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Univariate Extreme Shock Maintenance Model for a Repairable System Under Alpha Series Process.

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Abstract:- In this paper, univariate extreme shock maintenance model for a repairable system under Alpha series process. Assume that the shocks from the system environment. A shock is called a deadly shock, if the amount of damage of one shock to the system exceeds a specific threshold so that the system will fail. For a deteriorating system, we assume that the successive threshold values are non-decreasing after repair, and the consecutive repair times after failure form an increasing Alpha series process. For an improving system, we assume that the successive threshold values are decreasing after repair, and the consecutive repair times after failure form a decreasing Alpha series process. A replacement policy N is adopted by which we shall replace the system by an identical new one at the time following the N-th failure. Then for each of the deteriorating system and improving system, an explicit expression for the long run average cost per unit time under N policy is derived and an optimal policy N^* for minimizing the long run average cost per unit time is determined analytically. A numerical example is given for deteriorating system.

***Keywords**: Alpha Series Process, Geometric distribution, Replacement Policy, Shock Model, Exponential distribution..*

1. Introduction

In reliability, the study of maintenance problem is always an important topic. For the system of "repair as new" a lot of scholars had made many results. In this research work of repair replacement problems, in the early stages, a common assumption is "repair is perfect", i.e., the system after repair is "as good as new". Most of the maintenance models just concentrate on the internal source of the system failure, but not on an external cause. A system failure may be caused by some external causes, such as a shock. But shocks with a small level of damage are harmless for the system, while shocks with a large level of damage may result in failure of the system. The shock model has been successfully applied to many different fields, such as physics, communication, electronic engineering and medicine, etc. Chen and Li (2008) introduced and studied an extended extreme shock maintenance model for a deteriorating system under which the consecutive repair time is geometric process. In this paper, we shall study a univariate extreme shock maintenance model for a deteriorating system and for an improving system under Alpha series process and the system's repairable is shown in both the internal and the external.

Therefore we consider the system has been considered in two aspects- the internal and the external. First, if the system is failed by one shock, it is repaired or replaced by a new and identical one. In view of the ageing time and the continuous wear, the repair time will become longer and longer and tend to infinity. i.e., finally the system is non-repairable, repair times are not negligible. Therefore, for a deteriorating system, we model the repair times after the system failures as an increasing alpha series process and for an improving system we model the repair times after the system's failure as an decreasing Alpha series process.

Alpha series process was introduced in Braun et al. (2005) . Furthermore Braun et.al (2005) explained the main advantage of the Alpha series process compared with the Geometric process is that under certain conditions is always has a finite expected number of events at any arbitrary time. The expected number of events at any arbitrary time does not always exist for the decreasing Geometric process, which could be a drawback in its applications. The drawback is remedied in the decreasing version of the Alpha series process.Indeed, for any Alpha series process(increasind or decreasing) the expected number of events upto time t, $E(N(t))$ is always finite. The Alpha series process grows either as a polynomial in time or exponential in time. It also noted that the geometric process doesn't satisfy a central limit theorem, while the Alpha series process does.

The definition of Alpha series process is given below.

Definition 1.1. A stochastic process is called a Alpha series process(increasing and decreasing) process, if there exists a real $\alpha, \alpha < 0 (\alpha \geq 0)$ such that $\{n^{\alpha} X_n, n = 1,2,3 \ldots\ldots\}$ forms a renewal process. The real α is called the exponent of process.

Next the shocks from the system environment are studied. There were many papers which considered extreme shock models. See Zuckerman(1978), Shanthikumar and Sumita(1983,1984), and Gut and Husler(1999).In their models, the system will fail if the amount of shock damage by one "big" shock exceeds a specific threshold but a small level of damage is harmless for the system. In these models, a shock is termed as deadly shock, if the amount of damage of one shock to the system exceeds a specific threshold so that the system fails. This kind of shock models is called "extreme shock model".

The aim of the present paper is to provide a method to calculate the average cost rate and to determine an optimal policy N^* such that the Average cost rate is minimized analytically and numerically.

