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Super Lehmer - 3 Mean Labeling of Path Union Related Graphs

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Abstract

Let $f: V(G) \to \{1,2,\ldots,p+q\}$ be an injective function. For a vertex labeling f the induced edge labeling $f(e=uv)$ is defined by $f(e) = \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ or $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then f is called Super Lehmer -3 mean labeling, if $\{f\,(V(G))\} \cup \{f(e)/e \in E(G)\} = \{1,2,3,\ldots,p+q\}$. A graph which admits Super Lehmer – 3 Mean Labeling is called Super Lehmer – 3 Mean graphs. In this paper, we prove that path union related graphs such as path union of two cycle, path union of three cycle, $k-$ path union of two cycle with path P_k, path union of two crown, path union of three crown and $k-$ path union of two crown with path P_k are all super lehmer – 3 mean graph.

Keywords: Graph, Super lehmer – 3 mean labeling, Super lehmer - 3 mean graphs, Cycle, Crown.

I. INTRODUCTION

In this paper we consider the graphs which are simple, finite and undirected with p vertices and q edges. For a detailed survey of graph labeling, we refer to Gallain [1]. For all other standard terminology and notations, we follow Harary [2]. The concept of some more results on Lehmer – 3 mean labeling of graphs has been introduced by Somasundaram. S, Sandhya.S.S and Pavithra. T.S [3] and also proved that some Super Lehmer – 3 mean labeling and $k-$ Super Lehmer – 3 mean labeling graphs [4,5,6]. In this article, we investigate the Super Lehmer – 3 mean labeling of path union related graphs. Some new examples are presented and verified. We now give the definitions which are necessary for the present investigation.

Definition 1.1:

A graph $G=(V,E)$ with p vertices and q edges is called **Lehmer – 3 mean graph.** If it is possible to label vertices $x \in V$ with distinct labels $f(x)$ from $1,2,3,\ldots,q+1$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv) = \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ or $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then the edge labels are distinct. In this case f is called Lehmer – 3 mean labeling of G.

Definition 1.2:

Let $f: V(G) \to \{1,2,\ldots,p+q\}$ be an injective function. For a vertex labeling f the induced edge labeling $f(e=uv)$ is defined by $f(e) = \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ or $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then f is called Super Lehmer -3 mean labeling, if $\{f\,(V(G))\} \cup \{f(e)/e \in E(G)\} = \{1,2,3,\ldots,p+q\}$. A graph which admits Super Lehmer – 3 mean labeling is called **Super Lehmer – 3 Mean graph.**

Definition 1.3:

A walk in which $u_1u_2 \ldots u_m$ are distinct is called a path. A path on m vertices is denoted by P_m.

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Definition 1.4:

A closed path is called a cycle. A cycle on m vertices is denoted by C_m.

Definition 1.5:

The Corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 1.6:

The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $= E_1 \cup E_2$.

Definition 1.7:

Let $G_1, G_2, \ldots, G_n$, $n \geq 2$ be n copies of a fixed graph G. The graph G obtained by adding an edge between G_i and G_{i+1} for $i = 1,2, \ldots, n-1$ is called a path union of G.

Definition 1.8:

The $k-$ path union of two cycles C_m is the graph obtained by joining two vertices from two copies of C_m by a path P_k of length $k-1$.

II. Main Results

In this paper, we investigate the Super Lehmer – 3 mean labeling of path union related graphs.

Theorem 2.1

Path union of two cycles is a super lehmer – 3 mean graph.

Proof:

Let $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_m$ be the vertices of two cycles C_m in G.

Let $V(G) = \{a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_m\}$

$E(G) = \{a_i a_{i+1} \,/\, 1 \leq i \leq m-1\} \cup \{b_i b_{i+1} \,/\, 1 \leq i \leq m-1\}$
$\cup \{a_m a_1, b_m b_1, a_1 b_1\}$.

Define a function $f: V(G) \to \{1,2, \ldots, 4m-1\}$ by

$$f(a_i) = 2i + 1 \quad \text{for } 1 \leq i \leq m-1$$
$$f(a_m) = 1$$
$$f(b_i) = 2m + 2i + 1 \quad \text{for } 1 \leq i \leq m-1$$
$$f(b_m) = 2m + 1$$

Then the edge labels are distinct.

Hence f is a super lehmer – 3 mean labeling of G.

