4-Total Geometric Mean Cordial Labelling of Some Disconnected Graphs

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Abstract:	
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Let G be a (p, q) graph. Let $f: V(G) \square \{1, 2, 3, ..., k\}$ be a function where $k \square N$ and $k \square 1$. For each edge uv, assign the label $f(uv) = \lceil \sqrt{f(u)f(v)} \rceil$. f is called k-Total geometric mean cordial labeling of G if $\square t_{mf}(i) - t_{mf}(j) \square \le 1$, for all $i, j \square \{1, 2, 3, ..., k\}$ where $t_{mf}(x)$ denotes the total number of vertices and edges labeled with $x, x \square \{1, 2, 3, ..., k\}$. A graph that admits the ktotal geometric mean cordial labeling is called k-total geometric mean cordial graph.

In this paper we investigate 4- Total geometric mean cordiality of graphs like $P_n \square C_n$, $P_n \square K1$, $P_n \square Bn$, $P_n \square C_n$, P_n

 $\textbf{Keywords:} \ path, \ cycle, \ star, \ bistar, \ Y_n\text{-tree} \ and \ comb \ graph.$

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1. Introduction

Finite, simple and undirected graphs are considered here. Graph labeling we refer to Gallian [1]. We pursue [2] for symbols and phrases. Cordial labeling was introduced by Cahit [3]. Geometric mean cordial labeling of graphs was introduced in [4]. k- total mean cordial labeling of graphs was introduced in [5]. 4-total mean cordial labeling of some graphs derived from path and cycle was introduced in [6]. The notation of k-total geometric mean cordial labeling of some graphs was introduced in [7]. 4-total geometric mean cordial graphs was introduced in [8].

2. k-Total Geometric Mean Cordial Graph

Definition 2.1.

Let G be a (p, q) graph. Let $f: V(G) \ \Box \ \{1, 2, 3, ..., k\}$ be a function where $k \Box N$ and $k \Box 1$. For each edge uv, assign the label $f(uv) = \boxed{\sqrt{f(u)f(v)}}$. f is called k-Total geometric mean cordial labeling of G if \Box $t_{mf}(i) - t_{mf}(j) \ \Box \le 1$, for all i, $j \Box \{1, 2, 3, ..., k\}$ where $t_{mf}(x)$ denotes the total number of vertices and edges labeled with x, $x \Box \{1, 2, 3, ..., k\}$. A graph that admits the k-total geometric mean cordial labeling is called k-total geometric mean cordial graph.

3. Preliminaries

Definition 3.1.

The union of two graphs G_1 and G_2 is the graph $G_1 \square G_2$ with $V(G_1 \square G_2) = V(G_1) \square V(G_2)$ and $E(G_1 \square G_2) = E(G_1) \square E(G_2)$.

Definition 3.2.

A walk in which all the vertices say $u_1, u_2, \dots u_n$ are distinct is called a path. A path is denoted by P_n . The path P_n has n vertices and n-1 edges.

Definition 3.3.

A cycle is a closed walk in which all the vertices are distinct, except the last and the first.

Definition 3.4.

The graph obtained by joining a single pendant edge to each vertex of a path is called a Comb.

Definition 3.5.

The complete bipartite graph $K_{1,n}$ is called a star.

Definition 3.6.

The Bistar $B_{m,n}$ is the graph obtained by joining the two central vertices of $K_{1,m}$ and $K_{1,n}$.

Definition 3.7.

A Y_n - tree is obtained from a path P_n by attaching a pendant vertex to the $(n-1)^{th}$ vertex of P_n .

A tree on n+1 vertices is denoted by Y_n .

4. Main Results

Theorem 4.1.

The union of path P_n and cycle C_n , $P_n \square C_n$ is 4-total geometric mean cordial graph.

Proof:

Let $x_1, x_2, x_3, \dots, x_n$ be the vertices of the path P_n . Let $y_1, y_2, y_3, \dots, y_n y_1$ be the vertices of the cycle C_n .

