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The Effect of Oblateness on the mean motion of rotating Cyclic Kite Configuration.

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Abstract: This paper deals with the effect of oblateness of the first point mass of the cyclic kite configuration on the mean motion of the rotating frame lying on plane of the kite configuration by using python language .Graphical studies show that the mean motion increases with the increase of the values of oblateness parameter of the primary of mass $m_1 = (1-\mu)/2$ where $\mu \in (0, 3^{-1})$. The oblateness of other bodies also may affect the mean motion of the rotating frame.

Keywords: Kite configuration, Cyclic kite configuration, Mass parameter, Rotating frame, Mean motion.

1. **Definition**: Before to introduce the exact problem; let us define first the kite configuration with classifications. Kite configuration means a quadrilateral with two pairs of equal adjacent sides and cyclic kite configuration is a kite whose vertices lie on the circumference of a common circle. From different articles of previous authors; it is to be noted that a quadrilateral is a kite if and only if their diagonals are perpendicular to each other and at least one of the two diagonals is a line of symmetry. In some cases both the diagonals may be the lines of symmetry.

Thus, the cyclic kite configuration can be classified into two classes.

i. The kite formed by the combination of one equilateral and one isosceles triangle.
ii. The kite formed by the combination of two congruent isosceles right angled triangles.

2. Introduction:

The researchers of celestial mechanics used to discuss the particular cases of both the branches such as Three-body, Four-body, Five-body (configuration or problem). In the field of four body configuration MacMillon etal (1932) proved two theorems in detail for the existence of quadrilateral configuration. Brumberg (1957) studied permanent configuration of the Four-body problem.

In (1996) the symmetric central configuration of equal masses was first studied by Albouy. Long et al (2002) proved that a convex non-collinear planar four body central configuration with three equal masses must be a kite .He also analyzed the four-body central configuration with two pairs of equal masses. Chavela et al (2007) invented that if in a convex four body central configuration two equal masses are located at opposite vertices of a quadrilateral and at most the mass of one of the remaining particles is larger than the equal masses then the configuration must be a kite shaped quadrilateral. Further Albouy et al (2008) discussed some properties of a quadrilateral configuration and establish some relations among the masses of the system and it was proved that the planar four-body configuration is a convex central configuration and is symmetric about it's one diagonal if and only if the masses of two particles at the end of the other diagonal are equal. To construct a general four-body problem Pina et al (2009) developed a new algorithm with the ratios of directed areas and the corresponding scalar areas and in (2010) they developed co-ordinates of four particles of kite configuration in terms of principle moments of inertia and Eulerian angles independently .In Newtonian four-body problem ,the cyclic central configuration has been studied for the first time by Cors etal (2012)in which they considered the six mutual distances of the particles as their co-ordinates and showed that the four point masses forming a kite lying on a two-dimensional plane. Also they invented the kite configuration and isosceles trapezoidal configuration .Further they have shown that if any central configuration contains two equal masses then the configuration must have a line of symmetry. Balint Erdi et al (2016) extended the work of Cors etal (2012) in three cases (one in convex case and two in concave cases) by expressing the masses of the central configuration in terms of angle coordinates Further they claimed that formulas derived represents the exact analytical solutions of the four-body configuration . Deng etal (2017) used mutual distances as the coordinates and proved that the diagonals of a cyclic central quadrilateral cannot be perpendicular except that the configuration is a kite. Corbera etal (2017) proved that the diagonals of a four body central configuration are perpendicular then the configuration must be a kite. Further they verified the same theorem in the four-vertex convex central configuration Hassan (2023) studied cyclic kite configuration by three theorems and he expressed the masses of the four particles in terms of a mass parameter μ and the total mass M of the system .Also he developed the co-ordinates of the four masses and justified all the results of previous authors by using his results .Not only that ,he also derived the mean motion of the rotating frame lying in the plane of the kite .Presently we have proposed to study the effect of the oblateness of the first body of mass $m_1 = \frac{M(1-\mu)}{2}$ on the mean motion of the rotating frame .

3: Theorem:- **When the cyclic kite configuration is formed by one equilateral triangle and one isosceles triangle with the common base** P_2P_4 **:**

Statement:-The necessary and sufficient conditions for the existence of cyclic kite configuration with four positive point masses $m_k\ (k=1,2,3,4)$ at the vertices P_k of the kite are that there exists a non-dimensional mass parameter $\mu \in \left(0, 3^{-1}\right)$ such that $\frac{m_2}{M} = \frac{m_4}{M} = \mu$, then $m_1 = \frac{M(1-\mu)}{2}, m_3 = \frac{M(1-3\mu)}{2}$, where $m_1 + m_2 + m_3 + m_4 = M$.

Proof.

Let $P_k\ (k=1,2,3,4)$ be the positions (ref. fig 1) of the four positive point masses m_k moving on a common circular orbit with center at O, radius R and diameter $P_1OP_3 = 2R$ as the axis of symmetry. Considering the center of the circle as the origin, the axis of symmetry as the x-axis and a line YOY' perpendicular to P_1OP_3 as the y-axis. Using the properties of cyclic quadrilateral $P_1P_2\ P_3P_4$ in the form of cyclic kite configuration and the cyclic equilateral triangle $P_1P_2P_4$, the coordinates of the four point masses can be written as $P_k(R\cos\theta_k, R\sin\theta_k)$, where $\theta_1 = 0^0, \theta_2 = 120^0,\ \theta_3 = 180^0, \theta_4 = 240^0$.