Example 2.1.1:

The super lehmer – 3 mean labeling of path union of two cycles C_6 is given below:

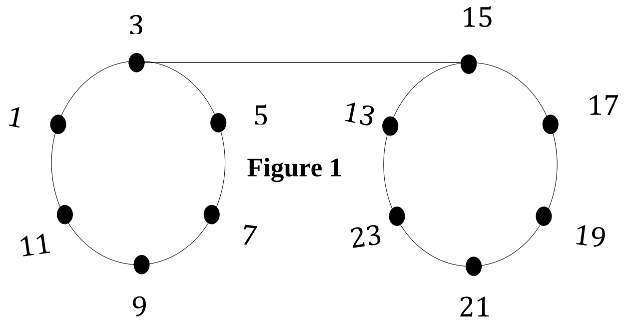


Figure 1

Theorem 2.2

Path union of three cycles is a super lehmer – 3 mean graph.

Proof:

Let $a_1, a_2, \dots a_m$, $b_1, b_2, \dots, b_m$ and $c_1, c_2, \dots, c_m$ be the vertices of three cycles C_m in G.
Let $V(G) = \{a_1, a_2, \dots a_m, b_1, b_2, \dots, b_m, c_1, c_2, \dots, c_m\}$
$E(G) = \{a_i a_{i+1} \,/\, 1 \le i \le m-1\} \cup \{b_i b_{i+1} \,/\, 1 \le i \le m-1\}$
$\cup \{c_i c_{i+1} \,/\, 1 \le i \le m-1\} \cup \{a_m a_1, a_1 b_1, b_m b_1, b_1 c_1, c_m c_1\}$.

Define a function $f: V(G) \to \{1,2,\dots,6m-1\}$ by

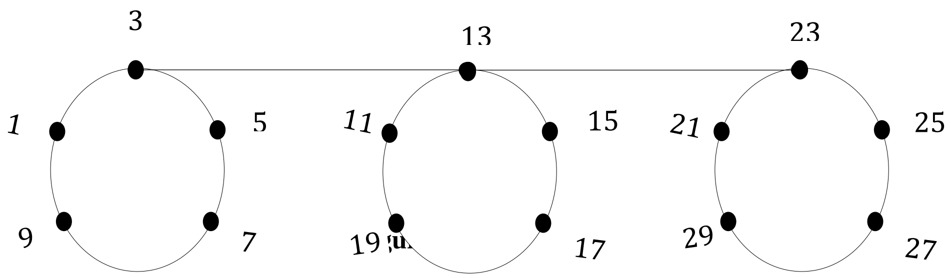
$f(a_i) = 2i + 1$ for $1 \le i \le m-1$
$f(a_m) = 1$
$f(b_i) = 2m + 2i + 1$ for $1 \le i \le m-1$
$f(b_m) = 2m + 1$
$f(c_i) = 4m + 2i + 1$for $1 \le i \le m-1$
$f(c_m) = 4m + 1$

Then the edge labels are distinct.

Hence f is a super lehmer – 3 mean labeling of G.

Example 2.2.1:

The super lehmer – 3 mean labeling of path union of three cycles C_5 is given below:



Theorem 2.3

$k -$ path union of two cycles with path $P_k$is a super lehmer – 3 mean graph.

Proof:

Let $a_1, a_2, \dots, a_m$ and $b_1, b_2, \dots, b_m$ be the vertices of two cycles C_m in G.
Let $a_1 = c_1, c_2, \dots, c_k = b_1$ be the vertices of the path P_k.
Define a function $f: V(G) \to \{1,2,\dots,2m+k\}$ by

$f(a_i) = i + 1$ for $1 \le i \le m-1$
$f(a_m) = 1$
$f(b_i) = m + k + i$ for $1 \le i \le m-1$
$f(b_m) = m + k$
$f(c_i) = m + i - 1$for $2 \le i \le k-1$

Then the edge labels are distinct.

Hence f is a super lehmer – 3 mean labeling of G.

Example 2.3.1:

The super lehmer – 3 mean labeling of k −path union of C_5 is given below:

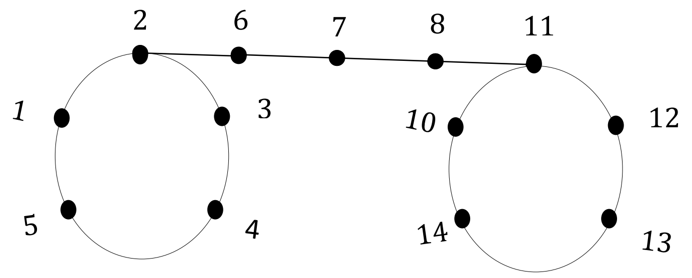


Figure 3

Theorem 2.4

Path union of two crowns is a super lehmer – 3 mean graph.