Clearly
$$|V(P_n \cup C_n)| + |E(P_n \cup C_n)| = 4n - 1$$
.
The $f: V(P_n \cup C_n) \to \{1, 2, 3, 4\}$ function defined by

Case 1: n is odd

$$f(x_1) = f(x_2) = \dots = f\left(x_{\frac{n+1}{2}}\right) = 1;$$

$$f\left(x_{\frac{n+3}{2}}\right) = f\left(x_{\frac{n+5}{2}}\right) = f\left(x_{\frac{n+7}{2}}\right) = \dots = f(x_n) = 2;$$

$$f(y_1) = f(y_2) = f(y_3) = \dots = f\left(y_{\frac{n+1}{2}}\right) = 3;$$

$$f\left(y_{\frac{n+3}{2}}\right) = f\left(y_{\frac{n+5}{2}}\right) = f\left(y_{\frac{n+7}{2}}\right) = \dots = f(y_n) = 4.$$

Case 2: n is even

$$\begin{split} f(x_1) &= f(x_2) = f(\ x_3) = \dots = f\left(x_{\frac{n}{2}}\right) = 1; \\ f\left(x_{\frac{n+2}{2}}\right) &= f\left(x_{\frac{n+4}{2}}\right) = f\left(x_{\frac{n+6}{2}}\right) = \dots = f(x_n) = 3; \\ f(y_1) &= f(y_2) = f(y_3) = \dots = f\left(y_{\frac{n}{2}}\right) = 4; \\ f\left(y_{\frac{n+2}{2}}\right) &= f\left(y_{\frac{n+4}{2}}\right) = f\left(y_{\frac{n+6}{2}}\right) = \dots = f(y_{n-1}) = 2; \\ f(y_n) &= 1. \end{split}$$

Hence the function f is 4-total geometric mean cordial labeling and $P_n \square C_n$ is 4-total geometric mean cordial graph.

Theorem 4.2.

The graph $P_n \square K_{1,n}$ is 4-total geometric mean cordial graph.

Proof:

Let $x_1 x_2 x_3 \dots x_n$ be the path P_n . Let y be the centre vertex and $y_1, y_2, y_3, \dots, y_n$ be the pendant vertices of the star $K_{1,n}$.

$$|V(P_n \cup K_{1,n})| + |E(P_n \cup K_{1,n})| = 4n$$
 Clearly.

Define
$$f: V(P_n \cup K_{1,n}) \rightarrow \{1,2,3,4\}$$
 by

Case 1: n is odd

$$f(x_1) = f(x_2) = \dots = f\left(x_{\frac{n+1}{2}}\right) = 1;$$

$$f\left(x_{\frac{n+3}{2}}\right) = f\left(x_{\frac{n+5}{2}}\right) = f\left(x_{\frac{n+7}{2}}\right) = \dots = f(x_n) = 2;$$

$$f(y) = 2;$$

$$f(y_1) = f(y_2) = f(y_3) = \dots = f(y_n) = 4.$$

Case 2: n is even

$$\begin{split} f(x_1) &= f(x_2) = f(x_3) = \dots = f\left(x_{\frac{n}{2}}\right) = 3; \\ f\left(x_{\frac{n+2}{2}}\right) &= f\left(x_{\frac{n+4}{2}}\right) = f\left(x_{\frac{n+6}{2}}\right) = \dots = f(x_n) = 4; \\ f(y) &= 3; \\ f(y_1) &= f(y_2) = f(y_3) = \dots = f(y_n) = 1. \end{split}$$

Hence f is 4-total geometric mean cordial labeling and $P_n \square K_{1,n}$ is 4-total geometric mean cordial graph .

Theorem 4.3.

