) = 0; \quad \rho^*(l_1l_2) = 0; \quad \rho^*(l_2l_3) = 1$$

We find that the induced edge labeling satisfies the condition$|e_\rho(1) - e_\rho(0)| \leq 1$.

Thus, Spider graph **SP (1^m,2^n,3^1)** is Divided Square Difference Cordial Labeling.

Example 3.3: SP (1^3,2^3,3^1)



Theorem 3.4: Spider graph **SP (1^m,2^n,4^1)** is Divided Square Difference Cordial Labeling.

Proof: Let the graph G=SP (1^m,2^n,4^1) with vertices and edges where

V= {a, a_i, b_j, c_j, l_k ; $1\leq i,j \leq n, 1 \leq k \leq 4$} and E={($aa_i$) ; $1\leq i \leq n$} $\cup \{(ab_j); (b_jc_j); 1 \leq j \leq n\} \cup \{(al_1)\} \cup$

$\{(l_{k+1}l_k); 1 \le k \le 3\}$

The vertices are computed as below

$\rho(a) = 1;$ $\rho(a_1) = 2;$ $\rho(a_{2i}) = 6i + 3;$ $\rho(a_{2i+1}) = 6i + 5;$ $1 \le i \le n$

$\rho(b_1) = 3;$ $\rho(b_{2j+1}) = 6j + 6;$ $\rho(b_{2j}) = 6j + 4;$ $1 \le j \le n$

$\rho(c_1) = 4;$ $\rho(c_{2j+1}) = 6j + 7;$ $\rho(c_{2j}) = 6j + 2;$ $1 \le j \le n$

$\rho(l_1) = 5;$ $\rho(l_2) = 8;$ $\rho(l_3) = 6;$ $\rho(l_4) = 7$

The edges are computed as below

$\rho^*(aa_1) = 1;$ $\rho^*(aa_i) = 0;$ $2 \le i \le n$

$\rho^*(ab_1) = 0;$ $\rho^*(ab_j) = 1;$ $2 \le j \le n$

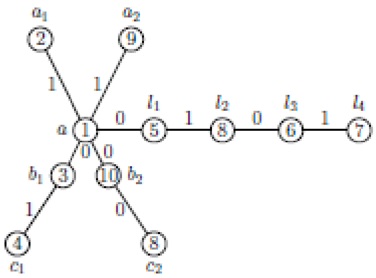
$\rho^*(b_jc_j) = 1;$ $j \equiv 1(mod2)$ $\rho^*(b_jc_j) = 0;$ $j \equiv 0(mod2)$

$\rho^*(al_1) = 0;$ $\rho^*(l_1l_2) = 1;$ $\rho^*(l_2l_3) = 0;$ $\rho^*(l_3l_4) = 1$

We find that the induced edge labeling satisfies the condition $|e_\rho(1) - e_\rho(0)| \le 1$.

Thus, Spider graph **SP (1^m,2^n,4^1)** is Divided Square Difference Cordial Labeling.

Example 3.4: SP (1^3,2^3,4^1)



Theorem 3.5: Spider graph **SP (1^m,2^n,5^1)** is Divided Square Difference Cordial Labeling.

Proof: Let the graph G=SP (1^m,2^n,5^1) with vertices and edges where

V= $\{a, a_i, b_j, c_j, l_k ; 1\le i, j \le n, 1 \le k \le 5\}$ and E=$\{(aa_i) ; 1\le i \le n\} \cup \{(ab_j); (b_jc_j); 1 \le j \le n\} \cup \{(al_1)\} \cup \{(l_{k+1}l_k); 1 \le k \le 5\}$

The vertices are computed as below

$\rho(a) = 1;$ $\rho(a_1) = 2;$ $\rho(a_{2i}) = 6i + 5;$ $\rho(a_{2i+1}) = 6i + 7;$ $1 \le i \le n$

$\rho(b_1) = 3;$ $\rho(b_{2j+1}) = 6j + 8;$ $\rho(b_{2j}) = 6j + 4;$ $1 \le j \le n$

$\rho(c_1) = 4;$ $\rho(c_{j+1}) = 3j + 9;$ $1 \le j \le n$

$\rho(l_1) = 5;$ $\rho(l_2) = 7;$ $\rho(l_3) = 6;$ $\rho(l_4) = 8;$ $\rho(l_5) = 9$

The edges are computed as below

$\rho^*(aa_1) = 1;$ $\rho^*(aa_i) = 0;$ $2 \le i \le n$

$\rho^*(ab_j) = 1;$ $1 \le j \le n$

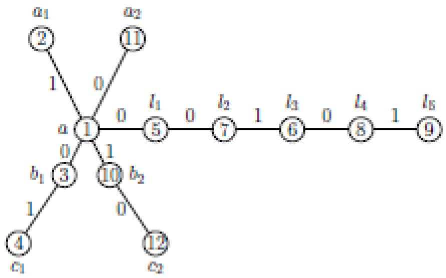
$\rho^*(b_{2j-1}c_{2j-1}) = 1;$ $j \equiv 1(mod2)$ $\rho^*(b_{2j}c_{2j}) = 0;$ $j \equiv 0(mod2)$

$\rho^*(al_1) = 0;$ $\rho^*(l_1l_2) = 0;$ $\rho^*(l_2l_3) = 1;$ $\rho^*(l_3l_4) = 0;$ $\rho^*(l_4l_5) = 1$

We find that the induced edge labeling satisfies the condition$\left|e_\rho(1) - e_\rho(0)\right| \leq 1$.

Thus, Spider graph **SP (1^m,2^n,5^1)** is Divided Square Difference Cordial Labeling.

Example 3.5: SP(1^3,2^3,5^1)



4 Conclusion

In understanding the different labeling techniques and its applications we in the paper have found some classes of spider graph satisfying the divided square difference cordial labeling conditions. We in our future discussion intend to find applications to the labeling techniques employed through spider graphs in this paper. We also are in the process of identifying some special classes of graphs which can be proved to be Divided square difference cordial labeling. We also further look for the applications through this labeling in computers.

References

1. A. Alfred . Leo, R.Vikramaprasad and R.Dhavaseelan; Divided Square Difference Cordial labeling Graphs, International Journal of Mechanical Engineering and Technology ,9(1) Jan (2018) PP 1137-1144.
2. J.A Gallian, A Dyamic Survey of graph labeling, The Electronic .J .Combin,18(2015) D56.
3. P.Jeyanthi and T.Saratha Devi; Journal of Algorithms and Computation 45 (2014) PP 25-34.
4. J.A .Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (1967) PP349-355.
5. J.Shiama ,Square difference labeling for certain graph, International Journal of computer Applications (0975-08887),44(4)(2012).
6. S.Sriram and K.Thirusangu ; i-cordial labeling of spider graphs-Journal Application and Engineering Math V II Special Issue ,2021 , pp 164-171.