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Total Chromatic Number of Comb Product of Arrow Graph with Certain Graphs

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Abstract

The total chromatic number of a graph G is defined to be the minimum number of colors needed to color the vertices and edges of a graph in such a way that no two adjacent vertices, no two adjacent edges and no edge and its end its end vertices given the same color. In this paper, we have obtained the total coloring and total chromatic number of comb product of arrow graph with path, star, fan and cycle.

Keywords: Arrow graph, path, star, cycle, fan, comb product, Total colouring.

INTRODUCTION

Chromatic number of graphs is a special area in a Graph theory. Bezhad [1] introduced the concept of total coloring and found the chromatic number of some simple graphs.

Total coloring of a graph G is coloring all the vertices and the edges of G simultaneously in such a way that no two adjacent vertices have the same color, no two adjacent edges have the same color, and that each edge and end vertices does not receive the same color. The total chromatic number of a graph G is the minimum number of colors that required to produce a total coloring and is denoted by $\chi_{tc}(G)$.

Bezhad [2] conjectured that for any graph of maximum degree $\Delta(G)$ has a total chromatic number satisfying the condition

$\Delta(G) + 1 \leq \chi_{tc}(G) \leq \Delta(G) + 2$.

S. Sudha , K.Manikanda [4] have discussed the total coloring and total chromatic number of the central graph of a path, a cycle and a star. Muthuramakrishnan and G. Jayaraman [5] have discussed the total coloring of middle graph, total graph of path and sunlet graph. Also,they have obtained the total chromatic number of those graph.

We found the total chromatic number of the following graphs in [6]: Middle graph of Extended duplicate graph of path graph, Total graph of Extended duplicate graph of path graph, Middle graph of Extended duplicate graph of star, Total graph of extended duplicate of star, Central graph of Extended duplicate graph of star graph.

In this paper, we investigated about total chromatic number of comb product of arrow graph with the path graph, star graph, fan graph, and cycle graph.

DEFINITIONS:

2.1 PATH

A path in a graph is a finite or infinite sequence of edges which joins a sequence of n vertices which are all distinct and is denoted by P_n.

2.2 CYCLE

A cycle graph or circular graph is a path P_{n+1} (n>3, if the graph is simple) whose end vertices are joined to form a closed chain. The cycle graph with n vertices is denoted by C_n.

2.3 STAR GRAPH

A Star graph is a complete bipartite graph $K_{1,n}$ for $n \geq 1$.

2.4 FAN GRAPH

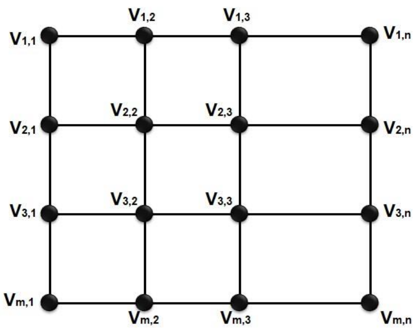
Let $P_n = v_1 \cdots v_n$ be the path of order n. The fan graph $F_n = K_1 \vee P_n$ is the join of and P_n.

2.5 COMB GRAPH

A comb graph is a graph obtained by joining a single pendant edge to each vertex of a path.

2.6 SUPERIOR VERTICES

Let Graph, $P_m \times P_n$ (grid graph on m n vertices) vertices $v_{1,1}, v_{2,1}, v_{3,1}, \ldots, v_{m,1}$ and vertices $v_{1,n}, v_{2,n}, v_{3,n}, \ldots, v_{m,n}$ are called as superior vertices.



2.7 ARROW GRAPH

An arrow graph $A^t n$ with width t and length n is obtained by joining a vertex u with superior vertices of $P_m \times P_n$ by m new edges from one end. Let G= $A^2 n$ be an arrow graph obtained by joining a vertex u with superior vertices of $P_2 \times P_n$ by 2 new edges.