The graph $P_n \square B_{n,n}$ is 4-total geometric mean cordial graph. Proof:

$$x_1 \ x_2 \ x_3 \ \dots \ x_n$$
 be the path. Let $y_1, y_2, y_3, \dots, y_n$, $z_1, z_2, z_3, \dots, z_n$ Let be the y, z be the centre vertices of $B_{n,n}$.

$$|V(P_n \cup B_{n,n})| + |E(P_n \cup B_{n,n})| = 6n + 2.$$

$$f: V(P_n \cup B_{n,n}) \rightarrow \{1, 2, 3, 4\}$$
 by

Define

Case 1: n is odd

$$f(x_1) = f(x_2) = f(x_3) = \dots = f\left(x_{\frac{n+1}{2}}\right) = 3;$$

$$f\left(x_{\frac{n+s}{2}}\right) = f\left(x_{\frac{n+s}{2}}\right) = f\left(x_{\frac{n+7}{2}}\right) = \dots = f(x_n) = 4;$$

$$f(y) = 3;$$

$$f(z) = 4;$$

$$f(y_1) = f(y_2) = f(y_3) = \dots = f(y_n) = 1;$$

$$f(z_1) = f(z_2) = f(z_3) = \dots = f\left(z_{\frac{n+1}{2}}\right) = 1;$$

$$f\left(z_{\frac{n+s}{2}}\right) = f\left(z_{\frac{n+s}{2}}\right) = f\left(z_{\frac{n+7}{2}}\right) = \dots = f(z_n) = 3.$$
Case 2: n is even
$$f(x_1) = f(x_2) = f(x_3) = \dots = f\left(x_{\frac{n+2}{2}}\right) = 3;$$

$$f\left(x_{\frac{n+4}{2}}\right) = f\left(x_{\frac{n+6}{2}}\right) = \dots = f(x_n) = 4;$$

$$f(y) = f(z) = 4;$$

$$f(y_1) = f(y_2) = f(y_3) = \dots = f(y_n) = 1;$$

$$f(z_1) = f(z_2) = f(z_3) = \dots = f\left(z_{\frac{n}{2}}\right) = 1;$$

$$f\left(z_{\frac{n+2}{2}}\right) = f\left(z_{\frac{n+4}{2}}\right) = \dots = f(z_n) = 3.$$

Hence f is 4-total geometric mean cordial labeling and $P_n \square B_{n,n}$ is 4-total geometric mean cordial graph.

Theorem 4.4.

The graph $C_n \square \ C_n$ is a 4-total geometric mean cordial graph.

Proof:

$$x_1, x_2, x_3, \dots, x_n$$
 and $y_1, y_2, y_3, \dots, y_n$ Let be the vertices of two copies of cycle C_n . $|V(C_n \cup C_n)| + |E(C_n \cup C_n)| = 4n$. Clearly

Define $f: V(C_n \cup C_n) \rightarrow \{1, 2, 3, 4\}$ by

Case

1: n is odd

$$f(x_1) = f(x_2) = \dots = f\left(x_{\frac{n+1}{2}}\right) = 1;$$

$$f\left(x_{\frac{n+3}{2}}\right) = f\left(x_{\frac{n+5}{2}}\right) = \dots = f(x_n) = 2;$$

$$f(y_1) = f(y_2) = \dots = f\left(y_{\frac{n+1}{2}}\right) = 3;$$

$$f\left(y_{\frac{n+3}{2}}\right) = f\left(y_{\frac{n+5}{2}}\right) = \dots = f(y_n) = 4.$$

Case 2: n is even

Subcase (i):
$$n = 4$$

$$f(x_1) = f(x_2) = f(y_1) = 1;$$

$$f(x_3) = f(x_4) = f(y_2) = 3;$$

$$f(y_3) = f(y_4) = 4.$$

Subcase (ii): $n \ge 6$

$$f(x_{1}) = f(x_{2}) = f(x_{3}) = \dots = f\left(x_{\frac{n}{2}}\right) = 1;$$

$$f(x_{n-1}) = 1;$$

$$f(x_{n}) = 3;$$

$$f\left(x_{\frac{n+2}{2}}\right) = f\left(x_{\frac{n+4}{2}}\right) = f\left(x_{\frac{n+6}{2}}\right) = \dots = f(x_{n-2}) = 2;$$

$$f(y_{1}) = f(y_{2}) = f(y_{3}) = \dots = f\left(y_{\frac{n}{2}}\right) = 4;$$

$$f\left(y_{\frac{n+2}{2}}\right) = f\left(y_{\frac{n+4}{2}}\right) = f\left(y_{\frac{n+6}{2}}\right) = \dots = f(y_{n-1}) = 3;$$

$$f(y_{n}) = 2.$$

Hence f is 4-total geometric mean cordial labeling and $C_n \square C_n$ is a 4-total geometric mean cordial graph.