Proof:

Let $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_m$ be the vertices of two cycles C_m in G.

Let $a_1', a_2', \ldots a_m'$ be the pendent vertices attached at $a_1, a_2, \ldots, a_m$ respectively and $b_1', b_2', \ldots, b_m'$ be the pendent vertices attached at $b_1, b_2, \ldots, b_m$ respectively.

Define a function $f: V(G) \to \{1,2,\ldots,8m-1\}$ by

$$f(a_i) = 4i + 1 \quad \text{for } 1 \le i \le m-1$$
$$f(a_m) = 1$$
$$f(b_i) = 4m + 4i + 1 \quad \text{for } 1 \le i \le m-1$$
$$f(b_m) = 4m + 1$$
$$f(a_i') = 4i + 3 \text{for } 1 \le i \le m-1$$
$$f(a_m') = 3$$
$$f(b_i') = 4m + 4i + 3 \text{for } 1 \le i \le m-1$$
$$f(b_m') = 4m + 3$$

Then the edge labels are distinct.

Hence f is a super lehmer – 3 mean labeling of G.

Example 2.4.1:

The super lehmer – 3 mean labeling of path union of two crown C_4^* is given below:

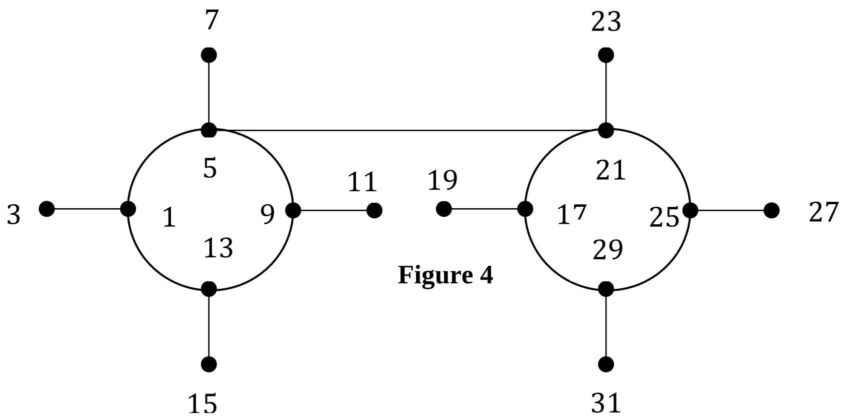


Figure 4

Theorem 2.5

Path union of three crown is a super lehmer – 3 mean graph.

Proof:

Let $a_1, a_2, \dots a_m, b_1, b_2, \dots, b_m$ and $c_1, c_2, \dots, c_m$ be the vertices of three cycles C_m in G.

Let $a'_1, a'_2, \dots a'_m$ be the pendant vertices attached at $a_1, a_2, \dots a_m$ respectively and $b'_1, b'_2, \dots, b'_m$ be the pendant vertices attached at $b_1, b_2, \dots, b_m$ respectively and $c'_1, c'_2, \dots, c'_m$ be the pendant vertices attached at $c_1, c_2, \dots, c_m$respectively.

Define a function $f: V(G) \to \{1,2, \dots ,12m - 1\}$ by

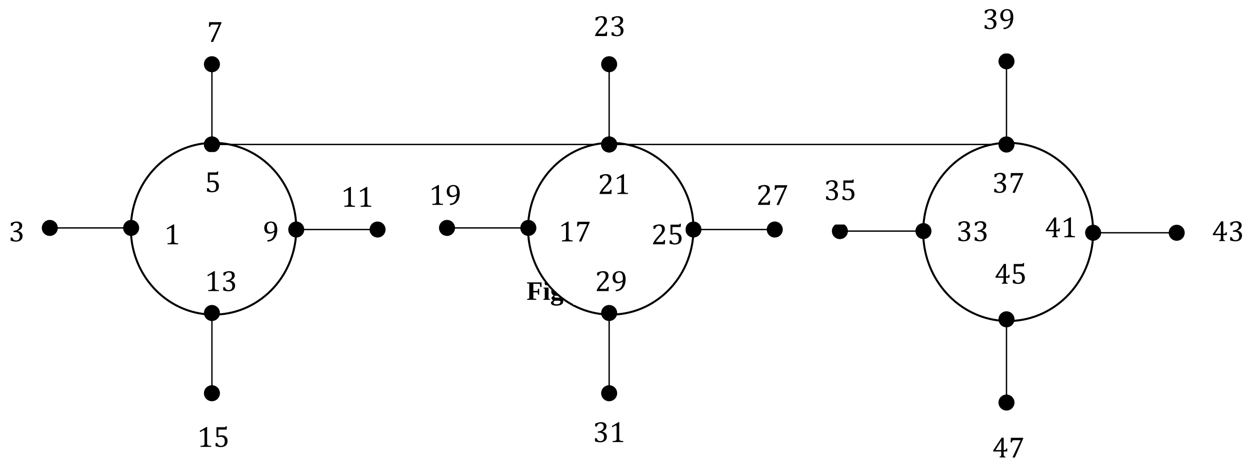
$$f(a_i) = 4i + 1 \quad \text{for } 1 \le i \le m - 1$$
$$f(a_m) = 1$$
$$f(a'_i) = 4i + 3 \quad \text{for } 1 \le i \le m - 1$$
$$f(a'_m) = 3$$
$$f(b_i) = 4m + 4i + 1 \quad \text{for } 1 \le i \le m - 1$$
$$f(b_m) = 4m + 1$$
$$f(b'_i) = 4m + 4i + 3 \quad \text{for } 1 \le i \le m - 1$$
$$f(b'_m) = 4m + 3$$
$$f(c_i) = 8m + 4i + 1 \quad \text{for } 1 \le i \le m - 1$$
$$f(c_m) = 8m + 1$$
$$f(c'_i) = 8m + 4i + 3 \quad \text{for } 1 \le i \le m - 1$$
$$f(c'_m) = 8m + 3$$