Let G= $A^3 n$ be an arrow graph obtained by joining a vertex u with superior vertices of

$P_3 \times P_n$ by 3 new edges. [7]

2.8 COMB PRODUCT:

Let G and H be two connected graphs and o be a vertex of H. The comb product between G and H denoted by G $\rhd$ H, is a graph obtained by taking one copy of G and |V (G)| copies of H and grafting the vertex o of i-th copy of H with the i-th vertex of G. [3].

3. MAIN RESULTS:

Theorem: 3.1. The total chromatic number of Comb product of Arrow graph with

path graph is $\chi_{tc}(A_n^2 \rhd P_n)$=6 for $n \geq 2$.

Proof:

Let G be an arrow graph(A_n^2) and H be path graphP_n.

Let $V(A_n^2) = \{x\} \cup \{u_i : 1 \leq i \leq n\} \cup \{v_j : 1 \leq j \leq n\}$

$$E(A_n^2) = \{xu_1\} \cup \{xv_1\} \cup \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{v_j v_{j+1} : 1 \leq j \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$$

and

$V(P_n) = \{w_i : 1 \le i \le n\}$

$E(P_n) = \{w_i w_{i+1} : 1 \le i \le n-1\}$

By definition of comb product of arrow graph with path graph the vertex w_1 of path P_n is identified with each vertex of the arrowgraph A_n^2 the vertex set and the edge set of $(A_n^2 \rhd P_n)$ as follows:

$$V(A_n^2 \rhd P_n) = \{x\} \cup \{u_i : 1 \le i \le n\} \cup \{v_j : 1 \le j \le n\} \cup$$
$$\{x_{ik} : 1 \le i \le n, 1 \le k \le n-1\} \cup$$
$$\{y_{jk} : 1 \le j \le n, 1 \le k \le n-1\} \cup$$
$$\{z_i : 1 \le i \le n-1\}$$

$$E(An^2 \rhd P_n) = \{xu_1\} \cup \{xv_1\} \cup \{xz_1\} \cup$$
$$\{u_i u_{i+1} : 1 \le i \le n-1\} \cup$$
$$\{v_j v_{j+1} : 1 \le j \le n-1\} \cup$$
$$\{u_i v_i : 1 \le i \le n\} \cup \{u_i x_{i1} : 1 \le i \le n\} \cup$$
$$\{v_j y_{j1} : 1 \le j \le n\} \cup \{z_i z_{i+1} : 1 \le i \le n-2\} \cup$$
$$\{x_{ik} x_{ik+1} : 1 \le i \le n, 1 \le k \le n-2\} \cup$$
$$\{y_{jk} y_{jk+1} :: 1 \le j \le n, 1 \le k \le n-2\}$$

The number of Vertices and Edges of $(A_n^2 \rhd P_n)$ is $2n^2 + \mathrm{n}$ and $2n^2 + 2n - 1$

respectively.

Let $S = V(A_n^2 \rhd P_n) \cup E(A_n^2 \rhd P_n)$ and $C = \{1,2,3,4,5,6\}$

Now we define total coloring f: S $\rightarrow$ C by

$$f(x) = 6$$

1 $f(u\ ) = \{$ if i is odd or $1 \le i \le n$,
i 2 if i is e en

is odd $f(v_j) = \begin{cases} 2 & \text{if } j \\ 1 & \text{if } j \end{cases}$ or $1 \le j \le n$
is e en

if i is odd or $1 \le i \le n$ $f(z_i) = \begin{cases} 1 \\ 2 & \text{if } i \text{ is e en} \end{cases}$

For $1 \le i \le n, 1 \le k \le n-1$

ik $\{$ 1 if i is e en, is odd and if i is odd, is e en
$f(x\ \ ) =$
2 if i is e en, is e en and if i is odd, is odd

For $1 \le j \le n, 1 \le k \le n-1$

$f(y_{jk}) = \{$ 1 if j is odd , is odd and if j is e en, is e en
2 if j is odd, is e en and if j is e en, is odd

$$f(xu_1) = 5$$
$$f(xv_1) = 4$$
$$f(xz_1) = 3$$

For $1 \le i \le n-1$ $f(u\ u\ \ \ ) = \{4$ if i is odd
$i\ \ i+1$ 5 if i is e en