Theorem4.5.

The graph $C_n \square K_{1,n}$ is a 4-total geometric mean cordial graph .

Proof:

$$x_1 x_2 x_3 \dots x_n x_1$$
 Let be the cycle C_n. Let y be the

 $y_1, y_2, y_3, ..., y_n$ be the pendant vertices of the star $K_{1,n}$. centre vertex and

Clearly
$$|V(C_n \cup K_{1,n})| + |E(C_n \cup K_{1,n})| = 4n + 1$$

Define
$$f: V(C_n \cup K_{1,n}) \rightarrow \{1,2,3,4\}$$
 by

$$f(y) = 3;$$

$$f(y_1) = f(y_2) = f(y_3) = \dots = f(y_n) = 1.$$

Case 1: n is odd

$$f(x_1) = f(x_2) = f(x_3) = \dots = f\left(x_{\frac{n+1}{2}}\right) = 3;$$

$$f\left(x_{\frac{n+3}{2}}\right) = f\left(x_{\frac{n+5}{2}}\right) = f\left(x_{\frac{n+7}{2}}\right) = \dots = f(x_n) = 4.$$

Case 2: n is even

$$f(x_1) = f(x_2) = f(x_3) = \dots = f\left(x_{\frac{n}{2}}\right) = 3;$$

 $f\left(x_{\frac{n+2}{2}}\right) = f\left(x_{\frac{n+4}{2}}\right) = f\left(x_{\frac{n+6}{2}}\right) = \dots = f(x_n) = 4.$

Hence f is 4-total geometric mean cordial labeling and $C_n \square K_{1,n}$ is a 4-total geometric mean cordial graph.

Theorem 4.6.

The graph $C_n \square B_{n,n}$ is 4-total geometric mean cordial graph .

Proof:

$$x_1 \ x_2 \ x_3 \ ... \ ... \ x_n x_1$$
 be the cycle C $_n$. Let $y_1, y_2, y_3, ..., y_n$, $z_1, z_2, z_3, ..., z_n$ pendant vertices and y, z be the centre vertices of $B_{n,n}$.

Let $|V(C_n \cup B_{n,n})| + |E(C_n \cup B_{n,n})| = 6n + 3$ be the cycle C $_n$. Let $(C_n \cup B_{n,n}) \to \{1, 2, 3, 4\}$ by

Clearly

Define

Case 1: n is odd

$$\begin{split} f(x_1) &= f(x_2) = f(x_3) = \dots = f\left(x_{\frac{n+1}{2}}\right) = 3; \\ f\left(x_{\frac{n+3}{2}}\right) &= f\left(x_{\frac{n+5}{2}}\right) = f\left(x_{\frac{n+7}{2}}\right) = \dots = f(x_n) = 4; \\ f(y) &= 3; \\ f(z) &= 4; \\ f(y_1) &= f(y_2) = f(y_3) = \dots = f(y_n) = 1; \\ f(z_1) &= f(z_2) = f(z_3) = \dots = f\left(z_{\frac{n+1}{2}}\right) = 1; \\ f\left(z_{\frac{n+3}{2}}\right) &= f\left(z_{\frac{n+5}{2}}\right) = f\left(z_{\frac{n+7}{2}}\right) = \dots = f(z_n) = 3. \end{split}$$