Now we have the following assumptions about the model for a repairable system(deteriorating or improving) subject to shocks.

2. Model Assumptions

We consider univariate extreme shock the maintenance model for a repairable system under the following assumptions.

A1. Initially a new system is installed. Whenever the system fails, it is repaired or replaced by a new and identical one.

A2. Once the system starts operating, the shocks from the environment arrive according to a renewal process. Let $\{W_{ki}, i = 1,2,3 \ldots\ldots\}$ be the intervals between the$(i-1)^{st}$ and the i^{th} shock after the $(k-1)^{st}$ repair. Let $E(W_{11}) = \lambda$ and Let $\{D_{ki}, i = 1,2,3 \ldots.\}$ be the sequence of the amount of shock damage produced by the i^{th} shock after the $(k-1)^{st}$ repair. Let $E(D_{11}) = \beta$. Assume that $\{W_{ki}, i = 1,2,3 \ldots\ldots\}$ and $\{D_{ki}, i = 1,2,3 \ldots.\}$ are identically independent distributed sequences for all k.

In the k^{th} operating stage, i.e., after the $(k-1)^{st}$ repair , the system will fail, if the amount of the a shock damage first exceeds $a^{k-1}M$ where M is a positive constant, if the system fails, it is closed so that the random shocks have no effect on the system during the repair time.

A3. Let Z be the replacement time with $E(Z) = \tau$.

A4. The process $\{D_{ki}, i = 1,2,3 \ldots.\}$, $\{W_{ki}, i = 1,2,3 \ldots\ldots\}$, $\{Y_k, k = 1,2,3 \ldots\ldots\}$ and Z are independent.

A5. The repair cost rate is c, the reward rate is r and the replacement cost is R

A6. Let Y_k be the repair time after the k^{th} failure. Then the distribution function of Y_k is assumed to be $F(k^{\alpha} y)$ for $y \geq 0, \alpha$ is a real number. That is the consecutive repair times of the system after failures form a Alpha Series Process. Moreover, assume that $E(Y_1) = \mu \geq 0, \mu = 0$ means that the repair time is negligible.$N_n(t)$is the counting processof the number of shocks after the $(k-1)$st repair. It is clear that $E(Y_k) = \frac{\mu}{k^{\alpha}}$.

For a deteriorating system , an additional assumption A8 is made.

A7. $\alpha < 0$ and $0 < a \leq 1$.

Then under Assumptions A1 to A7, univariate Extreme shock maintenance model for a deteriorating system.

For an improving system, Assumption A7 will be replaced by Assumption A7a

A7a. . $\alpha > 0$ and $a > 1$.

Then under Assumptions A1 to A6 and A7a, univariate extreme shock maintenance model for an improving system.

Remarks.

In practice, many systems are deteriorating because of the ageing effect and accumulated wearing. For a deteriorating system after repair should be more fragile and easier to be broken down. As a result, the threshold of a deadly shock of the system will be increasing in n, the number of repairs taken. In other words, as the number of repairs n increases, the threshold of a deadly shock of the system will increase accordingly. Furthermore, for a deteriorating system, the successive operating times of the system will be shorter and shorter, while the consecutive repair times of the system will be longer and longer. Therefore under Assumptions A1 to A8, univariate extreme shock maintenance model for a deteriorating system.

However, in real life, there do have some improving systems. For examples some systems could be improved, this might be due to the fact that the operator can accumulate the operating experience so that the damage caused by a shock will be lightened, this might be due to the repair facility becoming more familiar with the system so that the successive repair times might be decreasing, then for an improving system, the older the system, the more solid the system is. Thus, the threshold shock values should be decreasing Alpha series process. Therefore under Assumptions A1 to A7 and A8a, univariate extreme shock maintenance model for an improving system.

Now we shall first study the model under Assumptions A1 to A7.

3. Average Cost Rate

Let T_1 be the first replacement time, in general, for $k \geq 2$. Let T_n be the time between the $(k-1)^{st}$ and the k th replacement. Then obviously $\{T_n, n = 1,2, \ldots\}$ forms a renewal process.