$\{$ For $1 \le\ \ \le n\ \ \ 1$ $f(v\ v\ \ \ ) =$ 5 if j is odd
j – $j\ \ j+1$ 4 if j is e en

$For 1 \le i \le n$ $f(u_i v_i) = 6$
$For\ 1 \le i \le n$ $f(u_i x_{i1}) = 3$
$_j y_{j1}$ $For\ \ 1 \le j \le n$ $f(v\ \ \ \ ) = 3$

$For 1 \le i \le n-2$ $f(z\ z\ \ \ ) = \{4$ if i is odd
$i\ \ i+1$ 3 if is e en

For $1 \le i \le n, 1 \le k \le n-2$

$f(x\ \ x\ \ \ )$ $\{3$ $ik\ \ ik_{+1}4$ ii if i is odd , is e en and if i is e en, is e en
f is odd, is odd and if i is e en, is odd

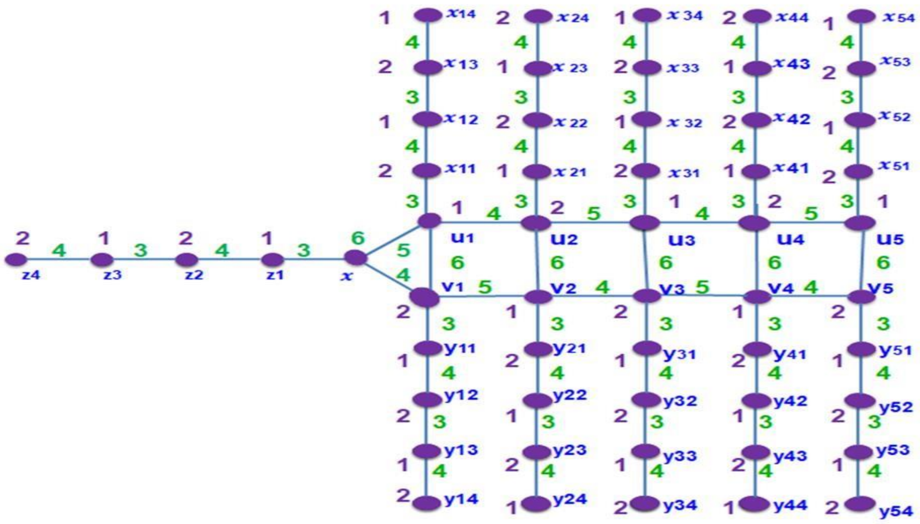
$For\ 1 \leq j \leq n, 1 \leq k \leq n-2$

$f(y_{jk}y_{jk}\ \ ) \begin{cases} 3 \\ \ _{+1} \quad 4ij \end{cases}$ if j is odd , is e en and if j is e en, is e en
f is odd, is odd and if j is e en, is odd

The above defined function f gives the total coloring for the graph

Hence $\chi_{tc}(A_n^2 \rhd P_n)$ =6 for $n \geq 2$.

Illustration 1: Consider the Arrow graph with path graph $A_5^2 \rhd P_5$ Figure 1



$\chi_{tc}(A_5^2 \rhd P_5)$=6

Theorem 3.2.

The total chromatic number of comb product of Arrow graph with Star graph is $\chi_{tc}(A_n^2 \rhd S_n)$=n+5 for $n \geq 2$.

Proof

Let G be an arrow graph(A_n^2) and H be a star graphS_n.

