
Analyzing Performance Measure of Fuzzy Queueing Problem with Trapezoidal Fuzzy Numbers by LR Method

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Abstract:
LR method has been emphasized in this document to analyses the queueing model that directs for evaluation of system performance measure. LR method has been proposed to FM/FM/1 queueing pattern having limited capacity and unlimited calling source population. The inter arrival time and service time are fuzzy and expressed as trapezoidal fuzzy numbers. A numerical example has been taken and compared with DSW Algorithm. The numerical example shows the applicability and efficiency of this model.

Keywords: Fuzzy Queueing, system performance, LR method, Trapezoidal fuzzy numbers.

1. INTRODUCTION

Queueing models have wider application under fuzzy area in the actual existence. Performance measure in Queueing system helps to study the nature of arrival and service rates under different circumstances depending upon the type of queueing models. System Performance measure for Queueing models are calculated by many methods such as alpha-cut method using non-linear parametric programming approach, Robust ranking technique, DSW algorithm, LR method. Fuzzy Queueing theory has been researched by Negi and Lee[1], Li and Lee[2], Chen SP [3] by using parametric non-linear programing approach, J. P. Mukeba Kanyinda et al [5] , W. Ritha and S. Josephine Vinnarasi [6] by using LR method, B. Palpandi, G.Geetharamani [7] by using Robust Ranking technique, S. Shanmugasundaram et al [8] , R. Srinivasan [9] by using DSW Algorithm.

In this paper, $(\mathrm{FM/FM/1}) : (\infty/\mathrm{FCFS})$ queueing system is studied with the inter arrival time and service time following exponential distributions with ratio 'λ' and 'μ' under FCFS pattern having infinite source population. To emphasize the validity of the LR method, a numerical example is derived by LR method and compared with the problem under DSW algorithm.

2. PRELIMINARIES

A. Fuzzy set:

A fuzzy set $\widetilde{\mathrm{A}}$ defined on R is characterized by a membership function having the components of a domain space 'X' in the interval [0, 1]. It is represented as

$$\widetilde{\mathrm{A}} = \{ (\mathrm{Z}, \mu_{\widetilde{\mathrm{A}}}(\mathrm{z})) \,;\, \mathrm{z} \in \mathrm{Z} \}$$

B. LR – type trapezoidal fuzzy number:

A fuzzy number $\widetilde{\mathrm{M}} = (m, n, \alpha, \beta)_{LR}$ is said to be LR – type trapezoidal fuzzy numbers if its membership function is given by

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$$\mu_{\widetilde{M}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right); \text{ if } x \le m;\ \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right); \text{ if } x \ge n;\ \beta > 0 \\ 1 \quad ; \text{ otherwise} \end{cases}$$

C. Basic operations in LR – type trapezoidal fuzzy numbers:

Let $\check{T} = (t_1, t_2, t_3, t_4)$ & $\tilde{V} = (v_1, v_2, v_3, v_4)$ & 'λ' is a parameter, then

(i) $\check{T} + \tilde{V} = (t_1, t_2, t_3, t_4) + (v_1, v_2, v_3, v_4) = (t_1+v_1, t_2+v_2, t_3+v_3, t_4+v_4)$

(ii) $\check{T} - \tilde{V} = (t_1, t_2, t_3, t_4) - (v_1, v_2, v_3, v_4) = (t_1-v_2, \ t_2-v_1, t_3+v_4, t_4+v_3)$

(iii) $\check{T}\tilde{V} = (t_1, t_2, t_3, t_4).(v_1, v_2, v_3, v_4) = (t_1v_1, t_2v_2, t_1v_3 + t_3v_1, t_2v_4+v_2t_4)$

(iv) $\frac{\check{T}}{\tilde{V}} = \frac{(t_1,t_2,t_3,t_4)}{(v_1,v_2,v_3,v_4)} = \left(\frac{t_1}{v_2}, \frac{t_2}{v_1}, \frac{t_3}{v_4}, \frac{t_4}{v_3}\right)$

(v) $\lambda\,(t_1, t_2, t_3, t_4) = (\lambda t_1, \lambda t_2, \lambda t_3, \lambda t_4)$

(vi) $1/\lambda\,(t_1, t_2, t_3, t_4) = (t_1/\lambda, t_2/\lambda, t_3/\lambda, t_4/\lambda)$

D. Representation of LR – type trapezoidal fuzzy number:

Trapezoidal Fuzzy numbers are noted as $\widetilde{A}$(a, b, c, d). In LR representation, this can be written as

$\widetilde{A}$(a, b, c, d) = < b, c, b-a, d-c > Where b, c are left and right spreads.

