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A Study of Homogeneous and Isotropic Cosmological Models

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Abstract

Cosmology is the study of universe as a whole. Cosmos means universe and logos means science or study; therefore, the science of universe is known as cosmology. Gravitation can be described by a space-time (metric). The metric is related to the matter and energy in the Universe through Einstein's field equation, which contains matter as well as energy part. The left-hand side of the Einstein's field equations is the Einstein's tensor, which describes the geometry of the Universe. The energy momentum tensor present in the right-hand side of the Einstein's equations, describes the energy part of the Universe.The Einstein theory is not the only geometric theory of gravitation that can be constructed upon the Newton theory and special relativity. Almost all of the assumptions made in constructing new theories can be modified so that another set of equations will result. In this research work we have investigated Bianchi cosmological models, under the framework of general relativity, and some alternative theories of gravity. In the framework of General Relativity and assuming the Copernican principle, accounted for by the Friedman-Lemaitre-Robertson-Walker metric, a spatially flat universe is consistent with the cosmographic conversely, this condition, which is fulfilled by astrophysical measurements, necessarily requires spatial flatness. Here, we will construct some cosmological models assuming the validity of General Relativity, of Copernican principle (homogeneity and isotropy at large scale) and on dark energy pictured either by some non-ideal fluids or by canonical scalar fields interacting with dark matter. A theorist cannot appeal to this theory in order to justify their views.

Keywords: Homogeneous, Isotropic Cosmological Models, space-time, geometric theory.

INTRODUCTION

The field of cosmology is concerned with the study of the structure, history, and potential future evolution of the universe on a galactic scale, spanning billions of light-years. To investigate the physics of the cosmos, cosmologists develop theoretical models within the context of general relativity. They focus on the big picture by contrasting the seen cosmos with the models. Before the advent of general relativity, cosmologists looked on Newton's theory of gravitation to explain the cosmos. A variety of issues arose while attempting to deal with the dynamics of the cosmos using Newtonian cosmological models. The hypothesis relies on the idea that a gravitational disturbance can spread instantly, which is a controversial idea, especially when extrapolated across huge distances. This stymied the development of Newtonian theory. The general theory of relativity proposed by Einstein ultimately superseded Newtonian theory. Einstein's general theory of relativity was a major influence on modern cosmology. For the first time, this theory offers a physical and mathematical framework of general relativity to address issues of galactic proportions.

It has been discovered that most of the universe's matter is concentrated into large structures like galaxies, clusters of galaxies, and groups of stars. The cosmos is described in the continuum approximation, as if it were a cosmic fluid, and its distribution is viewed as a fine structure. Approximately 106 of the total volume of the universe is taken up by the galaxies. This clarifies why cosmological models treat galaxies as though they were points. Cosmological parameters, such as Hubble's parameter, density parameter, deceleration parameter, curvature, etc., are used to verify the physical feasibility of cosmological theories that accurately describe our universe and its history. Before embracing any cosmological model, there have been robust agreements between theoretical predictions and experimental observations. In addition to its roots in physics and astronomy, cosmology has traditionally captivated the imagination of religious thinkers, philosophers, and poets. Cosmology is the study of the cosmos on a grand scale, yet our observational techniques and understanding of the laws of nature are not yet sophisticated enough to comprehend the scientific data we have collected thus far. The unfathomable thing about the universe is that it is incomprehensible; hence the answers to these problems are evident. In his general theory of relativity, published in 1915, Albert Einstein provided a full explanation of gravity as geometric phenomena, described by a single field equation that describes the fundamental interactions and motions of any particle. The first physically relevant solution to Einstein's field equations was obtained by Chodos [1]. A spherically symmetrical distribution of matter was used to demonstrate the bending of space-time. The results of these three tests give us strong evidence that general relativity is an improvement over Newtonian gravity and provides a satisfactory treatment of the field from close to a star to galactic distances. Therefore, at the current moment, general relativity provides the only viable theory of gravitation that might be applied to examine the behavior of enormous areas of the universe, and so we are compelled to utilies this theory if we are to engage in any cosmological speculations at all.