Case 2: n is even

$$\begin{split} f(x_1) &= f(x_2) = f(x_3) = \dots = f\left(x_{\frac{n+2}{2}}\right) = 3; \\ f\left(x_{\frac{n+4}{2}}\right) &= f\left(x_{\frac{n+6}{2}}\right) = f\left(x_{\frac{n+8}{2}}\right) = \dots = f(x_n) = 4; \\ f(y) &= 3; \\ f(z) &= 4; \\ f(y_1) &= f(y_2) = f(y_3) = \dots = f(y_n) = 1; \\ f(z_1) &= f(z_2) = f(z_3) = \dots = f\left(z_{\frac{n+2}{2}}\right) = 1; \\ f\left(z_{\frac{n+4}{2}}\right) &= f\left(z_{\frac{n+6}{2}}\right) = \dots = f(z_n) = 3. \end{split}$$

Hence f is 4-total geometric mean cordial labeling.

Theorem 4.7.

The graph $P_n \square Y_n$ is 4-total geometric mean cordial graph.

Proof:

Let $x_1 x_2 x_3 \dots x_n$ be the path P_n . Let $y_1, y_2, y_3, \dots, y_n$ be the vertices of the path in the Y graph and z_1 be the pendant vertex adjacent to y_2

$$|V(P_n \cup Y_n)| + |E(P_n \cup Y_n)| = 4n$$
 Clearly .

Define $f: V(P_n \cup Y_n) \rightarrow \{1, 2, 3, 4\}$ by

Case 1: n is odd

$$f(x_1) = f(x_2) = f(x_3) = \dots = f\left(x_{\frac{n-1}{2}}\right) = 3;$$

$$\begin{split} f\left(x_{\frac{n+1}{2}}\right) &= f\left(x_{\frac{n+3}{2}}\right) = f\left(x_{\frac{n+5}{2}}\right) = \dots = f(x_{n-1}) = 4; \\ f(x_n) &= 3; \\ f(y_1) &= f(y_2) = f(y_3) = \dots = f\left(y_{\frac{n+1}{2}}\right) = 1; \\ f\left(y_{\frac{n+3}{2}}\right) &= f\left(y_{\frac{n+5}{2}}\right) = f\left(y_{\frac{n+7}{2}}\right) = \dots = f(y_n) = 2; \\ f(z_1) &= 3. \end{split}$$

Case 2: n is even

$$\begin{split} f(x_1) &= 1; \\ f(x_2) &= f(x_3) = \dots = f\left(x_{\frac{n+2}{2}}\right) = 4; \\ f\left(x_{\frac{n+4}{2}}\right) &= f\left(x_{\frac{n+6}{2}}\right) = \dots = f(x_n) = 3; \\ f\left(y_{\frac{n+2}{2}}\right) &= f\left(y_{\frac{n+4}{2}}\right) = \dots = f(y_{n-1}) = 2; \\ f(y_n) &= 3; \\ f(z_1) &= 3. \end{split}$$

Hence f is 4-total geometric mean cordial labeling and $P_n \square Y_n$ is 4-total geometric mean cordial graph.

Theorem 4.8.

 $P_n \square P_n \odot K_1$ is 4-total geometric mean cordial graph for $n \ge 2$.

Proof:

$$u_{1}, u_{2}, u_{3}, ..., u_{n}$$

$$y_{1}, y_{2}, y_{3}, ..., y_{n}$$

$$f: V(P_{n} \cup P_{n} \odot K_{1}) \rightarrow \{1, 2, 3, 4\}$$

Let be the vertices of the path. Let $x_1, x_2, x_3, \dots, x_n$ be the vertices of the path and vertices of the comb graph.