Then the edge labels are distinct.

Hence f is a super lehmer – 3 mean labeling of G.

Example 2.5.1:

The super lehmer – 3 mean labeling of path union of three crownC_4^* is given below:



Theorem 2.6

k −Path union of two crownwith path P_k is a super lehmer – 3 mean graph.

Proof:

Let $a_1, a_2, \dots, a_m$ and $b_1, b_2, \dots, b_m$ be the vertices of two cycles C_m in G.

Let $a_1 = c_1, c_2, \dots, c_k = b_1$be the vertices of the path P_k.

Let $a'_1, a'_2, \dots a'_m$ be the pendent vertices attached at $a_1, a_2, \dots, a_m$ respectively and $b'_1, b'_2, \dots, b'_m$ be the pendent vertices attached at $b_1, b_2, \dots, b_m$ respectively.

Define a function $f: V(G) \to \{1,2, \dots ,4m + k - 1\}$ by

$$f(a_i) = 2i - 1 \text{for } 1 \le i \le m$$

$f(a_i') = 2i$for $1 \leq i \leq m$
$f(b_i) = 2m + k + 2i - 2$for $1 \leq i \leq m$
$f(b_i') = 2m + k + 2i - 1$for $1 \leq i \leq m$
$f(c_i) = 2m + i$for $2 \leq i \leq k - 1$

Then the edge labels are distinct.
Hence f is a super lehmer – 3 mean labeling of G.

Example 2.6.1:

The super lehmer – 3 mean labeling k –path union of C_4^* is given below::

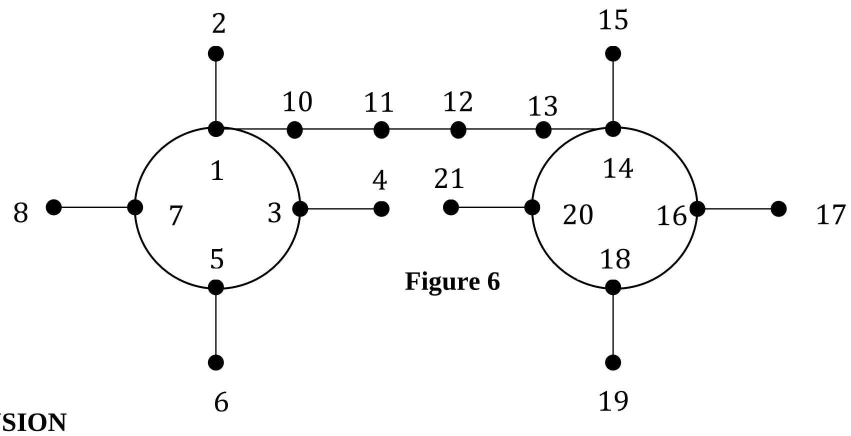


Figure 6

III CONCLUSION

As all graphs are not super lehmer – 3 mean graphs, it is very interesting to investigate graphs which admits super lehmer – 3 mean labeling. In this paper,we prove that path union of some cycle, crownare super lehmer 3 mean graphs. Then, we present six new results on super lehmer – 3 mean labeling of graphs. It is possible to investigate similar results for several other graphs.

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