Let V$(A_n^2) = \{x\} \cup \{u_i: 1 \leq i \leq n\} \cup \{v_j: 1 \leq j \leq n\}$

$E(A_n^2) = \{xu_1\} \cup \{xv_1\} \cup \{u_iu_{i+1}: 1 \leq i \leq n-1\} \cup \{v_jv_{j+1}: 1 \leq j \leq n-1\} \cup \{u_iv_i: 1 \leq i \leq n\}$

and

V (S_n)={u} $\cup \{w_i: 1 \leq i \leq n\}$

$E(S_n) = \{uw_i: 1 \leq i \leq n\}$

By definition of comb product of arrow graph with star graph the vertex u, of star S_n is identified with each vertex of the arrow graph $A_n^2$the vertex set and the edge set of$(A_n^2 \rhd S_n)$ as follows:

$V(A_n^2 \rhd S_n) = \{x\} \cup \{u_i : 1 \le i \le n\} \cup$ for
$\{v_j : 1 \le j \le n\} \cup$
$\left\{x_{ik} : \begin{matrix} 1 \le i \le n \\ 1 \le k \le n \end{matrix}\right\} \cup$ $\{z_i : 1 \le$
$\left\{y_{jk} : \begin{matrix} 1 \le j \le n \\ \text{for } 1 \le k \le n \end{matrix}\right\} \cup$ $i \le n$ $E(A_n^2 \rhd S_n) = \{xu_1\}$
$\cup \{x\, v$ $\{u_i u_{i+1}\}$
$\{v_j v_{j+1}\ 1\} \cup$
$: \cup$ $\{u_i v_i : 1$ $1 \le i \le n-1\}$
$\cup$ $:$ $\{u_i x_{ik} :$ $1 \le j \le n-1\}$
$\le$ i $\le n\} \cup$
1 $\left\{v_j y_{jk} : \begin{matrix} \le i \le n \\ \le k \le n \end{matrix}\right\} \cup$
1 for
1 $\begin{matrix} \le j \le n \\ \le k \le n \end{matrix}\} \cup \{xz$
$_i ; 1 \le i \le n\}$ 1 for

for for

The number of Vertices and Edges of $(A_n^2 \rhd S_n)$ is $2n^2 + 3n + 1$ and $2n^2 + 4n$ respectively.

Let $S = V(A_n^2 \rhd S_n) \cup E(A_n^2 \rhd S_n)$ *and* $C =$ n+

Now we define total coloring $f : S \to C$ *by*

$f(x) = n + 1$

$For \le i \le n$ $f(u_i) = \begin{cases} n+3 \\ n+4 \end{cases}$ if i is odd 1 i if is e en

$For 1 \le j \le n$ $f(v_j) = \begin{cases} n+4 & \text{if } j \\ n+3 & \text{i } j \end{cases}$ is odd

is e en

or $1 \le i \le n, 1 \le k \le n$ $f(x_{ik}) = \mathrm{n} + 5$

$1 \le j \le n, 1 \le k \le n$ $f(y_{jk}) = \mathrm{n} + 5$ or

For $1 \le i \le n$ $f(z_i) = \mathrm{n} + 5$

$f(\mathrm{x}u_1) = n + 2$
$f(\mathrm{x}v_1) = n + 3$

$For 1 \le i \le n$ $f(u_i u_{i+1}) = \begin{cases} n+1 \\ n+2 \end{cases}$ if i is odd if i is e en if j is odd

1

if is e en

$For \le j \le n$ $f(v\ v\ \ ) = \begin{cases} n+1 \end{cases}$ $_j\ _{j+1}$ $n + 2_j$