Thus the position vectors of the four point masses are given by $\vec{r}_k = R\cos\theta_k\hat{i} + R\sin\theta_k\hat{j}$

$$\Rightarrow \vec{r}_1 = R\hat{i}, \vec{r}_2 = -\frac{R}{2}\hat{i} + \frac{\sqrt{3}}{2}R\hat{j}, \vec{r}_3 = -R\hat{i}, \vec{r}_4 = -\frac{R}{2}\hat{i} - \frac{\sqrt{3}}{2}R\hat{j}. \tag{1}$$

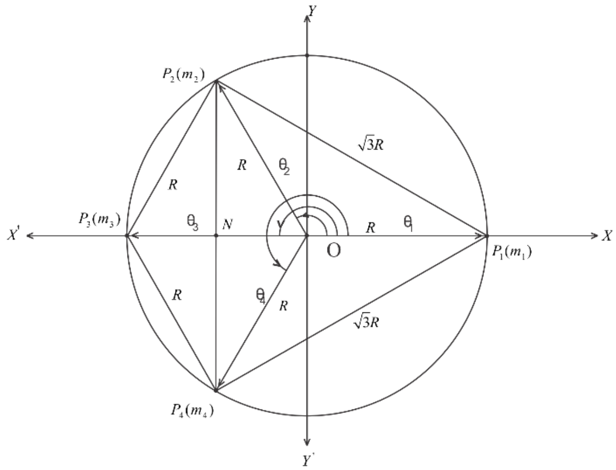

Figure-1

The position vector of the center of mass of the system of four- point masses is given by

$$\vec{c} = M^{-1}\sum_{k=1}^{4} m_k\vec{r}_k. \tag{2}$$

If we consider the center' of mass as the origin, then $\vec{c} = \hat{0} \Rightarrow m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4 = \hat{0}$

$$\Rightarrow m_1R\hat{i} + m_2(-\frac{R}{2}\hat{i} + \frac{\sqrt{3}}{2}R\hat{j}) + m_3(-R\hat{i}) + m_4(-\frac{R}{2}\hat{i} - \frac{\sqrt{3}}{2}R\hat{j}) = 0$$

$$\Rightarrow R\{2(m_1 - m_3) - (m_2 + m_4)\}\hat{i} + \sqrt{3}R\{m_2 - m_4\}\hat{j} = 0$$

Taking scalar product of $\hat{i}$ and $\hat{j}$ with above equations we get

$$\left.\begin{aligned} 2(m_1 - m_3) - (m_2 + m_4) = 0 \\ m_2 = m_4 \end{aligned}\right\} \tag{3}$$

Also we know

$$m_1 + m_2 + m_3 + m_4 = M \tag{4}$$

The equations (3) and (4) yield

$$m_1 - m_2 - m_3 = 0 \tag{5}$$

and

$$m_1 + 2m_2 + m_3 = M \tag{6}$$

Let us introduce a non- dimensional mass parameter $\mu \in \left(0, 3^{-1}\right)$ such that $m_2 = m_4 = M\mu$ then from (5) & (6) we get

$$m_1 = \frac{M(1-\mu)}{2}, m_3 = \frac{M(1-3\mu)}{2}.$$

Thus the necessary conditions for four positive point masses m_k situated at the vertices of a cyclic kite formed by an equilateral triangle $P_1P_2P_4$ and an isosceles triangle $P_2P_3P_4$ with the common base P_2P_4 are

$$\left.\begin{aligned}
&(a)\ \vec{r}_1 + \vec{r}_3 = 0 \qquad \vec{r}_1 + \vec{r}_2 + \vec{r}_4 = 0 \\
&(b)\ m_2 = m_4 = M\mu,\ m_1 = \frac{M(1-\mu)}{2}, m_3 = \frac{M(1-3\mu)}{2} \\
&(c)\ m_1 - m_2 = m_3 > 0 \Rightarrow m_1 > m_2 = m_4
\end{aligned}\right\} \tag{7}$$

Here 5(b) justified the work of Chavella (2007).

For converse part let us prove that the diagonals of the quadrilateral $P_1P_2P_3P_4$ are perpendicular to each other. For this we have to find magnitudes of the sides of the quadrilateral. From previous results we have

$$\left.\begin{aligned}
&|\vec{r}_{12}| = |\vec{r}_{21}| = |\vec{r}_2 - \vec{r}_1| = \left| -\frac{3R}{2}\hat{i} + \frac{\sqrt{3}R}{2}\hat{j} \right| = \sqrt{3}R = |\vec{r}_{14}| = |\vec{r}_{41}| = |\vec{r}_{24}| = |\vec{r}_{42}| \\
&|\vec{r}_{23}| = |\vec{r}_{32}| = |\vec{r}_{34}| = |\vec{r}_{43}| = R \ , \qquad |\vec{r}_{13}| = |\vec{r}_{31}| = 2R
\end{aligned}\right\} \tag{8}$$

Now in the kite $P_1P_2P_3P_4$

$$\vec{r}_{13} \cdot \vec{r}_{24} = (\vec{r}_3 - \vec{r}_1) \cdot (\vec{r}_4 - \vec{r}\ ) = (-R\vec{i} - R\vec{i}\ ) \cdot (-\frac{R}{2}\vec{i} - \frac{\sqrt{3}}{2}R\vec{j} + \frac{R}{2}\vec{i} - \frac{\sqrt{3}}{2}R\vec{j}\ ) = 0 \tag{9}$$

Therefore the diagonals P_1P_3 & P_2P_4 are perpendicular to each other and hence the quadrilateral $P_1P_2P_3P_4$ Is a cyclic kite configuration. Thus the results (7) & (9) satisfy the necessary and sufficient conditions for a quadrilateral to be a cyclic kite. These results can satisfy all the results of previous authors also.

4. The Equations of motion of four bodies of the kite in rotating frame about z- axis:

Before to derive the equations of motion of the four bodies forming cyclic kite configuration in synodic frame, let us derive first the gravitational force between two bodies (one is spherical and the other is oblate spheroid). Let A be the oblateness parameter of any oblate body then the potential V_{ij} between a spherical body of mass m_i and an oblate body of mass m_j separated by the distance r_{ij} is given by (McCuskey -1967,pp)

$$V_{ij} = -\frac{Gm_i m_j}{r_{ij}} - \frac{GA}{2r_{ij}^3} \quad (i \neq j = 1,2,3,4) \tag{10}$$

Where the oblateness parameter is given by $A = I_a - I_e$, I_a is the moment of inertia of the oblate body about the polar axis and I_e is the moment of inertia about the equatorial axis.

