Open Neighbourhood Coloring of Line, Middle and Total graphs of some graphs

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Abstract
In this paper, we determine the Open neighborhood chromatic number of line graph of Comb graph and double comb graph. Also, we obtain the open neighborhood chromatic number of the middle graph and total graph of path graph P_n, cycle graph C_n.

Keywords: simple graph, Open neighborhood coloring, line graph, middle graph,total graph.
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1 Introduction

All the graphs considered here are simple, connected and undirected graph $G = (V(G), E(G))$. For every vertex $a,b \in V(G)$, the edge connecting two vertices is denoted by $ab \in E(G)$. For all other standard concepts of graph theory, we see [1], [2], [5].
An open neighborhood k-coloring of a graph $G(V,E)$ is a k-coloring $f : V(G) \to \{1,2,\cdots k\}, k \in Z^+$ which admits the conditions such that for each $c \in V(G)$ and $\forall a,b \in N(c), f(a) 6= f(b)$. The minimum value of k for which G admits an open neighborhood k-coloring is called the open neighborhood chromatic number of G and denoted by $\chi_{onc}(G)$.
[3], [4] Geetha K. N. introduced the notion and discussed the open neighborhood chromatic number of some graphs. Also, Adjacent vertex distinguishing total coloring of line and splitting graph of some graph and line graph of snake graph family has been obtained in the literature [6],
[7],[8].
The line graph $L(G)$ is the graph that represents the adjacencies between the edges of G. The vertex set of the middle graph $M(G)$ of the graph G is $V(G) \cup E(G)$ and in which two vertices are adjacent in $M(G)$ if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it. The total graph $T(G)$ of G has vertex set $V(G) \cup E(G)$, and edges joining all elements of this vertex set which are adjacent or incident in G.

Definition 1. *The comb graph denoted by c^m is obtained from the path graph P_m with $\{v_1, v_2, \cdots v_m\}$ and the new vertices $\{u_1, u_2, \cdots u_m\}$ by joining the vertices $v_x u_x$ for $1 \le x \le m$.*

$$V[c^m] = \left(\bigcup_{i=1}^{m} u_x \cup v_x \right)$$

and

$$E[cm] = \left(\bigcup_{x=1}^{m-1} vxvx{+}1 \right) \cup \left(\bigcup_{x=1}^{m} uxvx \right)$$

Here $d(v_1) = d(v_m) = 2$, $d(v_x) = 3$ *for* $2 \le x \le m-1$ *and* $d(u_x) = 1$ *for* $1 \le x \le m$.

Definition 2. *The double comb graph denoted by Dc^m is obtained from the path graph P_m with $\{v_1, v_2, \cdots v_m\}$ and the new vertices $\{u_1, u_2, \cdots u_m\}$ and $\{w_1, w_2, \cdots w_m\}$ by joining the vertices $v_x u_x$ and $v_x w_x$ for $1 \le x \le m$.*

$$V[Dc^m] = \left(\bigcup_{x=1}^{m} u_x \cup v_x \cup w_x \right)$$

and

$$E[Dc^m] = \left(\bigcup_{x=1}^{m-1} (v_xv_{x+1} \cup u_xv_x \cup v_xw_x \right)$$

Here $d(v_1) = d(v_m) = 3$, $d(v_x) = 4$ *for* $2 \le x \le m-1$ *and* $d(u_x) = d(w_x) = 1$ *for* $1 \le x \le m$.

2 Open neighborhood coloring of Line graph of comb and double comb graph

In this section, An open neighborhood coloring of line graph of comb graph c^m and double comb graph Dc^m are discussed.

Theorem 2.1. *For Comb graph* c^m, $\chi_{onc}(c^m) = 3$, *for* $m \ge 3$.

Proof. Let we denote $\{v_1, v_2, \cdots, v_m, u_1, u_2, \cdots, u_m\}$ as the vertices of c^m. Hence $|V(c^m))| = 2m$ and $|E(c^m)| = 2m-1$. Now we define $f : V(c^m)) \to Z^+$ given by for $m \ge 3$,

$$\text{For } 1 \le x \le m, \quad m, \quad f(v_x) = \begin{cases} \{1\}, & \text{if } x \equiv 1,2 \ (mod\ 4) \\ \{2\}, & \text{if } x \equiv 0,3 \ (mod\ 4) \end{cases}$$

$$\text{For } 1 \le x \le m, \quad f(u_x) = 3$$

It is easy to verify that f is an open neighborhood 3-coloring of c^m.

$$\therefore \chi_{onc}(c^m) = 3, \qquad m \ge 3.$$

Hence the theorem. □

Theorem 2.2. *For Double Comb graph* Dc^m, $\chi_{onc}(Dc^m) = 4$, *for* $m \ge 3$.