By the renewal reward theorem, Ross(1983), the long-run average cost per unit time under $(C(N)$ is given by

$$C(N) = \frac{The\ expected\ cost\ incurred\ in\ a\ cycle}{The\ expected\ length\ of\ a\ cycle} \quad (1)$$

In our models, we do not specify the distribution of the shocks interarrivals, i.e., the distributions of $W_{11}$and D_{11} may be arbitrary, it uses the geometric distribution.

For evaluating the expected cost incurred in a cycle and the expected length of a cycle. We need to first calculate the geometric distribution $G(P_k)$ and the expectation of X_k, the real operating time of the system after the $(k-1)^{st}$ repair.

Denote

$$L_k = \min\{l: D_{kl} > a^{k-1}M\} \quad (2)$$

That is L_k is the number of shocks until the first deadly shock occured following the $(k-1)^{st}$ failure. Then

$$X_k = \sum_{i=1}^{L_k} W_{ki} \quad (3)$$

and L_k follows a geometric distribution $G(P_k)$, with

$$P(L_k = k) = P_k(1-P_k)^{k-1}, k = 1,2 \ldots \ldots \quad (4)$$

where

$$P_k = P(D_{k1} > a^{k-1}M) \tag{5}$$

By equation (4) , we have

$$E(L_k) = \frac{1}{P_k} \tag{6}$$

As $\{W_{ki}, i = 1,2,3 \ldots..\}$ and $\{D_{ki}, i = 1,2,3 \ldots.\}$ are independent, it is clear that L_k and $\{W_{ki}, i = 1,2,3 \ldots..\}$ are independent.

Then from equations (3) and (4) and by Wald's equation. We have

$$\lambda_k = E(X_k) = E(L_k)E(W_{k1}) = \frac{\lambda}{P_k} \tag{7}$$

From Equation (1) ,The Average cost rate ,$C(N)$ under policy N is given by

$$C(N) = \frac{E(c\sum_{k=1}^{N-1} Y_k + R - r\sum_{k=1}^{N} X_k)}{E(\sum_{k=1}^{N} X_k + \sum_{k=1}^{N-1} Y_k + Z\ )}$$

$$= \frac{c(\sum_{k=1}^{N-1} E(Y_k)) + R - r\sum_{k=1}^{N} \lambda_k)}{\sum_{k=1}^{N} \lambda_k + (\sum_{k=1}^{N-1} Y_k) + \tau}$$

$$= \frac{c\left(\sum_{k=1}^{N-1} \frac{\mu}{k^\alpha}\right) + R - r\sum_{k=1}^{N} \lambda_k\Big)}{\sum_{k=1}^{N} \lambda_k + \left(\sum_{k=1}^{N-1} \frac{\mu}{k^\alpha}\right) + \tau} \tag{8}$$

In this section, We determine an optimal replacement policy $N^*$for minimizing C(N)

From equation (8) we have

$$C(N) = \frac{(c+r)\mu\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + R + r\tau}{h(N)} - r \tag{9}$$

Where

$$h(N) = \sum_{k=1}^{N} \lambda_k + \mu\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \tau$$

In order to obtain the optimal policy N^*, we need to investigate the difference between $C(N+1) and\ C(N)$.

$$C(N+1) - C(N) = \left[\frac{(c+r)\mu\left(\sum_{k=1}^{N} \frac{1}{k^\alpha}\right) + R + r\tau}{h(N+1)} - r\right] - \left[\frac{(c+r)\mu\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + R + r\tau}{h(N)} - r\right]$$

$$= \frac{\left[\begin{array}{c}\left[\sum_{k=1}^{N} \lambda_k + \mu\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \tau\right]\left[(c+r)\mu\left(\sum_{k=1}^{N} \frac{1}{k^\alpha}\right) + R + r\tau\right] - \\ \left[\sum_{k=1}^{N+1} \lambda_k + \mu\left(\sum_{k=1}^{N} \frac{1}{k^\alpha}\right) + \tau\right]\left[(c+r)\mu\left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + R + r\tau\right]\end{array}\right]}{h(N+1)h(N)}$$