Let the plane of motion of the four point masses (one is oblate and the others are spherical) forming kite configuration rotate with the angular velocity $\vec{\omega} = n\hat{k}$ about the z-axis ; $\hat{r}_{ij}$ be the unit vector along the line $P_i P_j$ joining the i-th and j-th point masses, then the equation of motion of the i-th point mass relative to the other three point masses is given by

$$-m_i n^2 \vec{r}_i = \sum_{j=1}^{4} \frac{\partial V_{ij}}{\partial r_{ij}} \hat{r}_{ij} , \qquad (\hat{i} \neq \hat{j}) \tag{11}$$

$$m_i n^2 \hat{r}_i + \frac{\partial V_{i1}}{\partial r_{i1}} \hat{r}_{i1} + \frac{\partial V_{i1}}{\partial r_{i1}} \hat{r}_{i2} + \frac{\partial V_{i1}}{\partial r_{i1}} \hat{r}_{i3} + \frac{\partial V_{i1}}{\partial r_{i1}} \hat{r}_{i4} = 0. \tag{12}$$

Putting i=1,2,3,4 in the equation (12) ,we get

$$\left.\begin{aligned}
m_1 n^2 \vec{r}_1 + \frac{\partial V_{12}}{\partial r_{12}} \hat{r}_{12} + \frac{\partial V_{13}}{\partial r_{13}} \hat{r}_{13} + \frac{\partial V_{14}}{\partial r_{14}} \hat{r}_{14} &= 0 \\
m_2 n^2 \vec{r}_2 + \frac{\partial V_{21}}{\partial r_{21}} \hat{r}_{21} + \frac{\partial V_{23}}{\partial r_{23}} \hat{r}_{23} + \frac{\partial V_{24}}{\partial r_{24}} \hat{r}_{24} &= 0 \\
m_3 n^2 \vec{r}_3 + \frac{\partial V_{31}}{\partial r_{31}} \hat{r}_{31} + \frac{\partial V_{32}}{\partial r_{32}} \hat{r}_{32} + \frac{\partial V_{34}}{\partial r_{34}} \hat{r}_{34} &= 0 \\
m_4 n^2 \vec{r}_4 + \frac{\partial V_{41}}{\partial r_{41}} \hat{r}_{41} + \frac{\partial V_{42}}{\partial r_{42}} \hat{r}_{42} + \frac{\partial V_{43}}{\partial r_{43}} \hat{r}_{43} &= 0
\end{aligned}\right\} \tag{13}$$

As we take the first body of mass m_1 at P_1 of the kite configuration as an oblate spheroid so

$$\left.\begin{aligned}
V_{12} &= -\frac{Gm_1 m_2}{r_{12}} - \frac{GA}{2r_{12}^3} = V_{21}, \\
V_{13} &= -\frac{Gm_1 m_3}{r_{13}} - \frac{GA}{2r_{13}^3} = V_{31}, \\
V_{14} &= -\frac{Gm_1 m_4}{r_{14}} - \frac{GA}{2r_{14}^3} = V_{41}.
\end{aligned}\right\} \tag{14}$$

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Introducing equations (14) in equations (13) one can find

$$\left.\begin{aligned}
&m_1 n^2 \vec{r}_1 - G\frac{\partial}{\partial r_{12}}\left[\frac{m_1 m_2}{r_{12}} + \frac{A}{2r_{12}^3}\right]\vec{r}_{12} - G\frac{\partial}{\partial r_{13}}\left[\frac{m_1 m_3}{r_{13}} + \frac{A}{2r_{13}^3}\right]\vec{r}_{13} - G\frac{\partial}{\partial r_{14}}\left[\frac{m_1 m_4}{r_{14}} + \frac{A}{2r_{14}^3}\right]\vec{r}_{14} = 0\\
&m_2 n^2 \vec{r}_2 - G\frac{\partial}{\partial r_{21}}\left[\frac{m_2 m_1}{r_{21}} + \frac{A}{2r_{21}^3}\right]\vec{r}_{21} - G\frac{\partial}{\partial r_{23}}\left[\frac{m_2 m_3}{r_{23}}\right]\vec{r}_{23} - G\frac{\partial}{\partial r_{24}}\left[\frac{m_2 m_4}{r_{24}}\right]\vec{r}_{24} = 0\\
&m_3 n^2 \vec{r}_3 - G\frac{\partial}{\partial r_{31}}\left[\frac{m_3 m_1}{r_{31}} + \frac{A}{2r_{31}^3}\right]\vec{r}_{31} - G\frac{\partial}{\partial r_{32}}\left[\frac{m_3 m_2}{r_{32}}\right]\vec{r}_{32} - G\frac{\partial}{\partial r_{34}}\left[\frac{m_3 m_4}{r_{34}}\right]\vec{r}_{34} = 0\\
&m_4 n^2 \vec{r}_4 - G\frac{\partial}{\partial r_{41}}\left[\frac{m_4 m_1}{r_{41}} + \frac{A}{2r_{41}^3}\right]\vec{r}_{31} - G\frac{\partial}{\partial r_{42}}\left[\frac{m_4 m_2}{r_{42}}\right]\vec{r}_{42} - G\frac{\partial}{\partial r_{43}}\left[\frac{m_4 m_3}{r_{43}}\right]\vec{r}_{43} = 0
\end{aligned}\right\} \quad (15)$$