$For \ \leq i \leq n, \ \leq k \leq nf(u_i x_{ik}) = \text{k}$, 1 1

$For \ \leq j \leq n, \ \leq k \leq n \ f\left(v_j y_{jk}\right) = \text{k}$

11

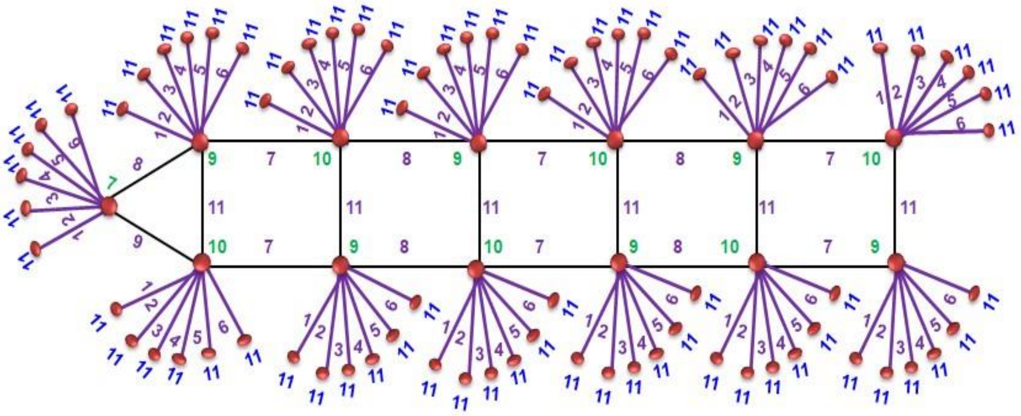
For $1 \leq i \leq n, f(xz_i) = \text{i}$

For $1 \leq i \leq n, f(u_i v_i) = \text{n} + 5$

The above defined function f gives the total coloring for the graph

Hence $\chi_{tc}(A_n^2 \rhd S_n) = n + 5$ for $n \geq 2$.

Illustration 2: Consider the Arrow graph with star graph $A_6^2 \rhd S_6$ Figure 2.



$\chi_{tc}(A_6^2 \rhd S_6)$=11

Theorem: 3.3

The total chromatic number of Comb Product of Arrow graph with fan graph

is $\chi_{tc}(A_n^2 \rhd F_n)$=n+5 for $n \geq 2$.

Proof:

Let G be an arrow graph (A_n^2) and H be a fan graph F_n.

Let V$(A_n^2) = \{ x \} \cup \{u_i: 1 \leq i \leq n\} \cup \{v_j: 1 \leq j \leq n\}$

$$E(A_n^2) = \{xu_1\} \cup \{xv_1\} \cup \{u_i u_{i+1}: 1 \leq i \leq n-1\} \cup$$

$$\{v_j v_{j+1}: 1 \leq j \leq n-1\} \cup \{u_i v_i: 1 \leq i \leq n\}$$

and

V (F_n)={u} $\cup \{w_i: 1 \leq i \leq n\}$

$$E(F_n) = \{uw_i: 1 \leq i \leq n\} \cup \{w_i w_{i+1}: 1 \leq i \leq n-1\}$$

By definition of comb product of arrow graph with fan graph

The vertex u, of star F_n is identified with each vertex of the arrow graph A_n^2

the vertex set and the edge set of $(A_n^2 \rhd F_n)$ as follows

$$V(A_n^2 \rhd F_n) = \{x\} \cup \{u_i: 1 \le i \le n\} \cup \{v_j: 1 \le j \le n\} \cup \left\{x_{ik}: \begin{matrix} 1 \le i \le n \\ 1 \le k \le n \end{matrix}\right\} \cup \left\{y_{jk}: \begin{matrix} 1 \le j \le n \\ 1 \le k \le n \end{matrix}\right\} \cup$$

$\{z_i: 1 \le$ $n\}.$ $i \le$

$E(A_n^2 \rhd F_n) = \{xu_1\}$ $v_1\} \cup$ $\{x$

$\{u_i u_{i+1}$ $\cup$ $i \le n-1\} \cup$ $: 1 \le$

$\{v_j v_{j+1}$ $j \le n-1\} \cup$ $1 \le$

$\{u_i v_i: 1$ $:$ $i \le n\} \cup$ $\le$

$\left\{u_i x_{ik}:\right.$ for $\left.\begin{matrix} 1 \le i \le n \\ 1 \le k \le n \end{matrix}\right\} \cup$

$\{x z_i:$ for $1 \le i \le n\} \cup$

$\{x_{ik} x$ $: \left.\begin{matrix} 1 \le i \le n \\ 1 \le k \le n-1 \end{matrix}\right\} \cup$ $_{+1}$

$\{y_{jk} y$ $; \left.\begin{matrix} 1 \le j \le n \\ 1 \le k \le n-1 \end{matrix}\right\} \cup$ $_{ik}$ $_{+1}$

$\{z_i z_{i+}$ $\le i \le n-1\}.$ $_{jk_1}: 1$

The number of vertices and edges of $(A_n^2 \rhd Fn)$ is $2n^2 + 3n + 1$ and $4n^2 + 3n - 1$ respectively.