$$= \frac{\begin{bmatrix} (c+r)\mu \sum_{k=1}^{N} \lambda_k \left(\frac{1}{N^\alpha}\right) - \lambda_{N+1}(c+r)\mu \left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) - R\lambda_{N+1} \\ -r\tau\lambda_{N+1} + \tau(c+r)\mu \left(\frac{1}{N^\alpha}\right) - R\mu \left(\frac{1}{N^\alpha}\right) - \mu r\tau \left(\frac{1}{N^\alpha}\right) \end{bmatrix}}{h(N+1)h(N)}$$

$$= \frac{\left[(c+r)\mu \left(\sum_{k=1}^{N} \lambda_k - N^\alpha \lambda_{N+1} \left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \tau\right) - \lambda_{N+1}(R+r\tau) - \mu \left(\frac{1}{N^\alpha}\right)(R+r\tau)\right]}{h(N)h(N+1)}$$

$$C(N+1) - C(N) = \frac{\left[(c+r)\mu \left(\sum_{k=1}^{N} \lambda_k - N^\alpha \lambda_{N+1} \left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \tau\right) - (R+r\tau)(N^\alpha \lambda_{N+1} + \mu)\right]}{h(N)h(N+1)} \quad (10)$$

As the denominator of $C(N+1) - C(N)$is always positive, it is clear that the sign of

$C(N+1) - C(N)$is the same as the sign of its numerator.

Thus we introduce the auxillary function $B(N)$ as follows.

$$B(N) = \frac{(c+r)\mu \left(\sum_{k=1}^{N} \lambda_k - N^\alpha \lambda_{N+1} \left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \tau\right)}{(R+r\tau)(N^\alpha \lambda_{N+1} + \mu)} \quad (11)$$

As a result we have the following lemma.

Lemma 3.1.

$$C(N+1) > C(N) \Leftrightarrow B(N) > 1$$
$$C(N+1) = C(N) \Leftrightarrow B(N) = 1$$
$$C(N+1) < C(N) \Leftrightarrow B(N) < 1 \quad (12)$$

Lemma (3.1) shows that the monotonicity of $C(N)$ can be determined by the value of $B(N)$.

Note that in this section, the results are developed under Assumptions A1 to A7 only. Therefore, all the results including Lemma 3.1 hold for the univariate extreme shock maintenance model for a deteriorating system and for an improving system.

4. Optimal Replacemet Policy N^*.

In this section, we shall determine an optimal replacement policy N^* analytically for minimizing $C(N)$ for a deteriorating system and for an improving system respectively. For this purpose at first , we haveto determine $B(N+1) - B(N)$

From equation (11) we have

$$B(N+1) - B(N) = \begin{bmatrix} \left(\frac{(c+r)\mu \left(\sum_{k=1}^{N+1} \lambda_k - (N+1)^\alpha \lambda_{N+2} \left(\sum_{k=1}^{N} \frac{1}{k^\alpha}\right) + \tau\right)}{(R+r\tau)((N+1)^\alpha \lambda_{N+2} + \mu)}\right) - \\ \left(\frac{(c+r)\mu \left(\sum_{k=1}^{N} \lambda_k - N^\alpha \lambda_{N+1} \left(\sum_{k=1}^{N-1} \frac{1}{k^\alpha}\right) + \tau\right)}{(R+r\tau)(N^\alpha \lambda_{N+1} + \mu)}\right) \end{bmatrix}$$

Let

$$A(N) = \frac{(c+r)\mu}{(R+r\tau)((N+1)^\alpha \lambda_{N+2} + \mu)(N^\alpha \lambda_{N+1} + \mu)}$$