$$\begin{aligned}
&n^2\vec{r}_1 + G\left[\frac{m_2}{r_{12}{}^2} + \frac{3A}{2m_1 r_{12}^4}\right]\frac{\vec{r}_{12}}{r_{12}} + G\left[\frac{m_3}{r_{13}^2} + \frac{3A}{2m_1 r_{13}^4}\right]\frac{\vec{r}_{13}}{r_{13}} + G\left[\frac{m_4}{r_{14}^2} + \frac{3A}{2m_1 r_{14}^4}\right]\frac{\vec{r}_{14}}{r_{14}} = 0\\
&n^2\vec{r}_2 + G\left[\frac{m_1}{r_{21}^2} + \frac{3A}{2m_2 r_{21}^4}\right]\frac{\vec{r}_{21}}{r_{21}} + G\left[\frac{m_3}{r_{23}^2}\right]\frac{\vec{r}_{23}}{r_{23}} + G\left[\frac{m_4}{r_{24}^2}\right]\frac{\vec{r}_{24}}{r_{24}} = 0\\
&n^2\vec{r}_3 + G\left[\frac{m_1}{r_{31}^2} + \frac{3A}{2m_3 r_{31}^4}\right]\frac{\vec{r}_{31}}{r_{31}} + G\left[\frac{m_2}{r_{32}^2}\right]\frac{\vec{r}_{31}}{r_{31}} + G\left[\frac{m_4}{r_{34}^2}\right]\frac{\vec{r}_{31}}{r_{31}} = 0\\
&n^2\vec{r}_4 + G\left[\frac{m_1}{r_{41}^2} + \frac{3A}{2m_4 r_{41}^4}\right]\frac{\vec{r}_{41}}{r_{41}} + G\left[\frac{m_2}{r_{42}^2}\right]\frac{\vec{r}_{42}}{r_{42}} + G\left[\frac{m_3}{r_{43}^2}\right]\frac{\vec{r}_{43}}{r_{43}} = 0
\end{aligned}$$

$$\begin{aligned}
&n^2\vec{r}_1 + G\left[\left\{\frac{m_2}{r_{12}^3} + \frac{3A}{2m_1 r_{12}^5}\right\}\vec{r}_{12} + \left\{\frac{m_3}{r_{13}^3} + \frac{3A}{2m_1 r_{13}^5}\right\}\vec{r}_{13} + \left\{\frac{m_4}{r_{14}^3} + \frac{3A}{2m_1 r_{14}^5}\right\}\vec{r}_{14}\right] = 0\\
&n^2\vec{r}_2 + G\left[\left\{\frac{m_1}{r_{21}^3} + \frac{3A}{2m_2 r_{21}^5}\right\}\vec{r}_{21} + \left\{\frac{m_3}{r_{23}^3}\right\}\vec{r}_{23} + \left\{\frac{m_4}{r_{24}^3}\right\}\vec{r}_{24}\right] = 0\\
&n^2\vec{r}_3 + G\left[\left\{\frac{m_1}{r_{31}^3} + \frac{3A}{2m_3 r_{31}^5}\right\}\vec{r}_{31} + \left\{\frac{m_2}{r_{32}^3}\right\}\vec{r}_{32} + \left\{\frac{m_4}{r_{34}^3}\right\}\vec{r}_{34}\right] = 0\\
&n^2\vec{r}_4 + G\left[\left\{\frac{m_1}{r_{41}^3} + \frac{3A}{2m_4 r_{41}^5}\right\}\vec{r}_{41} + \left\{\frac{m_2}{r_{42}^3}\right\}\vec{r}_{42} + \left\{\frac{m_3}{r_{43}^3}\right\}\vec{r}_{43}\right] = 0
\end{aligned}$$

Putting the values of m_1, m_2, m_3, m_4 from (5) we get,

$$n^2\vec{r}_1+G\left[\left\{\frac{M\mu}{3\sqrt{3}R^3}+\frac{A}{3\sqrt{3}M(1-\mu)R^5}\right\}\vec{r}_{12}+\left\{\frac{M(1-3\mu)}{16R^3}+\frac{3A}{32M(1-\mu)R^5}\right\}\vec{r}_{13}+\left\{\frac{M\mu}{3\sqrt{3}R^3}+\frac{A}{3\sqrt{3}M(1-\mu)R^5}\right\}\vec{r}_{14}\right]=\hat{0}$$

$$n^2\vec{r}_2+G\left[\left\{\frac{M(1-\mu)}{6\sqrt{3}R^3}+\frac{A}{6\sqrt{3}M\mu R^5}\right\}\vec{r}_{21}+\left\{\frac{M(1-3\mu)}{2R^3}\right\}\vec{r}_{23}+\left\{\frac{M\mu}{3\sqrt{3}R^3}\right\}\vec{r}_{24}\right]=\hat{0}$$

$$n^2\vec{r}_3+G\left[\left\{\frac{M(1-\mu)}{16R^3}+\frac{3A}{32M(1-3\mu)R^5}\right\}\vec{r}_{31}+\left\{\frac{M\mu}{R^3}\right\}\vec{r}_{32}+\left\{\frac{M\mu}{R^3}\right\}\vec{r}_{34}\right]=\hat{0}$$

$$n^2\vec{r}_4+G\left[\left\{\frac{M(1-\mu)}{6\sqrt{3}R^3}+\frac{A}{6\sqrt{3}M\mu R^5}\right\}\vec{r}_{41}+\left\{\frac{M\mu}{3\sqrt{3}R^3}\right\}\vec{r}_{42}+\left\{\frac{M(1-3\mu)}{2R^3}\right\}\vec{r}_{43}\right]=\hat{0}$$

Now choosing units of masses lengths and forces in such a way that

$$m_1+m_2+m_3+m_4=M=1, 2R=1, G=1,$$

then the above equations reduced to

$$n^2\vec{r}_1+\left[\left\{\frac{8\mu}{3\sqrt{3}}+\frac{32A}{3\sqrt{3}(1-\mu)}\right\}\vec{r}_{12}+\left\{\frac{(1-3\mu)}{2}+\frac{3A}{(1-\mu)}\right\}\vec{r}_{13}+\left\{\frac{8\mu}{3\sqrt{3}}+\frac{32A}{9\sqrt{3}(1-\mu)}\right\}\vec{r}_{14}\right]=\hat{0}$$

$$n^2\vec{r}_2+\left[\left\{\frac{4(1-\mu)}{3\sqrt{3}}+\frac{16A}{3\sqrt{3}\mu}\right\}\vec{r}_{21}+\{4(1-3\mu\}\vec{r}_{23}+\left\{\frac{8\mu}{3\sqrt{3}}\right\}\vec{r}_{24}\right]=\hat{0}$$

$$n^2\vec{r}_3+\left[\left\{\frac{(1-\mu)}{2}+\frac{3A}{(1-3\mu)}\right\}\vec{r}_{31}+\{8\mu\}\vec{r}_{32}+\{8\mu\}\vec{r}_{34}\right]=\hat{0}$$

$$n^2\vec{r}_4+\left[\left\{\frac{4(1-\mu)}{3\sqrt{3}}+\frac{16A}{3\sqrt{3}\mu}\right\}\vec{r}_{41}+\left\{\frac{8\mu}{3\sqrt{3}}\right\}\vec{r}_{42}+\{4(1-3\mu\}\vec{r}_{43}\right]=\hat{0}$$