The absence of gravity is assumed in Special Relativity, allowing us to explore the nature of physical reality. Mechanical and electromagnetic phenomena within a reference frame are the focus of the Special Theory of Relativity [15-17]. The impact of high-speed motion on an observer is investigated in this theory. Gravitational field is first described in General Relativity. simply said, cosmology is the study of the cosmos as a whole. Since cosmos means universe and logos means science or study, cosmology is the study of the cosmos. A spacetime (metric) can characterize gravity. Einstein's field equation, which includes both matter and energy parts, establishes a connection between the metric and the matter and energy in the universe. The Einstein tensor, which describes the geometry of the universe, is the left-hand side of Einstein's field equations. The right-hand side of Einstein's equations contains the energy momentum tensor, which characterizes the energy component of the universe. It is space itself, according to Einstein's general theory of relativity. Using Newton's theory and special relativity, one can build alternative geometric theories of gravitation in addition to Einstein's. Almost every premise used to build a new theory can be changed to provide a different set of equations. In this study, we have looked into possible alternatives to general relativity and the framework of general relativity with Bianchi cosmological models.

HOMOGENEOUS AND ISOTROPIC COSMOLOGICAL MODELS

Einstein's general theory of relativity, which provides new ways to think about and solve issues on a cosmic scale, is often credited as the inspiration for modern cosmology. The static cosmological models Bianchi [2] used were his own creations, and they were all filled with a perfect fluid with a uniform distribution. However, the model has a number of drawbacks that make it undesirable. This runs counter to observations made by Caroll [9] who found that the redshift of nebulae light increased at least very closely in a linear fashion with increasing distance. Bertolami, and Mortin’s [3] described the Einstein static universe with a pressure term and gave the field equations for this situation shortly after the publication of Einstein's static model. Nothing at all, not even radiation, exists in the de-sitter cosmos. In the de-sitter universe, we learn how Hubble and Humason's [9] measured redshift is really working. The observed contraction of nebulae is consistent with the de-sitter cosmos being entirely empty. The Einstein universe on the other hand is dense with stuff, but it fails to account for the observed receding of nebulae. Therefore, neither Einstein's nor de-sitter's universes are accurate representations of the real one. Non-static models where the metric tensor is inherently time-dependent are required to develop a model that combines the benefits of Einstein's and desitter's static models.

1. Standard Model and Cosmological Constant

Huang et al.,Akademie, [5, 6] used the Cosmological Principle to solve Einstein's field equations, and the resulting non-static cosmological solutions are consistent with an expanding universe. Therefore, the Friedmann-Robertson-Walker (FRW) metric is the best line element for representing a non-static and homogenous model of the cosmos. In standard spherical coordinates $(x_i) = (\mathrm{t}, \mathrm{r}, \theta, \varphi)$, a spatially homogeneous and isotropic FRW line element has the form (in units c=1)

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\emptyset^2)\right] \tag{1.0.1}$$

Where, $a(t)$ is the cosmic scale factor that describes the expansion or contraction of the universe in terms of distances (scales), and is related to the red shift of the 3-space; k is the curvature parameter that describes the geometry of the spatial section of space-time, with closed, flat, and open universes corresponding to $k = -1, 0$, and 1, respectively. Amazingly, the FRW models have been able to satisfactorily describe the observed characteristics of the cosmos.

The Einstein's field equations for the metric (1.0.1), in case of the energy momentum tensor, reduce to the following equations:

$$\frac{a^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{1.0.2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{1.0.3}$$

Where, an over dot denotes derivative with respect to the cosmic time t.

For the FRW space-time (1.0.1) and the perfect fluid energy-momentum tensor, yields a single conservation equation

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0 \tag{1.0.4}$$

In reality, this equation cannot exist without the Friedmann equations. This means that variations in the energy density at a given location are possible in the cosmos (as defined by the Hubble parameter $H = \dot{a}/a$). Due to the free flow of energy between matter and the space-time geometry, it is important to keep in mind that the concept of "total energy" does not hold.

The Friedmann-Robertson-Walker (FRW) models are fundamental in the study of the cosmos. These models may not be perfect representations of the cosmos, but they do provide useful global approximations of the universe as it is now. FRW models are characterized by (i) the universe being the same at all points in space (spatially homogeneous) and (ii) all spatial directions at a point being equivalent (isotropy).