Let $S = V(A_n^2 \rhd F_n) \cup E(A_n^2 \rhd F_n)$ and $C = n + 5$

Now we define the total colouring $f: S \to C$ by

$$f(x) = n + 1$$

For $1 \le i \le n$ $\quad f(u_i) = \begin{cases} n+3 & \text{if } i \text{ is odd} \\ n+4 & \text{if } i \text{ is even} \end{cases}$

For $1 \le j \le n$ $\quad f(v_j) = \begin{cases} n+4 & \text{if } j \text{ is odd} \\ n+3 & \text{if } j \text{ is even} \end{cases}$

or $1 \le i \le n, 1 \le k \le \mathrm{n}$ $f(x_{ik}) = k + 1$

$1 \le j \le n$ $1 \le k \le n$ $f(y_{jk}) = \mathrm{k} + 1$ or or

$1 \le i \le n$ $f(z_i) = k + 1$

$f(xv_1) = n+3$

$f(xu_1) = n+2$

For $1 \le i \le n-1$ $f(u_i u_{i+1})$ $\begin{cases} n+1 & \text{if } i \text{ is odd} \\ n+2 & \text{if } i \text{ is e en} \end{cases}$

For $1 \le i \le n-1$ $f(v\, v\quad) = \begin{cases} n+1 & \text{if } j \text{ is odd} \\ n+2 & \text{if } j \text{ is e en} \end{cases}$ $j\ j+1$

$For\ 1 \le i \le n, 1 \le k \le n$ $f(u_i x_{ik}) = \text{k}$

$For\ 1 \le j \le n, 1 \le k \le n$ $f(v_j y_{jk}) = k$

For $1 \le i \le n$ $f(\times z_i) = \text{i}$

For $1 \le i \le n$ $f(u_i v_i) = \text{n} + 5$

For $1 \le i \le n, 1 \le k \le n-1$ $f(x_{ik} x_{ik+1}) = \text{k} + 3$

For $1 \le j \le n, 1 \le k \le n-1$ $f(y_{jk} y_{jk+1}) = \text{k} + 3$

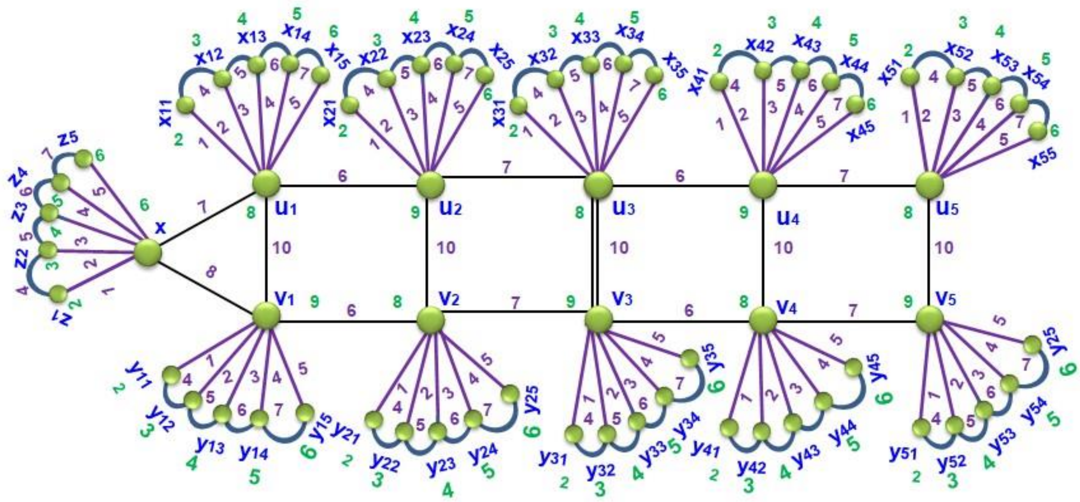
$For\ 1 \le i \le n-1$ $f(z_i z_{i+1}) = \text{k} + 3$

The above defined function f gives the total coloring for the graph $(A_n^2 \triangleright F_n)$

Hence $\chi_{tc}(A_n^2 \triangleright F_n)$=n+5 for $n \ge 2$.

