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Steiner Distance Related Parameters of Wheel graphs

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Abstract

For a connected graph$G$ of order at least 2 and $S \subseteq V(G)$, the Steiner distance $d_G(S)$ among the vertices of S is the minimum size among all the connected subgraphs whose vertex sets contain S. In this paper, we calculate the Steiner distance parameters such as Steiner degree distance, Steiner Gutman index, Steiner Harary index and Steiner reciprocal degree distance of a wheel graphs.

Keywords: *Steiner distance, Steiner degree distance, Steiner Gutman index, Steiner Harary index, Steiner reciprocal degree distance, Wheel Graphs.**Mathematics Subject Classification:** 05C10, 05C12, 05C07*

1. Introduction

All graphs considered in this paper are undirected, finite and simple. The basic concept of graph theory is distance. If G is a connected graph and $u, v \in V(G)$, then the distance $d(u, v)$ between u and v is the length of a shortest path between u and v. The degree of a vertex v in G is denoted by $deg_G(v)$.

The Steiner distance of a graph, introduced by Chartrand et al., is a natural generalization of the concept of classical graph distance [4]. For a graph $G(V, E)$ and a set $S \subseteq V(G)$ of at least two vertices, the S-steiner tree or Steiner tree connecting S is a subgraph $T(V, E)$ of G that is a tree with $S \subseteq V$. The Steiner distance $d_G(S)$ among the vertices of S is the minimum size among all connected subgraphs whose vertex set contains$S$.

Li et al. [5] introduced the concept of Steiner Wiener index by using thegeneralization of Wiener index. The *Steiner Wiener index*$SW_k(G)$of the graph G is defined by $SW_k(G) = \sum_{\substack{S\subseteq V(G),\\ |S|=k}} d_G(S)$.

Recently, Gutman[7] generalized the concept of Steiner degree distance by using Steiner distance. The *Steiner degree distance*$SDD_k(G)$ of G is defined by

$$SDD_k(G) = \sum_{\substack{S\subseteq V(G),\\ |S|=k}} \left[\sum_{v\in S} deg_G(v)\right] d_G(S).$$

Recently,Y. Mao et.al [9] generalized the concept of Steiner Gutman index by using Steiner distance. The *Steiner Gutman index*$SGut_k(G)$ of G is defined by

$$SGut_k(G) = \sum_{\substack{S\subseteq V(G),\\ |S|=k}} \left[\prod_{v\in S} deg_G(v)\right] d_G(S).$$

In 2018, Y.Mao[10]generalised the concept of Harary index in terms of Steiner distance. The *Steiner Harary index* of G is defined as

$$SH_k(G) = \sum_{\substack{S\subseteq V(G),\\ |S|=k}} \frac{1}{d_G(S)}.$$

In 2019, A.Babu et.al [1] introduced the concept of *Steiner reciprocal degree distance* of G as

$$SRDD_k(G) = \sum_{\substack{S\subseteq V(G)\\ |S|=k}} \frac{[\sum_{v\in S} deg_G(v)]}{d_G(S)}.$$

The *Wheel graph*W_n with n vertices is defined to be the join of K_1 and C_{n-1}, where K_n is the complete graph and C_n is the cycle on n vertices. Clearly, $|V(W_n)| = n$ and $|E(W_n)| = 2n - 2$.The vertex corresponding to K_1 is known as apex, while the vertices corresponding to C_{n-1} are known as rim vertices.(see Figure: 1)

In 2020, X.Li et al. [6] computed the Steiner wiener index of wheel graphs. Followed by the results obtained in [6], we obtain the Steiner parameters such as Steiner degree distance, Steiner Gutman index, Steiner Harary index and Steiner reciprocal degree distance of a Wheel graph.

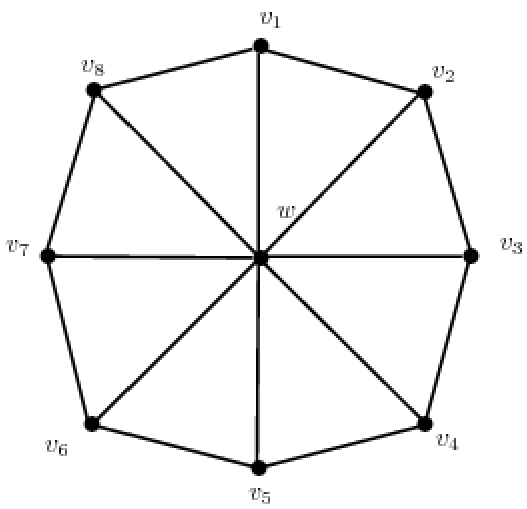

FIGURE 1 (W_8)

2. Steiner Degree Distance of Wheel graphs

In this section, we obtain the Steiner distance and Steiner degree distance of a Wheel graph.