3. PERFORMANCE MEASURE UNDER FUZZY QUEUEING SYSTEM WITH INFINITE SOURCE POPULATION:

We consider $(FM/FM/1) : (\infty/FCFS)$ queueing system having unlimited source population with the inter arrival time and the service time following exponential distributions with arrival rate 'λ' and service rate 'µ'respectively .

The system performance measures namely expected number of customers and waiting time in the system, queue are given by the respective formulas.

- $L_s = \frac{\lambda}{\mu-\lambda}$
- $W_s = \frac{1}{\mu-\lambda}$
- $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$
- $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

4. NUMERICAL EXAMPLE

A Queueing system FM/FM/1 is considered with the arrival rate and service rate given by λ = (1,2,3,4) and µ = (15,16,17,18) which are trapezoidal fuzzy numbers.

In LR type, the arrival rate and service rate are given by λ = (2,3,1,1) and µ= (16,17,1,1) which are LR type trapezoidal fuzzy numbers.

Expected customer count in the system (L_s):

$$L_s = \frac{\lambda}{\mu-\lambda}$$

$$= \frac{(2,3,1,1)}{(16,17,1,1)-(2,3,1,1)}$$

$$= \frac{(2,3,1,1)}{(13,15,2,2)}$$

$= (0.1333, 0.2307, 0.5, 0.5)$

The value of L_s lie between 0.1333 and 0.2307.

Expected delay period of customer in the system (W_s):

$$W_s = \frac{1}{\mu - \lambda}$$

$$= \frac{1}{(16,17,1,1) - (2,3,1,1)}$$

$$= \frac{1}{(13,15,2,2)}$$

$$= (0.0666, 0.0769, 0.5, 0.5)$$

The value of W_s lie between 0.0666 and 0.0769.

Expected customer count in the Queue (L_q):

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$= \frac{(2,3,1,1)\ (2,3,1,1)}{(16,17,1,1)[\ (16,17,1,1) - (2,3,1,1)]}$$

$$= \frac{(4,9,4,6)}{(208,255,45,49)}$$

$$= (0.0156, 0.0432, 0.0816, 0.1333)$$

The value of L_q lie between 0.0156 and 0.0432

Expected delay period of customer in the Queue (W_q):

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

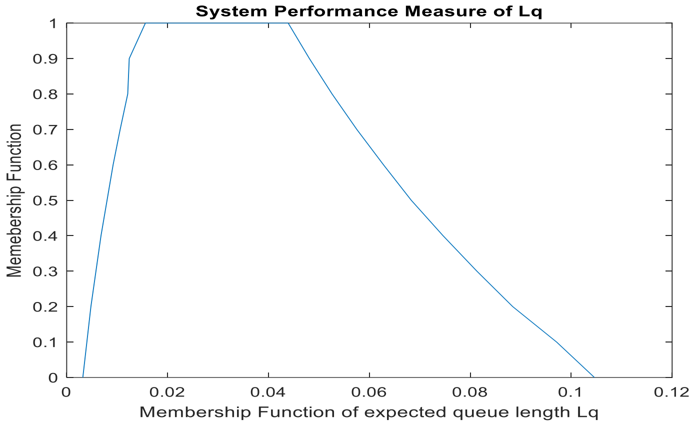
$$= \frac{(2,3,1,1)}{(16,17,1,1)[\ (16,17,1,1) - (2,3,1,1)]}$$