Proof. Let we denote $\{v_1, v_2, \cdots, v_m, u_1, u_2, \cdots, u_m, w_1, w_2, \cdots, w_m\}$ as the vertices of Dc^m. Hence $|V(Dc^m))| = 3m$ and $|E(Dc^m)| = 3m-1$. Now we define $f : V(Dc^m)) \to Z^+$ given by for $m \ge 3$,

$$\text{For } 1 \le x \le m, \quad f(v_x) = \begin{cases} \{1\}, & \text{if } x \equiv 1,2 \ (mod\ 4) \\ \{2\}, & \text{if } x \equiv 0,3 \ (mod\ 4) \end{cases}$$

$$\text{For } 1 \le x \le m, \qquad f(u_x) = 3 \text{ and } f(w_x) = 4.$$

It is easy to verify that f is an open neighborhood 4-coloring of Dc^m.

$$\therefore \chi_{onc}(Dc^m) = 4, \qquad m \ge 3.$$

Hence the theorem. □

Theorem 2.3. *For Line graph of Comb graph* c^m, $\chi_{onc}(L(c^m)) = 5$, *for* $m \ge 4$.

Proof. From the defn (1), The Line graph of c^m is obtained by replacing all edges as vertices, we have $v_xv_{x+1} = s_x$, for $1 \le x \le m-1$, $u_xv_x = t_x$ for $1 \le x \le m$. Here s_x, t_x are the vertices of $L(c^m)$. Hence, the vertex set and edge set of $L(c^m)$ is given by

$$V[L(c^m)] = \left(\bigcup_{x=1}^{m-1} s_x \ \cup \ \bigcup_{x=1}^{m} (t_x) \right)$$

$$E[L(c^m)] = \left(\bigcup_{x=1}^{m-2} (s_xs_x+1) \ \cup \ \bigcup_{x=1}^{m-1} (s_xt_x \cup s_xt_x+1) \right)$$

Therefore $|V(c^m))| = 2m-1$ and $|E(L(c^m))| = 3m-4$. Now we define $f : V[L(c^m)] \to 5$ given by for $m \ge 4$,

$$\text{For } 1 \le x \le m-1, \quad f(s_x) = \begin{cases} \{1\}, & \text{if } x \equiv 1 \ (mod\ 3) \\ \{2\}, & \text{if } x \equiv 2 \ (mod\ 3) \\ \{3\}, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases}$$

$$\text{For } 1 \le x \le m, \quad m, \quad f(t_x) = \begin{cases} \{4\}, & \text{if } x \equiv 1 \ (mod\ 2) \\ \{5\}, & \text{if } x \equiv 0 \ (mod\ 2) \end{cases}$$

It is easy to verify that f is an open neighborhood 5-coloring of $L(c^m)$.

$$\therefore \chi_{onc}(L(c^m)) = 5, \qquad m \ge 4.$$

Hence the theorem. □

Theorem 2.4. *For Line graph of double Comb graph Dc^m, $\chi_{onc}(L(Dc^m)) = 7$, for $m \geq 4$.*

Proof. From the defn (2), The Line graph of Dc^m is obtained by replacing all edges as vertices, we have $v_xv_{x+1} = s_x$, for $1 \leq x \leq m-1$. $u_xv_x = r_x$ for $1 \leq x \leq m$. $v_xw_x = t_x$ for $1 \leq x \leq m$. Here s_x, r_x, t_x are the vertices of $L(Dc^m)$. Hence, the vertex set and edge set of $L(Dc^m)$ is given by

$$V[L(Dc^m)] = \left(\bigcup_{x=1}^{m-1} s_x\right) \cup \left(\bigcup_{x=1}^{m} r_x\right) \cup \left(\bigcup_{x=1}^{m} t_x\right)$$

$$E[L(Dc^m)] = \left(\bigcup_{x=1}^{m-2} (s_xs_x+1)\right) \cup \left(\bigcup_{x=1}^{m-1} (s_xr_x \cup s_xr_x+1)\right) \cup \left(\bigcup_{x=1}^{m-1} (s_xt_x \cup s_xt_x+1)\right) \cup \left(\bigcup_{x=1}^{m} (t_xr_x)\right)$$