Theorem 2.1.Let$G$be a wheel graphwith$n$vertices. Let$k \geq 2$be an integer and$S$be a subset of$V(G)$with$|S| = k$.Then$k - 1 \leq d_G(S) \leq k$,if$2 \leq k \leq n - 2$.Also$d_G(S) = k - 1$,if$k = n - 1, n$.

Proof. Let w be the apex in G and$V(G) - \{w\} = \{v_1, v_2, \ldots, v_{n-1}\}$ be the vertices in the outer cycle C_{n-1}. Let $S \subseteq V(G)$ such that $|S| = k$, where $k \geq 2$ is an integer.
Case (i):$2 \leq k \leq n - 2$.
Let $S \subseteq V(G)$ with $|S| = k$. Clearly $w \in S$ or $w \notin S$. Suppose $w \in S$. As w is adjacent to all the remaining $k - 1$ vertices, which belongs to the outer cycle, we obtain an optimal steiner tree T, which is a star with w as centre and $k - 1$ vertices as leaves. This tree T contains exactly $k - 1$ edges.

Now if$w \notin S$, then S is the set of vertices on the outer cycle of G.

Suppose S is a set of k consecutive vertices, then $G[S]$ is a path with $k - 1$ edges and thus we obtain a steiner tree on k vertices with $k - 1$ edges.

Otherwise, suppose S contains a set of k vertices in which at least one vertex is non-consecutive. Then we obtain an optimal steiner tree T which is a star with w as centre and the vertices in S as leaves. Thus T contains k edges.

Case (ii): $k = n$

Let S be a set with $k = n$ vertices. By the structure of G, we see that there exists a steiner tree with w as centre and remaining $n-1$ vertices as leaves. Therefore this tree has $n-1 = k-1$ edges.

Case (iii): $k = n-1$

Let $S \subseteq V(G)$ with $k = n-1$ vertices. Clearly $w \in S$ or $w \notin S$. If $w \in S$ and as w is adjacent to all the remaining $n-2$ vertices, which belongs to the outer cycle, we obtain an optimal steiner tree T, which is a star with w as centre and $n-2 = k-1$ vertices as leaves. This tree T contains exactly $k-1$ edges.

Now if $w \notin S$, then S is the set of $n-1$ vertices on the outer cycle of G. Then the minimum connected subgraph $G[S]$ is a path P on $n-1$ vertices and $n-2$ edges. Thus we have an optimal steiner tree T which contains exactly $k-1$ edges.

This completes the proof.

Theorem 2.2. Let $G = W_n$ be a wheel graph with $n \geq 4$ vertices. Let k be an integer with $2 \leq k \leq n$. Let S be a subset of $V(G)$ with $|S| = k$. Then

$$SDD_k(G) = \begin{cases} 4(n-1)^2, & \text{if } k = n, \\ 4\big(n-1^2(n-2)\big), & \text{if } k = n-1. \end{cases}$$

and $SDD_k(G) = \frac{k^2(4n-7)+k(4-n)}{n}\binom{n}{k} - 3k(n-1), \quad \text{if} \quad 2 \leq k \leq n-2.$

Proof. Let $G = W_n$ be a wheel graph with n vertices, $n \geq 4$. Let w be the apex in G and $V(G) - \{w\} = \{v_1, v_2, \ldots, v_{n-1}\}$ be the vertices in the outer cycle C_{n-1}. Let $k \geq 2$ be an integer with $|S| = k$.

Case (i): $k = n$.