$\Rightarrow$

$$n^2\vec{r}_1+\left[\left\{\frac{8\mu}{3\sqrt{3}}+\frac{32A}{3\sqrt{3}(1-\mu)}\right\}(\vec{r}_2-\vec{r}_1)+\left\{\frac{(1-3\mu)}{2}+\frac{3A}{(1-\mu)}\right\}(\vec{r}_3-\vec{r}_1)+\left\{\frac{8\mu}{3\sqrt{3}}+\frac{32A}{9\sqrt{3}(1-\mu)}\right\}(\vec{r}_4-\vec{r}_1)\right]=\hat{0}$$

$$n^2\vec{r}_2+\left[\left\{\frac{4(1-\mu)}{3\sqrt{3}}+\frac{16A}{3\sqrt{3}\mu}\right\}(\vec{r}_1-\vec{r}_2)+\{4(1-3\mu\}(\vec{r}_3-\vec{r}_2)+\left\{\frac{8\mu}{3\sqrt{3}}\right\}(\vec{r}_4-\vec{r}_2)\right]=\hat{0}$$

$$n^2\vec{r}_3+\left[\left\{\frac{(1-\mu)}{2}+\frac{3A}{(1-3\mu)}\right\}(\vec{r}_1-\vec{r}_3)+\{8\mu\}(\vec{r}_2-\vec{r}_3)+\{8\mu\}(\vec{r}_4-\vec{r}_3)\right]=\hat{0}$$

$$n^2\vec{r}_4+\left[\left\{\frac{4(1-\mu)}{3\sqrt{3}}+\frac{16A}{3\sqrt{3}\mu}\right\}(\vec{r}_1-\vec{r}_4)+\left\{\frac{8\mu}{3\sqrt{3}}\right\}(\vec{r}_2-\vec{r}_4)+\{4(1-3\mu\}(\vec{r}_3-\vec{r}_4)\right]=\hat{0}$$

Now by arranging the above four equations in the form of linear combinations of $\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4$ one can find the equations of motion of four bodies forming kite as

$$\left.\begin{array}{l}
\left[n^2 - \frac{2}{3\sqrt{3}}(8\mu + \frac{32A}{1-\mu}) - (\frac{1-3\mu}{2} + \frac{3A}{1-\mu})\right]\vec{r}_1 + \frac{1}{3\sqrt{3}}(8\mu + \frac{32A}{1-\mu})\vec{r}_2 + (\frac{1-3\mu}{2} + \frac{3A}{1-\mu})\vec{r}_3 + \frac{1}{3\sqrt{3}}(8\mu + \frac{32A}{1-\mu})\vec{r}_4 = \hat{0} \\
\frac{4}{3\sqrt{3}}(1-\mu + \frac{4A}{\mu})\vec{r}_1 + \left[n^2 - \frac{4}{3\sqrt{3}}(1-\mu + \frac{4A}{\mu}) - 4(1-3\mu) - \frac{8\mu}{3\sqrt{3}}\right]\vec{r}_2 + 4(1-3\mu)\vec{r}_3 + \frac{8\mu}{3\sqrt{3}}\vec{r}_4 = \hat{0} \\
(\frac{1-\mu}{2} + \frac{3A}{1-3\mu})\vec{r}_1 + 8\mu\vec{r}_2 + \left[n^2 - (\frac{1-\mu}{2} + \frac{3A}{1-3\mu}) - 16\mu\right]\vec{r}_3 + 8\mu\vec{r}_4 = \hat{0} \\
\frac{4}{3\sqrt{3}}(1-\mu + \frac{4A}{\mu})\vec{r}_1 + 8\mu\vec{r}_2 + 4(1-3\mu)\vec{r}_3 + \left[n^2 - \frac{4}{3\sqrt{3}}(1-3\mu + \frac{4A}{\mu}) - 4(1-3\mu)\right]\vec{r}_4 = 0
\end{array}\right\} \quad (16)$$

5. Mean Motion of the rotating frame:

To find the mean motion 'n' of the rotating frame we have to express n in terms of other parameters. In the above four equations of (16) there are three parameters n, μ, A other than four position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4$.

Thus by eliminating $\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4$ from the above four equations we get

$$\begin{vmatrix}
a & \frac{8}{3\sqrt{3}}(\mu + \frac{4A}{1-\mu}) & (\frac{1-3\mu}{2} + \frac{3A}{1-\mu}) & \frac{8}{3\sqrt{3}}(\mu + \frac{4A}{1-\mu}) \\
\frac{4}{3\sqrt{3}}(1-\mu + \frac{4A}{\mu}) & b & 4(1-3\mu) & \frac{8\mu}{3\sqrt{3}} \\
\frac{1-\mu}{2} + \frac{3A}{1-3\mu} & 8\mu & c & 8\mu \\
\frac{4}{3\sqrt{3}}(1-\mu + \frac{4A}{\mu}) & 8\mu & 4(1-3\mu) & b
\end{vmatrix} = 0 \quad (17)$$

Where

$$a = n^2 - \frac{16}{3\sqrt{3}}(\mu + \frac{4A}{1-\mu}) - (\frac{1-3\mu}{2} + \frac{3A}{1-\mu}), \quad b = n^2 - \frac{4}{3\sqrt{3}}(1+\mu + \frac{4A}{\mu}) - 4(1-3\mu), c = n^2 - (\frac{1-\mu}{2} + \frac{3A}{1-3\mu}) - 16\mu.$$

The equation (16) gives the mean motion 'n' of the rotating (synodic) frame as a function **of** μ and A. i.e.
$n = f(\mu, A)$.