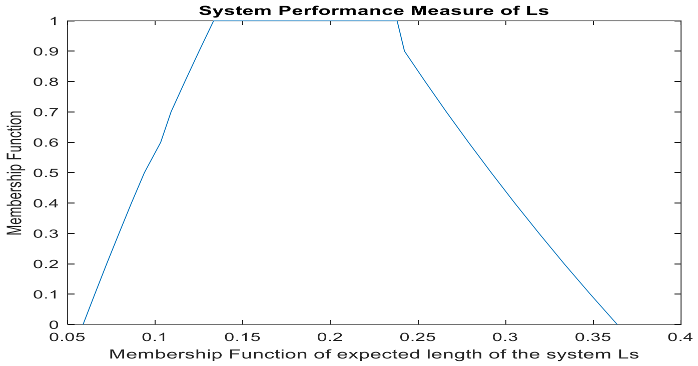
$$= \frac{(2,3,1,1)}{(208,255,45,49)}$$

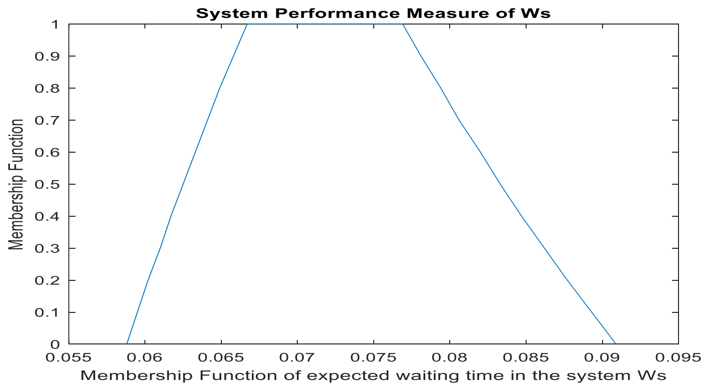
$$= (0.0078, 0.0144, 0.0204, 0.0222)$$

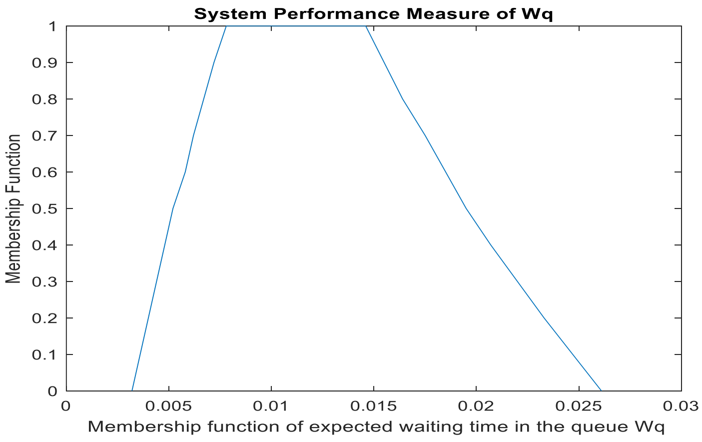
The value of W_q lie between 0.0078 and 0.0144.

The graphs of the system performance measure of L_q , L_s, W_s and W_q has been given using MATLAB R2023a.


System Performance Measure of Lq
Memebership Function
Membership Function of expected queue length Lq


System Performance Measure of Ls
Membership Function
Membership Function of expected length of the system Ls


System Performance Measure of Ws
Membership Function
Membership Function of expected waiting time in the system Ws


System Performance Measure of Wq
Membership Function
Membership function of expected waiting time in the queue Wq

Note:

For the above problem, results are compared with DSW Algorithm.

	DSW Algorithm	LR Method
	Interval Analysis Arithmetic is used in computing the algorithm which involves four steps for calculating system performance.	Simple manual calculation is carried out for system performance using LR type trapezoidal fuzzy number formulas.
L_q	Lie in the interval (0.0156 , 0.0439) and does not fall below 0.0032 and exceed 0.1046.	Lie in the interval (0.0156 , 0.0432)
L_s	Lie in the interval (0.1333 , 0.2380) and does not fall below 0.0588 and exceed 0.3636.	Lie in the interval (0.1333 , 0.2307)
W_s	Lie in the interval (0.0667, 0.0769 and does not fall below 0.0588 and exceed 0.0909	Lie in the interval (0.0666 , 0.0769)
W_q	Lie in the interval (0.0078, 0.0146) and does not fall below 0.0032 and exceed 0.0261.	Lie in the interval (0.0078 , 0.0144)

5. CONCLUSION

In this paper, the performance measure in FM/FM/1 queueing system with infinite source population under FCFS pattern is calculated using LR type trapezoidal fuzzy numbers. The left and right spreads are calculated using LR type trapezoidal fuzzy numbers by LR method.

The expected no of jobs in the queue lie between 0.0156 - 0.0432, expected number of jobs in the system lie between 0.1333 and 0.2307, the expected delay period in the system lie between 0.0666 and 0.0769 and the expected waiting time in the queue lie between 0.0078 and 0.0144. Therefore, LR method is simple and convenient in calculating the system performance measure of a queueing system.

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