Let S be a set of all vertices in the wheel graph. From the definition of G, we have $deg_G(w) = n-1$ and $deg_G(v_i) = 3$, $1 \leq i \leq n-1$. By Theorem 2.1, we have $d_G(S) = k-1 = n-1$ and hence $\sum_{v \in S} deg_G(v) = 3(n-1) + (n-1) = 4(n-1)$. Therefore,

$$\begin{aligned} SDD_k(G) &= \sum_{\substack{S \subseteq V(G), \\ |S|=k}} \left[\sum_{v \in S} deg_G(v)\right] d_G(S) \\ &= 4(n-1)(n-1) = 4(n-1)^2. \end{aligned}$$

Case (ii): $k = n-1$

Let $S \subseteq V(G)$ with $k = n-1$ vertices. Clearly $w \in S$ or $w \notin S$.

If $w \notin S$, then S contains the vertices of the outer $(n-1)$- cycle of G. By Theorem 2.1, we have $d_G(S) = k - 1 = n-2$ and the degree of each vertex in the outer $(n-1)$- cycle is 3. Then we have $\sum_{v \in S} deg_G(v) = 3(n-1)$.

Suppose $w \in S$. By Theorem 2.1, we have $d_G(S) = k-1 = n-2$. As $w \in S$, the remaining $k-1$ vertices can be chosen from the vertices of $(n-1)$- cycle.

Therefore $\sum_{v \in S} deg_G(v) = [3(n-2) + (n-1)]\binom{n-1}{n-2}$

$$\text{and hence} \quad SDD_k(G) = \sum_{\substack{S\subseteq V(G),\\ |S|=k}} \left[\sum_{v\in S} deg_G(v)\right] d_G(S)$$

$$= 4(n-1)^2(n-2).$$

Case (iii):$2 \leq k \leq n-2$.

Let $S \subseteq V(G)$ with k vertices. Then $w \in S$ or $w \notin S$.

If$w \in S$, then by Theorem 2.1, we have $d_G(S) = k-1$. Also as $w \in S$, the remaining $k-1$ vertices can be chosen from the vertices of the outer $(n-1)-$ cycle in $\binom{n-1}{k-1}$ ways. Therefore $\sum_{w\in S} deg_G(v) = [(n-1) + 3(k-1)]\binom{n-1}{k-1}$.

If $w \notin S$, then by Theorem 2.1, we have $k-1 \leq d_G(S) \leq k; S$ contains the vertices of the outer $(n-1)-$ cycle of G.

In this case if we choose$S$ to be k consecutive vertices in the outer cycle, then there are $n-1$ such sets for$n-1 > k$. and $d_G(S) = k-1$. The degree of each vertex in S is 3 and hence$\sum_{v\in S} deg_G(v) = 3k(n-1)$

Otherwise, if$S \subseteq V(G)$, is a set of k vertices in the outer $(n-1)-$ cycle which contains at least one non-consecutive vertex, then there are $\left[\binom{n-1}{k} - (n-1)\right]$ such sets with $d_G(S) = k$.

Hence, $\sum_{v\in S} deg_G(v) = 3k\left[\binom{n-1}{k} - (n-1)\right]$.

$$SDD_k(G) = \left[3k(n-1) + [(n-1) + 3(k-1)]\binom{n-1}{k}\right](k-1) + \left[3k\left[\binom{n-1}{k} - (n-1)\right]\right](k)$$

Therefore $SDD_k(G) = \frac{k^2(4n-7)+k(4-n)}{n}\binom{n}{k} - 3k(n-1)$, if $2 \leq k \leq n-2$.

This completes the proof.

3.SteinerGutman Index of Wheel Graphs

Theorem 3. Let$G = W_n$be awheel graphwith $n$vertices, $n \geq 4$.Let$k$be aninteger with$2 \leq k \leq n$. *Let* Sbe a subset of$V(G)$and let$|S| = k$.Then$SGut_k(G) = \begin{cases} 3^{(n-1)}(n-1)^2, \text{if } k = n. \\ 3^{(n-2)}[3+(n-1)^2](n-2), \text{if } k = n-1. \end{cases}$

and

$$SGut_k(G) = \frac{3^k(n-1)(k-1)}{3n}\left[k\binom{n}{k} + 3n\right] + \frac{3^k(nk-k^2)}{n}\binom{n}{k} - k3^k(n-1), \text{ if } 2 \leq k \leq n-2.$$