Now,

$$B(N+1)-B(N)=A(N)\left[\begin{array}{c}(N^{\alpha}\lambda_{N+1}+\mu)\left(\sum_{k=1}^{N+1}\lambda_k-(N+1)^{\alpha}\lambda_{N+2}\left(\sum_{k=1}^{N}\frac{1}{k^{\alpha}}\right)+\tau\right)\\ -((N+1)^{\alpha}\lambda_{N+2}+\mu)\left(\sum_{k=1}^{N}\lambda_k-N^{\alpha}\lambda_{N+1}\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\tau\right)\end{array}\right]$$

$$=A(N)\left[\begin{array}{c}\mu\lambda_{N+1}+\lambda_{N+1}N^{\alpha}\lambda_{N+1}-\lambda_{N+1}(N+1)^{\alpha}\lambda_{N+2}-\mu(N+1)^{\alpha}\lambda_{N+2}\left(\frac{1}{N^{\alpha}}\right)\\ +(N^{\alpha}\lambda_{N+1}-\lambda_{N+2}(N+1)^{\alpha})\left(\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\sum_{k=1}^{N}\lambda_k+\tau\right)\end{array}\right]$$

$$=A(N)\left[\begin{array}{c}\left(\mu\left(\frac{1}{N^{\alpha}}\right)+\lambda_{N+1}\right)(\lambda_{N+1}N^{\alpha}-\lambda_{N+2}(N+1)^{\alpha})+\\ (\lambda_{N+1}N^{\alpha}-\lambda_{N+2}(N+1)^{\alpha})\left(\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\sum_{k=1}^{N}\lambda_k+\tau\right)\end{array}\right]$$

$$=A(N)\left[(\lambda_{N+1}N^{\alpha}-\lambda_{N+2}(N+1)^{\alpha})\left(\mu\left(\frac{1}{N^{\alpha}}\right)+\lambda_{N+1}+\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\sum_{k=1}^{N}\lambda_k+\tau\right)\right]$$

$$B(N+1)-B(N)=\frac{(c+r)\mu\left[(\lambda_{N+1}N^{\alpha}-\lambda_{N+2}(N+1)^{\alpha})\left(\mu\left(\frac{1}{N^{\alpha}}\right)+\lambda_{N+1}+\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\sum_{k=1}^{N}\lambda_k+\tau\right)\right]}{(R+r\tau)((N+1)^{\alpha}\lambda_{N+2}+\mu)(N^{\alpha}\lambda_{N+1}+\mu)} \tag{13}$$

Then, we shall consider two models.

Model 1.

Under Assumptions A1 to A7, Univariate extreme shock maintenance model for a deteriorating system.

First of all, we have the following lemma.

Lemma 4.1. Under Assumptions A1 to A7, we have

(1) λ_k is decreasing in k.

(2) $B(N)$ is non-decreasing in N.

Proof. (1) Equations (5) and (7) are

$$P_k=P(D_{k1}>a^{k-1}M)$$

and

$$\lambda_k=E(X_k)=E(L_k)E(W_{k1})=\frac{\lambda}{P_k}$$

Since $a\le 1$ from equations (5) and (7). We can derive that λ_k is decreasing in k.

Proof (2). From Equation (13),we have

$B(N+1)-B(N)$

$$=\frac{(c+r)\mu\left[(\lambda_{N+1}N^{\alpha}-\lambda_{N+2}(N+1)^{\alpha})\left(\mu\left(\frac{1}{N^{\alpha}}\right)+\lambda_{N+1}+\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right)+\sum_{k=1}^{N}\lambda_k+\tau\right)\right]}{(R+r\tau)((N+1)^{\alpha}\lambda_{N+2}+\mu)(N^{\alpha}\lambda_{N+1}+\mu)}\ge 0$$

This shows that $B(N)$ is non-decreasing in N, because λ_k is decreasing in k and $\alpha<0$.

Using Lemma (3.1) and Lemma 4.1, we have the following theorem.

Theorem 4.1. The optimal replacement policy N^* is determined by

$$N_d^* = \min\{N/B(N) \geq 1\} \tag{14}$$

Moreover, the optimal replacement policy N^* is unique if and only if $B(N^*) > 1$.

We can apply Theorem 4.1 to determine an optimal policy analytically for the deteriorating system and also determine an optimal policy numerically.