6: Exact domain of the mass parameter μ in classical case (for A=0):

If we put A=0, $\mu = 1/3$ then $m_1 = m_2 = m_4 = 1/3 \,\&\, m_3 = 0.$ Thus the cyclic kite configuration converges to the cyclic equilateral triangular configuration and from (16) we have

$$\begin{vmatrix} n^2 - \frac{16}{9\sqrt{3}} & \frac{8}{9\sqrt{3}} & 0 & \frac{8}{9\sqrt{3}} \\ \frac{8}{9\sqrt{3}} & n^2 - \frac{64}{9\sqrt{3}} & 0 & \frac{8}{9\sqrt{3}} \\ \frac{1}{3} & \frac{8}{3} & n^2 - \frac{17}{3} & \frac{8}{3} \\ \frac{8}{9\sqrt{3}} & \frac{8}{3} & 0 & n^2 - \frac{64}{9\sqrt{3}} \end{vmatrix} = 0 \qquad (17)$$

$$\Rightarrow \quad n = 0.8726844 \;,\; 1.79568456 \;,\; 2.29162458 \;,\; 2.33047614 \;.$$

But equilateral triangular configuration rotates with unit mean motion so these values of mean motion are invalid and consequently the kite configuration does not exist for μ =1/3.Thus the exact domain of μ is $(0, 3^{-1})$.

7 . The data produced to analyze the effect of oblateness on the mean motion of the rotating frame by using Python.

S/No.	μ	A	n	S/No.	μ	A	n	S/No.	μ	A	n
1	0.01	0.01	1.094904	**18**	0.01	0.18	1.530321	**35**	0.02	0.15	1.544661
2	0.01	0.02	1.141577	**19**	0.01	0.19	1.550052	**36**	0.02	0.16	1.565094
3	0.01	0.03	1.177312	**20**	0.01	0.2	1.569512	**37**	0.02	0.17	1.585149
4	0.01	0.04	1.208292	**21**	0.02	0.01	1.120484	**38**	0.02	0.18	1.604858
5	0.01	0.05	1.236655	**22**	0.02	0.02	1.183872	**39**	0.02	0.19	1.624246
6	0.01	0.06	1.263339	**23**	0.02	0.03	1.230685	**40**	0.02	0.2	1.643336
7	0.01	0.07	1.288829	**24**	0.02	0.04	1.268953	**41**	0.03	0.01	1.137155
8	0.01	0.08	1.313402	**25**	0.02	0.05	1.302219	**42**	0.03	0.02	1.212038
9	0.01	0.09	1.337233	**26**	0.02	0.06	1.332265	**43**	0.03	0.03	1.268233
10	0.01	0.1	1.360438	**27**	0.02	0.07	1.360082	**44**	0.03	0.04	1.313568
11	0.01	0.11	1.3831	**28**	0.02	0.08	1.386267	**45**	0.03	0.05	1.352094
12	0.01	0.12	1.40528	**29**	0.02	0.09	1.411199	**46**	0.03	0.06	1.386074
13	0.01	0.13	1.427024	**30**	0.02	0.1	1.435133	**47**	0.03	0.07	1.416857
14	0.01	0.14	1.448371	**31**	0.02	0.11	1.458247	**48**	0.03	0.08	1.445293
15	0.01	0.15	1.46935	**32**	0.02	0.12	1.480672	**49**	0.03	0.09	1.471941
16	0.01	0.16	1.489988	**33**	0.02	0.13	1.502502	**50**	0.03	0.1	1.497184
17	0.01	0.17	1.510305	**34**	0.02	0.14	1.523813	**51**	0.03	0.11	1.521293