Define

by

Case1: n is odd

$$\begin{split} f(u_1) &= f(u_2) = \dots = f\left(u_{\frac{n+1}{2}}\right) = 1; \\ f\left(u_{\frac{n+s}{2}}\right) &= f\left(u_{\frac{n+s}{2}}\right) = \dots = f(u_n) = 3; \\ f(x_1) &= f(x_2) = \dots = f\left(x_{\frac{n-1}{2}}\right) = 2; \\ f\left(x_{\frac{n+1}{2}}\right) &= f\left(x_{\frac{n+s}{2}}\right) = \dots = f(x_n) = 4; \\ f(y_1) &= f(y_2) = \dots = f\left(y_{\frac{n-1}{2}}\right) = 1; \\ f\left(y_{\frac{n+1}{2}}\right) &= f\left(y_{\frac{n+s}{2}}\right) = \dots = f(y_{n-1}) = 3; \\ f(y_n) &= 2. \end{split}$$

Case2: n is even

$$f(u_1) = f(u_2) = f(u_3) = \dots = f\left(u_{\frac{n}{2}}\right) = 1;$$

$$f\left(u_{\frac{n+2}{2}}\right) = f\left(u_{\frac{n+4}{2}}\right) = \dots = f(u_n) = 3;$$

$$f(x_1) = f(x_2) = f(x_3) = \dots = f\left(x_{\frac{n}{2}}\right) = 2;$$

$$f\left(x_{\frac{n+2}{2}}\right) = f\left(x_{\frac{n+4}{2}}\right) = \dots = f(x_n) = 4;$$

$$f(y_1) = f(y_2) = f(y_3) = \dots = f\left(y_{\frac{n}{2}}\right) = 1;$$

$$f\left(y_{\frac{n+2}{2}}\right) = f\left(y_{\frac{n+4}{2}}\right) = \dots = f(y_n) = 3.$$

Hence f is 4- total geometric mean cordial labeling.

Theorem 4.9.

 $C_n \square P_n \Theta K_1$ is 4-total geometric mean cordial graph.

Proof:

$$u_1, u_2, u_3, ..., u_n u_1$$

 $y_1, y_2, y_3, ..., y_n$
 $f : V(C_n \cup P_n \odot K_1) \rightarrow \{1, 2, 3, 4\}$

Let be the vertices of the cycle. Let $x_1, x_2, x_3, \dots, x_n$ be the vertices of the path and be the pendant vertices of the comb graph.

Define by

Case 1: n is odd

$$\begin{split} f(u_1) &= f(u_2) = \dots = f\left(u_{\frac{n+1}{2}}\right) = 1; \\ f\left(u_{\frac{n+5}{2}}\right) &= f\left(u_{\frac{n+5}{2}}\right) = \dots = f(u_n) = 4; \\ f(x_1) &= f(x_2) = \dots = f\left(x_{\frac{n+1}{2}}\right) = 3; \\ f\left(x_{\frac{n+5}{2}}\right) &= f\left(x_{\frac{n+5}{2}}\right) = \dots = f(x_n) = 2; \\ f(y_1) &= f(y_2) = \dots = f\left(y_{\frac{n-1}{2}}\right) = 1; \\ f\left(y_{\frac{n+1}{2}}\right) &= f\left(y_{\frac{n+5}{2}}\right) = \dots = f(y_n) = 4. \end{split}$$

Case 2: n is even

$$\begin{split} f(u_1) &= f(u_2) = \ldots \ldots = f\left(u_{\frac{n+2}{2}}\right) = 1; \\ f\left(u_{\frac{n+4}{2}}\right) &= f\left(y_{\frac{n+6}{2}}\right) = \ldots \ldots = f(u_n) = 4; \\ f(x_1) &= f(x_2) = \ldots \ldots = f\left(x_{\frac{n}{2}}\right) = 3; \\ f\left(x_{\frac{n+2}{2}}\right) &= f\left(x_{\frac{n+4}{2}}\right) = \ldots \ldots = f(x_n) = 2; \\ f(y_1) &= f(y_2) = \ldots \ldots = f\left(y_{\frac{n-2}{2}}\right) = 1; \\ f\left(y_{\frac{n}{2}}\right) &= f\left(y_{\frac{n+2}{2}}\right) = \ldots \ldots = f(y_n) = 4. \end{split}$$

Hence f is 4- total geometric mean cordial labeling and $C_n \square P_n O K_1$ is 4-total geometric mean cordial graph.

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