Therefore $|V(Dc^m))| = 3m - 1$ and $|E(L(Dc^m))| = 6(m - 1)$. Now we define $f : V[L(Dc^m)] \to 7$ given by for $m \geq 4$,

$$\text{For } 1 \leq x \leq m-1, \quad f(s_x) = \begin{cases} \{1\}, & \text{if } x \equiv 1 \ (mod\ 3) \\ \{2\}, & \text{if } x \equiv 2 \ (mod\ 3) \\ \{3\}, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases}$$

$$\text{For } 1 \leq x \leq m, \quad f(r_x) = \begin{cases} \{6\}, & \text{if } x \equiv 1 \ (mod\ 2) \\ \{4\}, & \text{if } x \equiv 0 \ (mod\ 2) \end{cases}$$

$$\text{For } 1 \leq x \leq m, \quad f(t_x) = \begin{cases} \{7\}, & \text{if } x \equiv 1 \ (mod\ 2) \\ \{5\}, & \text{if } x \equiv 0 \ (mod\ 2) \end{cases}$$

It is easy to verify that f is an open neighborhood 7-coloring of $L(Dc^m)$.

$$\therefore \chi_{onc}(L(Dc^m)) = 7, \quad m \geq 4.$$

Hence the theorem. □

3 Open neighborhood coloring of Middle graph of Path and cycle graph

In this section, An open neighborhood coloring of middle graph of path P_n and cycle graph C_n are discussed.

Theorem 3.1. *For middle graph of path graph $M(P_n)$, $\chi_{onc}(M(P_n)) = 5$, for $n \geq 4$*

Proof. The middle graph of P_n is obtained from the path graph P_n with the vertices $\{v_1, v_2, \cdots v_n\}$ and let take the edges of P_n as vertices of $M(P_n)$, we have $v_xv_{x+1} = e_x$, for $1 \leq x \leq n-1$.

$$V[M(P_n)] = \left\{\left(\bigcup_{x=1}^{n} v_x\right) \cup \left(\bigcup_{x=1}^{n-1} e_x\right)\right\}$$

$$n \geq 6, \quad \chi_{onc}(M(C_n)) = \begin{cases} \{5\}, & if\ n \equiv 0, 5\ (mod\ 6) \\ \{6\}, & otherwise \end{cases}$$

$$E[M(P_n)] = \left\{\left(\bigcup_{x=1}^{n-2} e_xe_{x+1}\right) \cup \left(\bigcup_{x=1}^{n-1} (v_xe_x)\right) \cup \left(\bigcup_{x=1}^{n-1} v_{x+1}e_x\right)\right\}$$

and

Now we define $f : V[M(P_n)] \to 5$ given by for $n \geq 4$,

$$\text{For } 1 \leq x \leq n-1, \quad f(e_x) = \begin{cases} \{1\}, & \text{if } x \equiv 1 \ (mod\ 3) \\ \{2\}, & \text{if } x \equiv 2 \ (mod\ 3) \\ \{3\}, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases}$$

$$\text{For } 1 \leq x \leq n, \quad n, \quad f(v_x) = \begin{cases} \{4\}, & \text{if } x \equiv 1\ (mod\ 2) \\ \{5\}, & \text{if } x \equiv 0\ (mod\ 2) \end{cases}$$

It is easy to verify that f is an open neighborhood 5-coloring of $M(P_n)$.

$$\therefore \chi_{onc}(M(P_n)) = 5, n \geq 4.$$

Hence the theorem. □

Theorem 3.2. *For middle graph of cycle graph $M(C_n)$, for*

Proof. The middle graph of C_n is obtained from C_n with the vertices $\{v_1, v_2, \cdots v_n\}$ and let take the edges of C_n as vertices of $M(C_n)$, we have $v_x v_{x+1} = e_x$, for $1 \leq x \leq n-1$ and $v_n v_1 = e_n$.

$$V[M(C_n)] = \left(\bigcup_{x=1}^{n} v_x \cup e_x \right)$$

and

$$E[M(C_n)] = \left(\bigcup_{x=1}^{n-1} e_x e_{x+1} \cup \bigcup_{x=1}^{n} v_x e_x \cup \bigcup_{x=1}^{n-1} v_{x+1} e_x \right) \cup (e_n e_1) \cup (v_1 e_n)$$

Then $|V(M(C_n))| = 2n$ and $|E(M(C_n))| = 3n$.