2. Spatially Homogeneous and Anisotropic Models

There are no observational facts that ensure in an era previous to the recombination that the cosmos was isotropic and homogenous at the time. It's not known what kinds of matter fields existed in the early universe. Assumptions of spherical symmetry and isotropy are not strictly valid near the big bang singularity;therefore, a smoothed-out depiction of the early cosmos is impossible. In order to understand the origins of modern-day local anisotropies in galaxies, clusters, and super clusters, studying anisotropy at early times is a natural next step. Several probable causes have been proposed for these anisotropies [14]. These include cosmic magnetic or electric fields, long-wavelength gravitational waves, Yang-Mills fields, and others. In addition, theoretical interest in the cosmological models with anisotropic background has been sparked by experimental studies of the isotropy of the CMBR and speculation about the amount of helium generated in the early stages of the evolution of the universe. Therefore, it seems reasonable to assume a geometry that is more generic than merely the isotropy and homogeneous FRW geometry in order to characterise the early evolution of the cosmos. Understanding the early behaviour of the universe is crucial, and anisotropic cosmological models play a crucial part in this. Understanding the evolution of the universe and the factors that will shape its future are primary goals of modern cosmology. The presence of an anisotropic phase that, over long periods of time, approaches to an isotropic one has been confirmed by recent cosmological studies.

2.1. SPACE-TIME

General relativity is a field theory for the metric $g_{\mu\nu}$on a space-time Manifold M which determines the line element$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$. The line element specifies the geometry of space-time. We can measure the length of a curve $c: R \to M, t \to x^\mu(t)$, using

$$l(c) = \int ds = \sqrt{g\mu\nu\big(x(t)\big)\dot{x}^\mu(t)\dot{x}^\nu(t)}\, dt \qquad (1.1.1)$$

The current mainstream model of cosmology is based on predictions from general relativity, and these predictions include both expanding and contracting universes [18]. It allowed the so-called Friedmann-Lemaitre-Robertson-Walker (FLRW) models [18] to be created by combining the ideas of Friedmann and Lemaitre about expanding universes [4]; Banerjee, and Pavon [7]) with the geometry of homogeneous and isotropic spacetimes proposed by Adhav [10] and Felice [11]. Cosmic structures were added to these models using cosmological perturbation theory, which described the background cosmological evolution.

Consider the choice

$$ds = f(x, dx) = (g_{\mu\nu x^\mu x^\nu})^{1/2} (1.1.2)$$

where, $ds$is the square root of a quadratic form. Here $g_{\mu\nu}$are functions of the coordinates; there are ten of $g_{\mu\nu}$, since without loss of generality we can assume $g_{\mu\nu} = g_{\nu\mu}$ [12].

It is known that we can always choose coordinates so that at some specified event a nonsingular quadratic form reduces to where C is a positive or negative integer.

$$g_{\mu\nu}dx^\mu dx^\nu = C_1(dx^1)^2 + C_2(dx^2)^2 + C_3(dx^3)^2 + C_4(dx^4)^2 \qquad (1.1.3)$$

Changing the coordinates will not alter the form's signature prevalence of plus and minus signs. (It is assumed that only real coordinates are being considered; changing the signs by using imaginary coordinates is possible, and we will do so in the future, but only as a notational device) [11].

The indicators that should appear in (1.1.3) can be determined in light of what has been discussed so far regarding the past, the present, and the future. Since coordinates might be swapped around, we count only five possible signs to use instead:

a) + + + +
b) + + + -
c) + + - - (1.1.4)
d) + - - -
e) - - - -

Since ds, as in (1.1.2), is a real measured quantity, those space-time directions in which a clock can move must satisfy

$g_{\mu v} dx^{\mu} dx^{v} < 0$ (1.1.5)

It is possible to divide the space-time around an event into the past B, the present P, and the future A. We can write (1.1.5) if and only if the permissible directions for world lines are found in the past and the future.

past and future: $g_{\mu v} dx^{\mu} dx^{v} < 0$

present: $g_{\mu v} dx^{\mu} dx^{v} > 0$ (1.1.6)

null cone: $g_{\mu v} dx^{\mu} dx^{v} = 0$

There must be a past, a present, and a future, but the present should be a simple 3-dimensional object. The present is the dividing line between the past and the future Abdalla et al. [13].