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S/No.	μ	A	n	S/No.	μ	A	n	S/No.	μ	A	n
52	0.03	0.12	1.544466	**90**	0.05	0.1	1.598497	**127**	0.07	0.07	1.575389
53	0.03	0.13	1.566852	**91**	0.05	0.11	1.625267	**128**	0.07	0.08	1.61423
54	0.03	0.14	1.588563	**92**	0.05	0.12	1.650594	**129**	0.07	0.09	1.649398
55	0.03	0.15	1.609687	**93**	0.05	0.13	1.674722	**130**	0.07	0.1	1.681628
56	0.03	0.16	1.630295	**94**	0.05	0.14	1.697839	**131**	0.07	0.11	1.711477
57	0.03	0.17	1.650442	**95**	0.05	0.15	1.720093	**132**	0.07	0.12	1.739373
58	0.03	0.18	1.670176	**96**	0.05	0.16	1.741602	**133**	0.07	0.13	1.765648
59	0.03	0.19	1.689534	**97**	0.05	0.17	1.762461	**134**	0.07	0.14	1.790563
60	0.03	0.2	1.708548	**98**	0.05	0.18	1.782747	**135**	0.07	0.15	1.814324
61	0.04	0.01	1.150072	**99**	0.05	0.19	1.802523	**136**	0.07	0.16	1.837096
62	0.04	0.02	1.233274	**100**	0.05	0.2	1.821842	**137**	0.07	0.17	1.859013
63	0.04	0.03	1.297213	**101**	0.06	0.01	1.170298	**138**	0.07	0.18	1.880183
64	0.04	0.04	1.348933	**102**	0.06	0.02	1.264858	**139**	0.07	0.19	1.900698
65	0.04	0.05	1.392536	**103**	0.06	0.03	1.340777	**140**	0.07	0.2	1.920631
66	0.04	0.06	1.430515	**104**	0.06	0.04	1.40338	**141**	0.08	0.01	1.186299
67	0.04	0.07	1.464449	**105**	0.06	0.05	1.456288	**142**	0.08	0.02	1.288356
68	0.04	0.08	1.495376	**106**	0.06	0.06	1.502015	**143**	0.08	0.03	1.373163
69	0.04	0.09	1.524002	**107**	0.06	0.07	1.54233	**144**	0.08	0.04	1.444697
70	0.04	0.1	1.55082	**108**	0.06	0.08	1.578491	**145**	0.08	0.05	1.505885
71	0.04	0.11	1.576187	**109**	0.06	0.09	1.61141	**146**	0.08	0.06	1.558951
72	0.04	0.12	1.600364	**110**	0.06	0.1	1.641756	**147**	0.08	0.07	1.605604
73	0.04	0.13	1.623549	**111**	0.06	0.11	1.670028	**148**	0.08	0.08	1.647163
74	0.04	0.14	1.645894	**112**	0.06	0.12	1.696601	**149**	0.08	0.09	1.684642
75	0.04	0.15	1.667516	**113**	0.06	0.13	1.721766	**150**	0.08	0.1	1.718827
76	0.04	0.16	1.68851	**114**	0.06	0.14	1.745747	**151**	0.08	0.11	1.750326
77	0.04	0.17	1.708952	**115**	0.06	0.15	1.768722	**152**	0.08	0.12	1.779614
78	0.04	0.18	1.728902	**116**	0.06	0.16	1.790831	**153**	0.08	0.13	1.807065
79	0.04	0.19	1.748412	**117**	0.06	0.17	1.812189	**154**	0.08	0.14	1.832974
80	0.04	0.2	1.767524	**117**	0.06	0.17	1.812189	**155**	0.08	0.15	1.857577
81	0.05	0.01	1.160882	**118**	0.06	0.18	1.832889	**156**	0.08	0.16	1.881063
82	0.05	0.02	1.250422	**119**	0.06	0.19	1.853008	**157**	0.08	0.17	1.903586
83	0.05	0.03	1.320836	**120**	0.06	0.2	1.872608	**158**	0.08	0.18	1.925272
84	0.05	0.04	1.378281	**121**	0.07	0.01	1.178694	**159**	0.08	0.19	1.946224
85	0.05	0.05	1.426677	**122**	0.07	0.02	1.277346	**160**	0.08	0.2	1.966529
86	0.05	0.06	1.468583	**123**	0.07	0.03	1.358013	**161**	0.09	0.01	1.193269
87	0.05	0.07	1.505708	**124**	0.07	0.04	1.42529	**162**	0.09	0.02	1.298203
88	0.05	0.08	1.539227	**125**	0.07	0.05	1.482454	**163**	0.09	0.03	1.386649
89	0.05	0.09	1.56996	**126**	0.07	0.06	1.531899	**164**	0.09	0.04	1.462074

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S/No.	μ	A	n	S/No.	μ	A	n	S/No.	μ	A	n
165	0.09	0.05	1.527069	**203**	0.11	0.03	1.409781	**241**	0.13	0.01	1.216902
166	0.09	0.06	1.583665	**204**	0.11	0.04	1.492027	**242**	0.13	0.02	1.329853
167	0.09	0.07	1.633476	**205**	0.11	0.05	1.564031	**243**	0.13	0.03	1.429154
168	0.09	0.08	1.677795	**206**	0.11	0.06	1.627409	**244**	0.13	0.04	1.517101
169	0.09	0.09	1.717653	**207**	0.11	0.07	1.683516	**245**	0.13	0.05	1.595319
170	0.09	0.1	1.753869	**208**	0.11	0.08	1.733504	**246**	0.13	0.06	1.66507
171	0.09	0.11	1.787095	**209**	0.11	0.09	1.778356	**247**	0.13	0.07	1.727406
172	0.09	0.12	1.817849	**210**	0.11	0.1	1.818909	**248**	0.13	0.08	1.783257
173	0.09	0.13	1.846541	**211**	0.11	0.11	1.855862	**249**	0.13	0.09	1.833466
174	0.09	0.14	1.873504	**212**	0.11	0.12	1.889798	**250**	0.13	0.1	1.878796
175	0.09	0.15	1.899001	**213**	0.11	0.13	1.921201	**251**	0.13	0.11	1.919934
176	0.09	0.16	1.923247	**214**	0.11	0.14	1.950468	**252**	0.13	0.12	1.957491
177	0.09	0.17	1.946418	**215**	0.11	0.15	1.977923	**253**	0.13	0.13	1.991995
178	0.09	0.18	1.968656	**216**	0.11	0.16	2.003836	**254**	0.13	0.14	2.023904
179	0.09	0.19	1.990081	**217**	0.11	0.17	2.028428	**255**	0.13	0.15	2.053604
180	0.09	0.2	2.01079	**218**	0.11	0.18	2.051879	**256**	0.13	0.16	2.081421
181	0.1	0.01	1.199724	**219**	0.11	0.19	2.074341	**257**	0.13	0.17	2.107628
182	0.1	0.02	1.307116	**220**	0.11	0.2	2.09594	**258**	0.13	0.18	2.132452
183	0.1	0.03	1.398777	**221**	0.12	0.01	1.211462	**259**	0.13	0.19	2.156084
184	0.1	0.04	1.477763	**222**	0.12	0.02	1.322809	**260**	0.13	0.2	2.178682
185	0.1	0.05	1.546362	**223**	0.12	0.03	1.419853	**261**	0.14	0.01	1.222149
186	0.1	0.06	1.606396	**224**	0.12	0.04	1.505079	**262**	0.14	0.02	1.336508
187	0.1	0.07	1.659359	**225**	0.12	0.05	1.580289	**263**	0.14	0.03	1.437824
188	0.1	0.08	1.706487	**226**	0.12	0.06	1.646911	**264**	0.14	0.04	1.528252
189	0.1	0.09	1.7488	**227**	0.12	0.07	1.706146	**265**	0.14	0.05	1.609282
190	0.1	0.1	1.787135	**228**	0.12	0.08	1.759043	**266**	0.14	0.06	1.682035
191	0.1	0.11	1.822175	**229**	0.12	0.09	1.806527	**267**	0.14	0.07	1.747428
192	0.1	0.12	1.854473	**230**	0.12	0.1	1.849407	**268**	0.14	0.08	1.806265
193	0.1	0.13	1.88448	**231**	0.12	0.11	1.888388	**269**	0.14	0.09	1.859291
194	0.1	0.14	1.912558	**232**	0.12	0.12	1.924071	**270**	0.14	0.1	1.9072
195	0.1	0.15	1.939004	**233**	0.12	0.13	1.956968	**271**	0.14	0.11	1.950643
196	0.1	0.16	1.964058	**234**	0.12	0.14	1.987505	**272**	0.14	0.12	1.99022
197	0.1	0.17	1.987918	**235**	0.12	0.15	2.016041	**273**	0.14	0.13	2.026469
198	0.1	0.18	2.010745	**236**	0.12	0.16	2.042871	**274**	0.14	0.14	2.05987
199	0.1	0.19	2.032673	**237**	0.12	0.17	2.068242	**275**	0.14	0.15	2.090835
200	0.1	0.2	2.053813	**238**	0.12	0.18	2.092358	**276**	0.14	0.16	2.119722
201	0.11	0.01	1.205761	**239**	0.12	0.19	2.115386	**277**	0.14	0.17	2.146831
202	0.11	0.02	1.315271	**240**	0.12	0.2	2.13747	**278**	0.14	0.18	2.172418