Define $f : V(M(C_n)) \to Z^+$ as follows.

Case-1. When $n \equiv 0 \ (mod\ 6)$.

$$For\ 1 \leq x \leq n, \quad f(e_x) = \begin{cases} 1, & \text{if } x \equiv 1 \ (mod\ 3) \\ 2, & \text{if } x \equiv 2 \ (mod\ 3) \\ 3, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases}$$

$$For\ \ 2 \leq x \leq n, \quad f(v_x) = \begin{cases} 4, & \text{if } x \equiv 0 \ (mod\ 2) \\ 5, & \text{if } x \equiv 1 \ (mod\ 2) \end{cases} \quad and \quad f(v_1) = 5.$$

$\therefore \chi_{onc}(M(C_n)) = 5$, *for* $n \equiv 0 \ (mod\ 6)$.

Case-2. When $n \equiv 1 \ (mod\ 6)$.

For $1 \leq x \leq n-4$,

$$f(e_x) = \begin{cases} 1, & \text{if } x \equiv 1 \ (mod\ 3) \\ 2, & \text{if } x \equiv 2 \ (mod\ 3) \\ 3, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases}$$

$f(e_{n-3}) = f(e_n) = 6, f(e_{n-2}) = 2, f(e_{n-1}) = 3$

For $2 \leq x \leq n-2$,

$$f(v_x) = \begin{cases} 5, & \text{if } x \equiv 1 \ (mod\ 2) \\ 4, & \text{if } x \equiv 0 \ (mod\ 2) \end{cases}$$

and $f(v_1) = 4, f(v_n) = 5, f(v_{n-1}) = 1$.

$\therefore \chi_{onc}(M(C_n)) = 6$, *for* $n \equiv 1 \ (mod\ 6)$.

Case-3. When $n \equiv 2 \ (mod\ 6)$.

For $1 \leq x \leq n-2$,

$$f(e_i) = \begin{cases} 1, & \text{if } x \equiv 1 \ (mod\ 3) \\ 2, & \text{if } x \equiv 2 \ (mod\ 3) \\ 3, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases}$$

$f(e_{n-1}) = 4, f(e_n) = 5$

For $2 \leq x \leq n-3$,

$$f(v_x) = \begin{cases} 4, & \text{if } x \equiv 0 \ (mod\ 2) \\ 5, & \text{if } x \equiv 1 \ (mod\ 2) \end{cases} ; \quad f(v_x) = \begin{cases} 1, & \text{if } x = n-1 \\ 2, & \text{if } x = n \\ 3, & \text{if } x = 1 \end{cases} \quad and \quad f(v_{n-2}) = 6.$$

$\therefore \chi_{onc}(M(C_n)) = 6,\ for\ n \equiv 2\ (mod\ 6).$

Case-4. When $n \equiv 3\ (mod\ 6)$.

For $1 \leq x \leq n$,

$$f(e_x) = \begin{cases} 1, & \text{if } x \equiv 1 \ (mod\ 3) \\ 2, & \text{if } x \equiv 2 \ (mod\ 3) \\ 3, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases}$$

For $2 \leq x \leq n$,

$$f(v_x) = \begin{cases} 4, & \text{if } x \equiv 0 \ (mod\ 2) \\ 5, & \text{if } x \equiv 1 \ (mod\ 2) \end{cases} \quad and\ f(v_1) = 6$$

$\therefore \chi_{onc}(M(C_n)) = 6,\ for\ n \equiv 3\ (mod\ 6).$

Case-5. When $n \equiv 4\ (mod\ 6)$.