A Numerical Example

We study a numerical example with the assumption that D_{11} has an exponential distribution with expectation β.

Then $$F(x) = P(D_{11} \leq x) = 1 - e^{-\left(\frac{1}{\beta}\right)x}$$

From equation (5), we have

$$P_k = P(D_{k1} > a^{k-1}M) = e^{-\left(\frac{1}{\beta}\right)a^{k-1}M} \tag{15}$$

Then by equation (7), we have

$$\lambda_k = E(X_k) = \frac{\lambda}{P_k} = \lambda e^{\left(\frac{1}{\beta}\right)a^{k-1}M} \tag{16}$$

Substituting equation (15) in equation (9) and (11), the explicit expression for $C(N)$ and $B(N)$ are

$$C(N) = \frac{\left[(c+r)\mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right) + R + r\tau\right]}{\left[\lambda\sum_{k=1}^{N} e^{\left(\frac{1}{\beta}\right)a^{k-1}M} + \mu\left(\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right) + \tau\right]} - r \tag{17}$$

$$B(N) = \frac{(c+r)\mu\left[\lambda\sum_{k=1}^{N} e^{\left(\frac{1}{\beta}\right)a^{k-1}M} + \tau - N^{\alpha}\lambda e^{\left(\frac{1}{\beta}\right)a^{N}M}\sum_{k=1}^{N-1}\frac{1}{k^{\alpha}}\right]}{(R+r\tau)\left[\mu + N^{\alpha}\lambda e^{\left(\frac{1}{\beta}\right)a^{N}M}\right]} \tag{18}$$

Let $c = 6,\ \mu = 10,\ r = 10,\ \tau = 10,\ a = 0.95,\ R = 6000,\ \lambda = 10,\ \beta = 10,\ \alpha = -0.98,\ M = 20$

The numerical results are presented in Table and the corresponding figures are plotted in Figure 1 and Figure 2 respectively.

Table : The values of $C(N)$ and $B(N)$ for different values of N.

N	$C(N)$	$B(N)$	N	$C(N)$	$B(N)$
1	62.71378243	0.02862918	10	3.64595261	0.69378903
2	28.94257705	0.07507138	11	3.43664941	0.77152329
3	17.25365310	0.13569171	12	3.31351515	0.84659007
4	11.59934997	0.20654302	13	3.25081222	0.91883968
5	8.42880843	0.28411102	14	3.23075356	1.00012229
6	6.50549464	0.36552940	15	3.24082219	1.05475939
7	5.28622768	0.44859551	16	3.27208457	1.11851447
8	4.49555734	0.53169253	17	3.31809507	1.17958295