Illustration 3: Consider the arrow graph with fangraph $A_5^2 \triangleright F_5$ **Figure 3.**

$\chi_{tc}(\qquad)$=10 for



Theorem: 4

The total chromatic number of Comb Product of Arrow graph with cycle graph is

$(A_n^2 \triangleright C_n)$=7 for $n \ge 2$.

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Proof:

Let G be an arrow graph(A_n^2) and H be a cycle graphC_n.

Let V$(A_n^2) = \{\ x\ \} \cup \{u_i : 1 \le i \le n\} \cup \{v_j : 1 \le j \le n\}$

$E(A_n^2) = \{xu_1\} \cup \{xv_1\} \cup \{u_i u_{i+1} : 1 \le i \le n-1\} \cup$

$\{v_j v_{j+1} : 1 \le j \le n-1\} \cup \{u_i v_i : 1 \le i \le n\}$

and

V (C_n)=$\{w_i : 1 \le i \le n\}$

$E(C_n) = \{w_i w_{i+1} : 1 \le i \le n\}$

By definition of comb product of arrow graph with cycle graph

The vertexw_1, of cycle C_n is identified with each vertex of the arrow graph A_n^2 the vertex set and the edge set of $(A_n^2 \rhd C_n)$ as follows:

$v(A_n^2 \rhd C_n) = \{x\} \cup \{u_i : 1 \le i \le n\} \cup$

$\{v_j : 1 \le j \le n\} \cup \{x_{ik}, 1 \le i \le n, 1 \le k \le n-1\ \} \cup$

$\{\ y_{jk} : 1 \le j \le n, 1 \le k \le n-1\ \} \cup \{z_i \quad 1 \le i \le n-1\}$

$E(A_n^2 \rhd C_n) = \{xu_1\} \cup \{xv_1\} \cup \{xz_1\} \cup$

$\{xz_n\} \cup \{u_i u_{i+1}; 1 \le i \le n-1\} \cup$

$\{v_j v_{j+1} : 1 \le j \le n-1\} \cup \{u_i v_i : 1 \le i \le n\}$

$\{u_i x_{i1} : 1 \le i \le n\} \cup \{v_j y_{j1} : 1 \le j \le n\} \cup \{u_i\, x_{in} : \qquad \le \text{i} \le n\}$

$\{v_j y_{jn}, 1 \le j \le n\} \cup \{x_{ik} x_{ik+1}; \quad 1 \le i \le n, 1 \le k \qquad n-2\}$

$\{y_{jk} y_{jk+1}; 1 \le j \le n, 1 \le k \le n-2\} \cup \{z_j z_{i+1} : 1 \le \qquad j \le n-2\}$

1

The number of vertices and Edges of $(A_n^2 \rhd C_n)$ is $2n^2 + \text{n}$ and $2n^2 + 4\text{n}$ $\le$ respectively. Let $S = V(A_n^2 \rhd C_n) \cup E(A_n^2 \rhd C_n)$ and $C = \{1,2,3,4,5,6,7\}$

Now we define the total colouring$f : S \to C$ by

$f(x) =$7

For $1 \le i \le n \quad f(u_i) = \begin{cases} 3 & \text{if } i \text{ is odd} \\ 4 & \text{if } i \text{ is e en} \end{cases}$