For $1 \leq x \leq n-1$,

$$f(e_x) = \begin{cases} 1, & \text{if } x \equiv 1 \ (mod\ 3) \\ 2, & \text{if } x \equiv 2 \ (mod\ 3) \\ 3, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases} \quad and\ f(e_n) = 4$$

For $2 \leq x \leq n-2$,

$$f(v_x) = \begin{cases} 4, & \text{if } x \equiv 1 \ (mod\ 2) \\ 5, & \text{if } x \equiv 0 \ (mod\ 2) \end{cases} \quad and \quad f(v_1) = f(v_{n-1}) = 6, \quad f(v_n) = 5$$

$\therefore \chi_{onc}(M(C_n)) = 6,\ for\ n \equiv 4\ (mod\ 6).$

Case-6. When $n \equiv 5\ (mod\ 6)$.

For $1 \leq x \leq n-2$,

$$f(e_x) = \begin{cases} 1, & \text{if } x \equiv 1 \ (mod\ 3) \\ 2, & \text{if } x \equiv 2 \ (mod\ 3) \\ 3, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases} \quad and\ f(e_{n-1}) = 4,\ f(e_n) = 5$$

For $2 \leq x \leq n-2$,

$$f(v_x) = \begin{cases} 4, & \text{if } x \equiv 0 \ (mod\ 2) \\ 5, & \text{if } x \equiv 1 \ (mod\ 2) \end{cases} ; \quad f(v_i) = \begin{cases} 1, & \text{if } x = n-1 \\ 2, & \text{if } x = n \\ 3, & \text{if } x = 1 \end{cases}$$

$\therefore \chi_{onc}(M(C_n)) = 5,\ for\ n \equiv 5\ (mod\ 6).$

It is easy to verify that f is an open neighborhood coloring of $M(P_n)$.

$$\chi_{onc}(M(C_n)) = \begin{cases} \{5\}, & \text{if } n \equiv 0,5 \ (mod\ 6) \\ \{6\}, & \text{otherwise} \end{cases}$$

Hence the theorem. □

4 Open neighborhood coloring of Total graph of Path and cycle graph

In this section, An open neighborhood coloring of total graph of path P_n and cycle graph C_n are discussed.

Theorem 4.1. *For total graph of path graph $T(P_n)$, $\chi_{onc}(T(P_n)) = 5$, for $n \geq 4$*

Proof. The total graph of P_n is obtained from the path graph P_n with the vertices $\{v_1, v_2, \cdots v_n\}$ and let take the edges of P_n as vertices of $T(P_n)$, we have $v_x v_{x+1} = e_x$, for $1 \leq x \leq n-1$.

$$V[T(P_n)] = \left\{ \left(\bigcup_{x=1}^{n} v_x \right) \cup \left(\bigcup_{x=1}^{n-1} e_x \right) \right\}$$

and

$$E[T(P_n)] = \left\{ \left(\bigcup_{x=1}^{n-1} v_x v_{x+1} \right) \cup \left(\bigcup_{x=1}^{n-2} e_x e_{x+1} \right) \cup \left(\bigcup_{x=1}^{n-1} v_x e_x \right) \cup \left(\bigcup_{x=1}^{n-1} v_{x+1} e_x \right) \right\}$$

Now we define $f : V[T(P_n)] \to 5$ given by for $n \geq 4$,

$$\text{For } 1 \leq x \leq n-1, \quad f(e_x) = \begin{cases} \{1,2\}, & \text{if } x \equiv 3, 4 \ (mod\ 5) \\ \{3\}, & \text{if } x \equiv 0 \ (mod\ 5) \\ \{4,5\}, & \text{if } x \equiv 1, 2 \ (mod\ 5) \end{cases}$$

For $1 \leq x \leq n$ *and* $1 \leq y \leq 4$, $f(v_x) = y$, if $x \equiv y \ (mod\ 5)$, $f(v_{5x}) = 5$

It is easy to verify that f is an open neighborhood 5-coloring of $T(P_n)$.

$$\therefore \chi_{onc}(T(P_n)) = 5, \quad n \geq 4.$$

Hence the theorem. □

Theorem 4.2. *For total graph of cycle graph* $T(C_n)$ *for* $n > 5$, $\chi_{onc}(T(C_n)) = \begin{cases} \{6\}, & \text{if } n \equiv 0, 2 \ (mod\ 3) \\ \{7\}, & \text{otherwise} \end{cases}$

Proof. The total graph of C_n is obtained from C_n with the vertices $\{v_1, v_2, \cdots v_n\}$ and let take the edges of C_n as vertices of $T(C_n)$, we have $v_x v_{x+1} = e_x$, for $1 \leq x \leq n-1$ and $v_n v_1 = e_n$.