Proof. Let $G = W_n$ be a wheel graph with n vertices, $n \geq 4$. Let w be the apex in G and $V(G) - \{w\} = \{v_1, v_2, \ldots, v_{n-1}\}$ be the vertices in the outer cycle C_{n-1}. Let $k \geq 2$ be an integer with $|S| = k$. **Case (i):**$k = n$.
From the definition of G, we have $deg_G(w) = n-1$ and $deg_G(v_i) = 3$, $1 \leq i \leq n-1$. By Theorem 2.1, we have $d_G(S) = k-1 = n-1$. And

$$\prod_{v\in S} deg_G(v) = 3^{(n-1)}(k-1).$$

$$\text{Therefore,} SGut_k(G) = \sum_{\substack{S\subseteq V(G),\\ |S|=k}} \left[\prod_{v\in S} deg_G(v)\right] d_G(S) = 3^{(n-1)}(n-1)^2.$$

$$\text{Hence } SGut_k(G) = 3^{(n-1)}(n-1)^2.$$

Case (ii):$k = n-1$

Let $S \subseteq V(G)$ with $k = n-1$ vertices. Clearly $w \in S$ or $w \notin S$.

If $w \notin S$, then S contains the vertices of the outer $(n-1)$- cycle of G. By Theorem 2.1, we have $d_G(S) = k-1$ and the degree of each vertex in the outer $(n-1)$- cycle is 3. Thus we have $\prod_{v\in S} deg_G(v) = 3^{(n-1)}$.

Suppose $w \in S$. By Theorem 2.1, we have $d_G(S) = k-1$.As$w \in S$, the remaining $k-1$ vertices can be chosen from the vertices of $(n-1)$- cycle.

$$\text{Therefore,} \prod_{v\in S} deg_G(v) = \left[3^{(n-2)}(k-1)\right]\binom{n-1}{k-1}$$

$$SGut_k(G) = \sum_{\substack{S\subseteq V(G),\\ |S|=k}} \left[\prod_{v\in S} deg_G(v)\right] d_G(S).$$

$$= \left[3^{(n-1)} + \left(3^{(n-2)}(k-1)\right)\binom{n-1}{n-2}\right](k-1).$$

$$= 3^{(n-2)}[3+(n-1)^2](n-2).$$

$$\text{Therefore } SGut_k(G) = 3^{(n-2)}[3+(n-1)^2](n-2).$$

Case (iii):$2 \le k \le n-2$.

Let $S \subseteq V(G)$ with k vertices. Then $w \in S$ or $w \notin S$.

If$w \in S$, then by Theorem 2.1, we have $d_G(S) = k-1$. Also as $w \in S$, the remaining $k-1$ vertices can be chosen from the vertices of $(n-1)-$ outer cycle in $\binom{n-1}{k-1}$ ways. Therefore $\prod_{w\in S} deg_G(v) = \left[3^{(k-1)}(n-1)\right]\binom{n-1}{k-1}$.

If $w \notin S$, then by Theorem 2.1, we have $k-1 \le d_G(S) \le k$; now let S contains the vertices of the outer $(n-1)-$ cycle of G.

In this case, if we choose S to be k consecutive vertices in the outer cycle, then there are $n-1$ such sets for$n-1 > k$.And$d_G(S) = k-1$. The degree of each vertex in S is 3 and hence

$$\prod_{v\in S} deg_G(v) = 3^k(n-1).$$

Otherwise, if$S \subseteq V(G)$, is a set of k vertices in the outer $(n-1)-$ cycle which contains at least one non-consecutive vertex, then there are $\left[\binom{n-1}{k} - (n-1)\right]$ such sets. And hence $d_G(S) = k$.

$$\text{Thus,} SGut_k(G) = [3^k(n-1)](k-1) + \left[3^k\left[\binom{n-1}{k} - (n-1)\right]\right](k) + \left[(n-1)3^{(k-1)}\binom{n-1}{k-1}\right](k-1)$$

$$= \frac{3^k(n-1)(k-1)}{3n}\left[k\binom{n}{k} + 3n\right] + \frac{3^k(nk-k^2)}{n}\binom{n}{k} - k3^k(n-1).$$

This completes the proof.