9	3.97953895	0.61368059	18	3.37416518	1.23807733

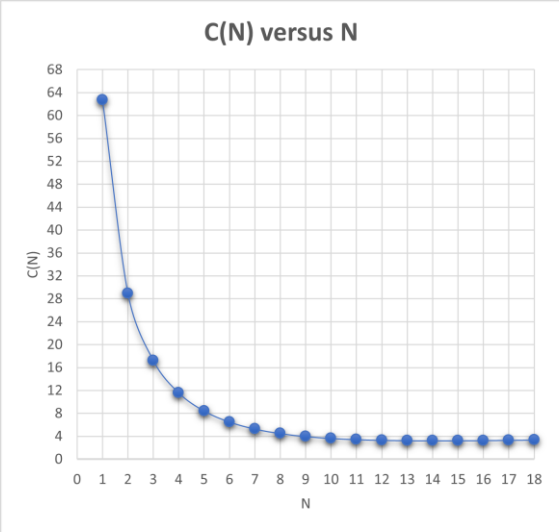

Figure 1

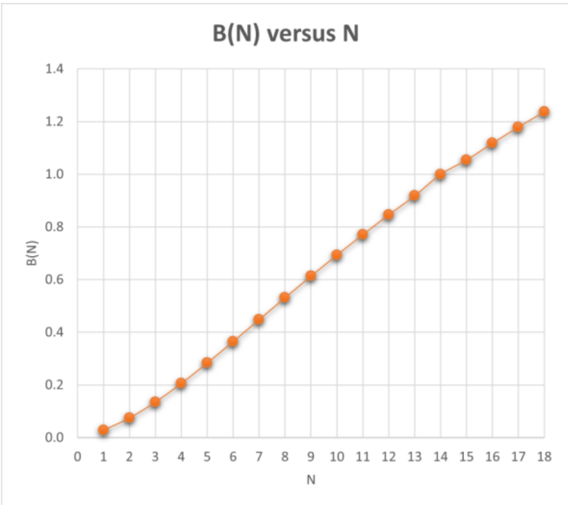

Figure2.

Clearly $C(14) = 3.23075356$ is the minimum of the long run average cost. On the other hand $B(14) = 1.00012229 > 1$ and

$$min\{N/B(N) \geq 1\} = 14$$

Model 2.

Under Assumptions A1 to A6 and A7a. Univariate extreme shock maintenance model for an improving system.

Now, because of Assumption A7a, instead of Lemma 4.1 we have the following Lemma.

Lemma 4.2. Under Assumptions A1 to A6 and A7a, we have

(1) λ_k is increasing in k.

(2) $B(N)$ is decreasing in N.

Proof. (1) Equations (5) and (7) are

$$P_k = P(D_{k1} > a^{k-1}M)$$

and

$$\lambda_k = E(X_k) = E(L_k)E(W_{k1}) = \frac{\lambda}{P_k}$$

Since $a > 1$ from equations (5) and (7). We can derive that λ_k is increasing in k.

Proof (2). From Equation (13), we have

$B(N+1) - B(N)$

$$= \frac{(c+r)\mu\left[(\lambda_{N+1}N^\alpha - \lambda_{N+2}(N+1)^\alpha)\left(\mu\left(\frac{1}{N^\alpha}\right) + \lambda_{N+1} + \mu\left(\sum_{k=1}^{N-1}\frac{1}{k^\alpha}\right) + \sum_{k=1}^{N}\lambda_k + \tau\right)\right]}{(R + r\tau)((N+1)^\alpha\lambda_{N+2} + \mu)(N^\alpha\lambda_{N+1} + \mu)} \leq 0$$

This shows that $B(N)$ is decreasing in N, because λ_k is increasing in k and $\alpha > 0$.

Using Lemma (3.1) and Lemma 4.2, we have the following theorem.

Theorem 4.2 Under Assumptions A1 to A6 and A7a, policy $N_i^* = \infty$ is the unique optimal policy for the improving system.

Proof.

Because $B(N)$ is decreasing in N, there exist an integer N_i such that

$$N_i = \min\{N | B(N) \leq 1\}. \quad (19)$$

In other words, we have

$B(N) > 1 <=> N < N_i$ and $B(N) \leq 1 <=> N \geq N_i$.

Therefore, lemma 3.1 implies that $C(N)$ is unimodel of N and take their maxima at N_i. Because of equation (9) and Assumption A7a, it is easy to check that the minimum of $C(N)$ will be given by $minC(N) = \min\{C(1), C(\infty)\}$

$$= \min\left\{\frac{R - r\lambda}{\lambda + \tau}, -r\right\} = -r. \quad (20)$$

Thus $N_i^* = \infty$ is the unique optimal replacement policy for the improving system.

Intuitively, it is interesting point out that the older the improving system is, the better the system is. This means that we shall repair the system when it fails without replacement. Therefore Theorem 4.2 agrees with this general knowledge.

5. Conclusion

By considering univariate extreme shock maintenance model for a repairable system, an explicit expression for the long-run average cost per unit of time under the replacement policy Nunder Alpha series process is derived. An optimal strategy N^* for minimizing the long run average cost per unit of time is determined analytically. A numerical example is given to illustrate the methodology developed in this research work for deteriorating system.On the other hand , it is interesting to point out that according to theorem 4.2 , the optimal replacement policy for an improving system is always $N_i^* = \infty$.

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