$$V[T(C_n)] = \left\{ \left(\bigcup_{x=1}^{n} v_i \cup e_x \right) \right\}$$

and

$$E[T(C_n)] = \left\{ \left(\bigcup_{x=1}^{n-1} e_x e_{x+1} \cup v_x v_{x+1} \right) \cup \left(\bigcup_{x=1}^{n} v_x e_x \right) \cup \left(\bigcup_{x=1}^{n-1} v_{x+1} e_x \right) \cup (e_n e_1) \cup (v_1 e_n) \cup (v_n v_1) \right\}$$

Then $|V(T(C_n))| = 2n$ and $|E(T(C_n))| = 4n$.

Define $f : V(T(C_n)) \to Z^+$ as follows.

Case-1. When $n \equiv 0 \ (mod\ 3)$.

$$\text{For } 1 \leq x \leq n, \quad f(e_x) = \begin{cases} 1, & \text{if } x \equiv 1 \ (mod\ 3) \\ 2, & \text{if } x \equiv 2 \ (mod\ 3) \\ 3, & \text{if } x \equiv 0 \ (mod\ 3) \end{cases}$$

$$\text{For } 2 \leq x \leq n, \quad f(v_x) = \begin{cases} 4, & \text{if } x \equiv 2 \ (mod\ 3) \\ 5, & \text{if } x \equiv 0 \ (mod\ 3) \\ 6, & \text{if } x \equiv 1 \ (mod\ 3) \end{cases} \text{ and } f(v_1) = 6.$$

$$\therefore \chi_{onc}(T(C_n)) = 6, \text{ for } n \equiv 0 \ (mod\ 3).$$

Case-2. When $n \equiv 1 \ (mod\ 3)$.

For $1 \leq x \leq n-4$,

1, if $x \equiv 1\ (mod\ 3)$ if x
□□□ $\equiv 2\ (mod\ 3)$ if $x \equiv 0$
$f(e_x) =$ 2, $(mod\ 3)$
□□□ 3,
$f(e_{n-3}) = 4, f(e_{n-2}) = 5, f(e_{n-1}) = 6, f(e_n) = f(v_{n-2}) = 7$

For $2 \leq x \leq n-3$,

$$f(v_x) = \begin{cases} 4, & \text{if } x \equiv 2\ (mod\ 3) \\ 5, & \text{if } x \equiv 0\ (mod\ 3) \\ 6, & \text{if } x \equiv 1\ (mod\ 3) \end{cases}$$

and $f(v_1) = 3, f(v_n) = 2, f(v_{n-1}) = 1$.

$\therefore \chi_{onc}(T(C_n)) = 7$, *for* $n \equiv 1\ (mod\ 3)$.

Case-3. When $n \equiv 2\ (mod\ 3)$.

For $1 \leq x \leq n-2$,

$$f(e_x) = \begin{cases} 1, & \text{if } x \equiv 1\ (mod\ 3) \\ 2, & \text{if } x \equiv 2\ (mod\ 3) \\ 3, & \text{if } x \equiv 0\ (mod\ 3) \end{cases}$$

$f(e_{n-1}) = 4, f(e_n) = 5$

For $2 \leq x \leq n-2$,

$$f(v_x) = \begin{cases} 4, & \text{if } x \equiv 2\ (mod\ 3) \\ 5, & \text{if } x \equiv 0\ (mod\ 3) \\ 6, & \text{if } x \equiv 1\ (mod\ 3) \end{cases}; \quad f(v_x) = \begin{cases} 1, & \text{if } x = n-1 \\ 2, & \text{if } x = n \\ 3, & \text{if } x = 1 \end{cases}$$

$\therefore \chi_{onc}(T(C_n)) = 6$, *for* $n \equiv 2\ (mod\ 3)$.

It is easy to verify that f is an open neighborhood coloring of $T(P_n)$.

$$\chi_{onc}(T(C_n)) = \begin{cases} \{6\}, & \text{if } n \equiv 0,2\ (mod\ 3) \\ \{7\}, & \text{otherwise} \end{cases}$$

Hence the theorem. □

Conclusion

In this paper, we have proved that the open neighborhood chromatic number of comb graph, double comb, line graph of comb, line graph of double comb graph. Also, we found the middle and total graph of path and cycle graph admits open neighborhood coloring conjecture.

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