4. Steiner Harary Index of Wheel graphs

Theorem 4.Let $G = W_n$ be a wheel graph with n vertices, $n \geq 4$. Let k be an integer with $2 \leq k \leq n$. Let S be a subset of $V(G)$ and let $|S| = k$. Then

$$SH_k(G) = \begin{cases} \frac{1}{n-1}, & \text{if } k = n. \\ \frac{n}{(n-2)}, & \text{if } k = n-1. \end{cases}$$

$$SH_k(G) = \left[\frac{k}{n(k-1)} + \frac{n-k}{k}\right]\binom{n}{k} + \frac{n-1}{k(k-1)}, \quad \text{if } 2 \leq k \leq n-2.$$

Proof. Let $G = W_n$ be a wheel graph with n vertices, $n \geq 4$. Let w be the apex in G and $V(G) - \{w\} = \{v_1, v_2, \ldots, v_{n-1}\}$ be the vertices in the outer cycle C_{n-1}. Let $k \geq 2$ be an integer with $|S| = k$.
Case (i): $k = n$.
From the definition of G, we have $deg_G(w) = n-1$ and $deg_G(v_i) = 3$, $1 \leq i \leq n-1$. By Theorem 2.1, we have $d_G(S) = k-1 = n-1$.

Therefore, $SH_k(G) = \frac{1}{n-1}$

Case (ii): $k = n-1$.
Let $S \subseteq V(G)$ with $k = n-1$ vertices. Clearly $w \in S$ or $w \notin S$.

If $w \notin S$, then S contains the vertices of the outer $(n-1)$- cycle of G. By Theorem 2.1, we have $d_G(S) = k-1$.

Suppose $w \in S$. By Theorem 2.1, we have $d_G(S) = k-1$. As $w \in S$, the remaining $k-1$ vertices can be chosen from the vertices of $(n-1)$- cycle in $\binom{n-1}{k-1}$ ways. Therefore,

$$\sum_{w \in S} \frac{1}{d_G(S)} = \binom{n-1}{n-2}(k-1)$$
$$SH_k(G) = \frac{1}{n(n-2)}.$$

Case (iii): $2 \leq k \leq n-2$.

$$SH_k(G) = \sum_{\substack{S \subseteq V(G), \\ |S|=k}} \frac{1}{d_G(S)}$$

Let $S \subseteq V(G)$ with k vertices. Then $w \in S$ or $w \notin S$.

If $w \in S$, then by Theorem 1, we have $d_G(S) = k-1$. Also as $w \in S$, the remaining $k-1$ vertices can be chosen from the vertices of $(n-1)-$ outer cycle in $\binom{n-1}{k-1}$ ways. Therefore

$$\sum_{w\in S}\frac{1}{d_G(S)} = \binom{n-1}{n-2}\frac{1}{(k-1)}.$$

If $w \notin S$, then by Theorem 2.1, we have $k-1 \le d_G(S) \le k$; S contains the vertices of the outer $(n-1)-$ cycle of G.

In this case if we choose S to be k consecutive vertices in the outer cycle, then there are $n-1$ such sets for $n-1 > k$. And $d_G(S) = k-1$.

And hence $\sum_{w\in S}\frac{1}{d_G(S)} = (n-1)\frac{1}{(k-1)}$.

Otherwise, if $S \subseteq V(G)$, is a set of k vertices in the outer $(n-1)-$ cycle which contains at least one non-consecutive vertex, then there are $\left[\binom{n-1}{k}-(n-1)\right]$ such sets. And hence $d_G(S) = k$.

$$\sum_{w\notin S}\frac{1}{d_G(S)} = \left[\binom{n-1}{k}-(n-1)\right]\frac{1}{k}.$$

$$\text{Therefore, } SH_k(G) = \left[\frac{k}{n(k-1)}+\frac{n-k}{k}\right]\binom{n}{k}+\frac{n-1}{k(k-1)}, \quad if \quad 2 \le k \le n-2.$$

This completes the proof.