For $1 \le \le n \qquad f(v\ ) = \{ \quad 4 \quad$ if j_j $\qquad j \qquad$ 3 is odd

if j is e en

For $1 \leq i \leq n, 1 \leq k \leq n-1$

ik 1 if i is odd, is e en and if i is e en, is e en $f(x \quad) =$

2 if i is e en, is odd and if i is odd, is odd

For $1 \leq j \leq n, 1 \leq k \leq n-1$

$f(\quad) = $ j y_{jk} 1 if is odd , is e en and if j is e en, is e en

2

if j is odd, is odd and if j is e en, is odd

For $1 \leq i \leq n$-1 $f(z \quad) = \begin{cases} 1 \\ i \end{cases}$ if i is odd

$2\, f(xv_1) = 6$ $f(xz_n) = 5$ if i is e en

$f(xu_1) = 2$ $f(xz_1) = 3$

$1 \leq i \leq n-1$ $f(u_i u_{i+1}) = \begin{cases} 5 & \text{if } i \\ 6 & \text{if } i \end{cases}$

is odd

$\begin{cases} 5 \\ \end{cases}$ $1 \leq j \leq n-1$ $f(v_j v_{j+1}) =$ $6j$

or

is e en if j is odd or

if is e en

. For $1 \leq i \leq n$ $f(u_i v_i) = 1$

For $1 \leq i \leq n$ $f(u_i x_{i1}) = \begin{cases} \\ 3 \end{cases}$ 4 f i is odd

f i is e en

For $1 \leq j \leq n$ $f(v_j y_{j1}) = \begin{cases} 3 \\ 4 \end{cases}$ j f is odd

f j is e en

For $1 \leq i \leq n, 1 \leq k \leq n-2$

$f(x \quad x \quad) \begin{cases} 3 \\ 4 \end{cases}$ if i is odd , is odd and if i is e en, is e en ik ik_{+1}

i f is odd, is e en and if i is e en, is odd

$For\ 1 \leq j \leq n, 1 \leq k \leq n-2$

$f(y \quad y \quad) \begin{cases} 3 \\ \end{cases}$ if j is odd , is e en and if j is e en, is odd

jk jk

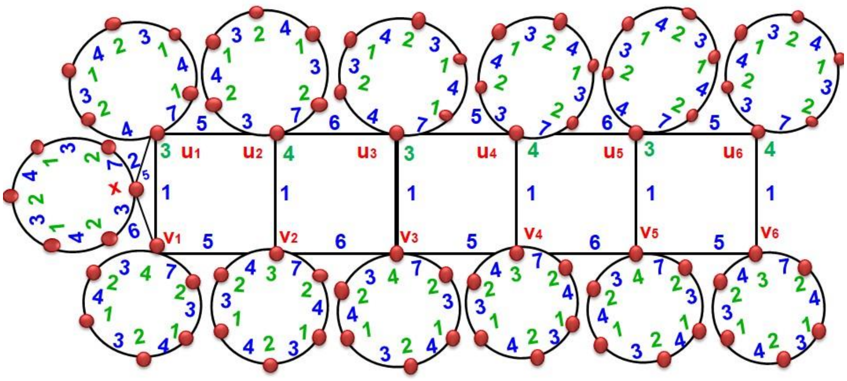
$_{+1}$ 4 f j is odd, is odd and if j is e en, is e en

For $1 \leq i \leq n$ $f(u_i\, x_{in}) = 7 = f(v_j\, v_{jn})$

For $1 \leq i \leq n-2$ $f(z z \quad) = \{4$ if i is odd $^{i \quad i+1}$ 3 if i is e en The above defined function f gives the total coloring for the graph.

Hence $\chi_{tc}(A_n^2 \triangleright C_n)=7$ for $n \geq 2$.

Illustration 4: Consider the arrow graph with cycle graph $A_6^2 \triangleright C_6$ Figure 4.



$\chi_{tc}(A_6^2 \triangleright C_6)=7$

CONCLUSION:

We have obtained the total chromatic number of comb product of arrow graph with path graph, star graph, cycle graph, and fan graph by defining total coloring for those graphs.

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