past and future: $g_{\mu v} dx^{\mu} dx^{v} = C_1 (dx^1)^2 + C_2 (dx^2)^2 - C_3 (dx^3)^2 - C_4 (dx^4)^2 < 0$

present:$g_{\mu v} dx^{\mu} dx^{v} = C_1 (dx^1)^2 + C_2 (dx^2)^2 - C_3 (dx^3)^2 - C_4 (dx^4)^2 > 0$(1.1.7)

In this case, the present is 4d space-time, but it's easy to see that it fails to separate the region "past and future" into a past and a future, as one can go from any event in "past and future" to any other event in "past and future" without ever coming into contact with the present. For the same reasons, we can't consider ((1.1.4) c) or (1.1.4) d). Therefore, ((1.1.4) b) is the only option left:

$g_{\mu v} dx^{\mu} dx^{v} = C_1 (dx^1)^2 + C_2 (dx^2)^2 + C_3 (dx^3)^2 - C_4 (dx^4)^2 > 0$(1.1.8)

There are three + and one - sign. We haves

past and future: $g_{\mu v} dx^{\mu} dx^{v} = C_1 (dx^1)^2 + C_2 (dx^2)^2 + C_3 (dx^3)^2 - C_4 (dx^4)^2 < 0$

present: = $C_1 (dx^1)^2 + C_2 (dx^2)^2 + C_3 (dx^3)^2 - C_4 (dx^4)^2 > 0$(1.1.9)

null cone:$C_1 (dx^1)^2 + C_2 (dx^2)^2 + C_3 (dx^3)^2 - C_4 (dx^4)^2 = 0$

The past being distinguished from the future by the condition that dx^4 is positive for one and negative for the other, but which is depends on the way in which x^4 is chosen. We shall choose it so that x^4 increases ($dx^4 > 0$)as we pass into the future [11].

At the chosen event and for the special coordinates used, we have then

$ds = f(x, dx) = [-(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + (dx^4)^2]^{1/2}$ (1.1.10)

This reduction of (1.1.2) is generally possible for only one occurrence, selected at random. For any given coordinate system x, if d s has the simple form (1.1.10) at some event E, then this simplicity is lost at other events, and is recovered at a second event E' only by changing the coordinate system and giving up the simplicity at E. At least, that's how things usually work, though we'll see that, in the absence of gravitational fields, a single coordinate system can achieve the basic form (1.1.10) everywhere in space and time.

Time-like vectors are conceivable tangents to the world lines of conventional clocks and can be used to travel back in time or into the future. Null vectors are those that lie on the null cone, whereas space-like vectors are those that are drawn into the present.

Thus, vectors are classified as follows:

time like: $g_{\mu v} V^{\mu} V^{v} < 0,$

spacelike: $g_{\mu v} V^{\mu} V^{v} > 0$ (1.1.11)

null: $g_{\mu v} V^{\mu} V^{v} = 0$

This quadratic form $g_{\mu v} dx^{\mu} dx^{v}$ is fundamental in general relativity [11].

2.2 HUBBLE PARAMETER

Questions about the universe's beginnings, ages, sizes, geometries, and final fate are central to the work of cosmologists. The field of study known as cosmology examines the origins, development, and fate of the cosmos as a whole. To begin our investigation, let's ask the obvious question: "Why is the sky dark at night?" If the universe is static, that is, it is neither expanding nor contracting, and the distribution of stars is uniform across the sky, then the sky should be bright everywhere, day or night (Fig. 1). This is for the same reason that every line of sight eventually intersects a tree in a hypothetical infinite forest. However, this is obviously not the case, resulting in a paradox (the dark sky conundrum) [12].

If we instead imagine an endless number of spheres with Earth at their centre, as shown in Fig. 2, we get a different picture. There is no difference in shell thickness, but the volume occupied by each further shell increases. There are more stars in the outer shells (due to their increasing volume), but the individual stars are dimmer are closer to the core. It's a model of a static cosmos in which space-age spheres of the same thickness surround a central Earthy sphere and contain an equal number of stars. Since the universe is eternal and there have always been an endless amount of stars, even at night, the sky should be extraordinarily luminous.