5. Steiner Reciprocal Degree Distance of Wheel graphs

Theorem 5. Let $G = W_n$ be a wheel graph with n vertices, $n \ge 4$. Let k be an integer with $2 \le k \le n$. Let S be a subset of $V(G)$ and let $|S| = k$. Then $SRD\ D_k(G) = \begin{cases} 4, & \text{if } k = n. \\ 4(n-1)^2, & \text{if } k = n-1. \end{cases}$ and

$$SRD\ D_k(G) = \left[(n-1)\frac{k}{n}+3\right]\binom{n}{k}+\frac{3k(n-1)}{k-1}-3(n-1), \text{ if } 2 \le k \le n-2.$$

Proof. Let $G = W_n$ be a wheel graph with n vertices, $n \ge 4$. Let w be the apex in G and $V(G)-\{w\} = \{v_1, v_2, \ldots, v_{n-1}\}$ be the vertices in the outer cycle C_{n-1}. Let $k \ge 2$ be an integer with $|S| = k$.
Case (i): $k = n$.
From the definition of G, we have $deg_G(w) = n-1$ and $deg_G(v_i) = 3$, $1 \le i \le n-1$. By Theorem 2.1, we have $d_G(S) = k-1 = n-1$.

And $\sum_{v\in S} deg_G(v) = 3(n-1)+(n-1) = 4(n-1)$.

$$\begin{aligned}\text{Therefore, } SRD\ D_k(G) &= \sum_{\substack{S\subseteq V(G),\\ |S|=k}}\frac{[\sum_{v\in S} deg_G(v)]}{d_G(S)} \\ &= \frac{4(n-1)}{(n-1)} \\ &= 4.\end{aligned}$$

$$\text{Therefore } SRD\ D_k(G) = 4.$$

Case (ii): $k = n-1$.
If $w \notin S$, then using the Theorem 2.1, we get $d_G(S) = k-1$ and $\sum_{v\in S} deg_G(v) = 3(n-1)$.

Suppose $w \in S$. By Theorem 2.1, we have $d_G(S) = k-1$.

Also $\sum_{v\in S} deg_G(v) = [3(n-2)+(n-1)]\binom{n-1}{n-2}$.

$$\sum_{\substack{S\subseteq V(G),\\ |S|=k}} \frac{[\sum_{v\in S} deg_G(v)]}{d_G(S)} = \left[3(n-1) + \left(3(n-2)+(n-1)\right)\binom{n-1}{n-2}\right]\frac{1}{(k-1)}.$$

$$= 4(n-1)^2$$

Therefore, $SRD \;\; D_k(G) = 4(n-1)^2$.

Case (iii): $2 \le k \le n-2$.

Let $S \subseteq V(G)$ with k vertices. Then $w \in S$ or $w \notin S$.

$$SRD \;\; D_k(G) = \sum_{\substack{S\subseteq V(G),\\ |S|=k}} \frac{[\sum_{w\in S} deg_G(v)]}{d_G(S)} + \sum_{\substack{S\subseteq V(G),\\ |S|=k}} \frac{[\sum_{w\notin S} deg_G(v)]}{d_G(S)}.$$

If $w \in S$, then by Theorem 2.1, we have $d_G(S) = k-1$. Also as $w \in S$, the remaining $k-1$ vertices can be chosen from the vertices of $(n-1)-$ outer cycle in $\binom{n-1}{k-1}$ ways. Therefore $\sum_{w\in S} deg_G(v) = [(n-1) + 3(k-1)]\binom{n-1}{k-1}$.

If $w \notin S$, by Theorem 2.1, we have $k-1 \le d_G(S) \le k$; S contains the vertices of the outer $(n-1)-$ cycle of G.

In this case if we choose S to be k consecutive vertices in the outer cycle, then there are $n-1$ such sets for $n-1 > k$. And $d_G(S) = k-1$. The degree of each vertex in S is 3 and hence $\sum_{v\in S} deg_G(v) = 3k(n-1)$

Otherwise, if $S \subseteq V(G)$, is a set of k vertices in the outer $(n-1)-$ cycle which contains at least one non-consecutive vertex, then there are $\left[\binom{n-1}{k} - (n-1)\right]$ such sets with $d_G(S) = k$.

$$\text{Therefore, } \sum_{v\in S} deg_G(v) = 3k\left[\binom{n-1}{k} - (n-1)\right].$$

$$\text{Thus, } SRD \;\; D_k(G) = \left[\frac{4nk - 3n - k}{n(k-1)}\right]\binom{n}{k} + 3\left[\frac{n-1}{k-1}\right] \quad if \quad 2 \le k \le n-2.$$

This completes the proof.

6. Conclusion

In this paper, we have obtained the Steiner distance of wheel graphs. By using the Steiner distance we have calculated the Steiner degree distance, Steiner Gutman index, Seiner Harary index and Steiner reciprocal degree distance of wheel graphs.

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