There may be several resolutions of Olbers's paradox for example

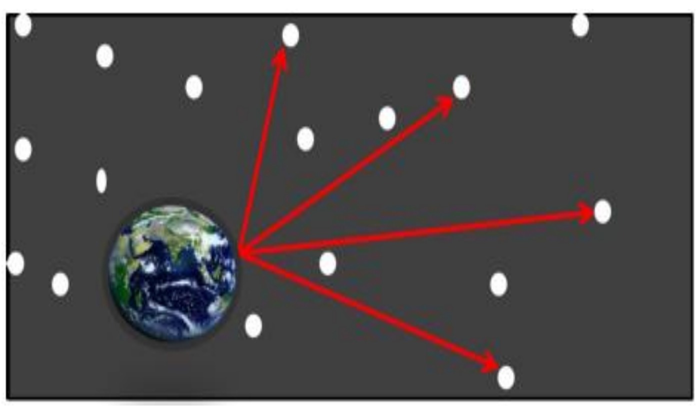

Figure 1: If we look far enough in any direction in an infinite universe, our line of sight should eventually hit the surface of star

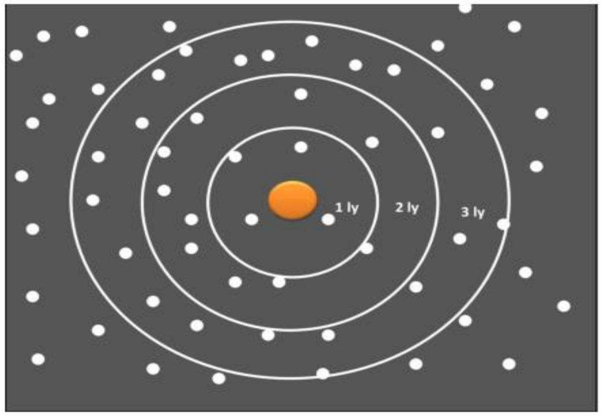

Figure 2: A model nonexpanding universe in which the stars are uniformly distributed in spherical shells, all of the same thickness, centered on the Earth

• The universe might have finite size.

• The universe has infinite size but with few or no stars for away.

• The universe has a finite age, etc.

The fact that the universe has an age, roughly 14 billion years, is a fundamental real solution to Olbers' paradox. We don't live in a static, unchanging universe. It is, in fact, growing. Olbers' paradox can potentially be explained by the expansion of the universe. When analysing galaxy spectra, Edwin Hubble arrived at the same conclusion about the expansion of the universe. Jamil, and Rashid, [8], Abdalla et al., [13]. In every direction that Hubble looked, galaxies' spectra were found to be red shifted towards longer wavelengths. Therefore, the current distance (d) of a galaxy is directly proportional to its measured redshift (z). If the observed redshift (z) is the result of a receding object, then the recession velocity (v) can be calculated using the Doppler Effect. Redshift z of an object is defined by $z = \Delta\lambda/\lambda_0$, where, $\Delta\lambda = \lambda - \lambda_0$is the observed wavelength of an absorption or emission line in the objects spectrum minus its laboratory(rest) wavelength.

CONCLUSION

The cosmological principle is supported by observations showing that, on very large sizes, the cosmos flattens out. Homogeneity and isotropy are two of the most notable characteristics of our large-scale universe. Isotropy is the claim that the universe appears uniform in all directions, whereas homogeneity is the claim that it appears uniform at all points. These are not necessary implications of one another. A world with a uniform magnetic field, for instance, is homogeneous since all points are the same, but it is not isotropic because directions parallel to the field lines and those along the field lines are distinguishable. A spherically symmetric distribution, on the other hand, is isotropic but not necessarily homogeneous when viewed from the Centre. However, homogeneity can also be enforced by insisting that a distribution be isotropic about every point. Due to the imprecision of the cosmological principle, homogeneity and isotropy are not strictly adhered to in our universe. The most exciting area of cosmological inquiry right now is the investigation of deviations from homogeneity. In 1922, a Russian mathematician named Alexander Friedmann created a dust model of Einstein field equations that was homogeneous, isotropic, and free of the-term. In 1927, the expanding universe hypothesis was independently discovered by a Belgian mathematician, physicist, and priest named George Lemaitre. In 1930, H.P. Robertson and A.G.Walker established that the most general line-element meeting homogeneity and isotropy of the universe could be generated from a non-static, homogeneous, and isotropic model.

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