S/No.	μ	A	n
279	0.14	0.19	2.196695
280	0.14	0.2	2.219841
281	0.15	0.01	1.227266
282	0.15	0.02	1.342872
283	0.15	0.03	1.445993
284	0.15	0.04	1.538682
285	0.15	0.05	1.622331
286	0.15	0.06	1.69795
287	0.15	0.07	1.766335
288	0.15	0.08	1.828174
289	0.15	0.09	1.884099
290	0.15	0.1	1.93472
291	0.15	0.11	1.980628
292	0.15	0.12	2.022393
293	0.15	0.13	2.060548
294	0.15	0.14	2.095584
295	0.15	0.15	2.12794
296	0.15	0.16	2.158
297	0.15	0.17	2.186095
298	0.15	0.18	2.21251
299	0.15	0.19	2.237482
300	0.15	0.2	2.261215
301	0.16	0.01	1.232311
302	0.16	0.02	1.349034
303	0.16	0.03	1.453781
304	0.16	0.04	1.548535
305	0.16	0.05	1.634618
306	0.16	0.06	1.712959
307	0.16	0.07	1.784258
308	0.16	0.08	1.849093
309	0.16	0.09	1.907986
310	0.16	0.1	1.961446
311	0.16	0.11	2.009986
312	0.16	0.12	2.054125
313	0.16	0.13	2.094372
314	0.16	0.14	2.131218
315	0.16	0.15	2.165115
316	0.16	0.16	2.196476
317	0.16	0.17	2.225662
318	0.16	0.18	2.252987
319	0.16	0.19	2.27872
320	0.16	0.2	2.303089
321	0.17	0.01	1.237342
322	0.17	0.02	1.35508
323	0.17	0.03	1.461305
324	0.17	0.04	1.557954
325	0.17	0.05	1.646303
326	0.17	0.06	1.727226
327	0.17	0.07	1.80135
328	0.17	0.08	1.869158
329	0.17	0.09	1.931068
330	0.17	0.1	1.987483
331	0.17	0.11	2.038821
332	0.17	0.12	2.08553
333	0.17	0.13	2.128076
334	0.17	0.14	2.166931
335	0.17	0.15	2.202555
336	0.17	0.16	2.235376
337	0.17	0.17	2.265785
338	0.17	0.18	2.294129
339	0.17	0.19	2.320707
340	0.17	0.2	2.345778
341	0.18	0.01	1.242415
342	0.18	0.02	1.361091
343	0.18	0.03	1.46868
344	0.18	0.04	1.567086
345	0.18	0.05	1.657561
346	0.18	0.06	1.740946
347	0.18	0.07	1.817815
348	0.18	0.08	1.888575
349	0.18	0.09	1.953544
350	0.18	0.1	2.013016
351	0.18	0.11	2.067306
352	0.18	0.12	2.116773
353	0.18	0.13	2.161826
354	0.18	0.14	2.202904
355	0.18	0.15	2.24046
356	0.18	0.16	2.274934
357	0.18	0.17	2.306738
358	0.18	0.18	2.336247
359	0.18	0.19	2.363791
360	0.18	0.2	2.389659
361	0.19	0.01	1.247584
362	0.19	0.02	1.367152
363	0.19	0.03	1.476024
364	0.19	0.04	1.576089
365	0.19	0.05	1.668595
366	0.19	0.06	1.754369
367	0.19	0.07	1.833953
368	0.19	0.08	1.907692
369	0.19	0.09	1.975809
370	0.19	0.1	2.038474
371	0.19	0.11	2.095872
372	0.19	0.12	2.148258
373	0.19	0.13	2.195972
374	0.19	0.14	2.239432
375	0.19	0.15	2.279093
376	0.19	0.16	2.315411
377	0.19	0.17	2.348816
378	0.19	0.18	2.379699
379	0.19	0.19	2.408408
380	0.19	0.2	2.43525
381	0.2	0.01	1.252907
382	0.2	0.02	1.373348
383	0.2	0.03	1.483463
384	0.2	0.04	1.585144
385	0.2	0.05	1.679655
386	0.2	0.06	1.767842
387	0.2	0.07	1.850249
388	0.2	0.08	1.927186
389	0.2	0.09	1.998781
390	0.2	0.1	2.065037
391	0.2	0.11	2.125914
392	0.2	0.12	2.181445

S/No.	μ	A	n	S/No.	μ	A	n
393	0.2	0.13	2.231847	**397**	0.2	0.17	2.392346
394	0.2	0.14	2.27755	**398**	0.2	0.18	2.424924
395	0.2	0.15	2.319139	**399**	0.2	0.19	2.455271
396	0.2	0.16	2.357225	**400**	0.2	0.2	2.483627

8 .Graphical study of the effect of oblateness on the mean motion of the rotating frame

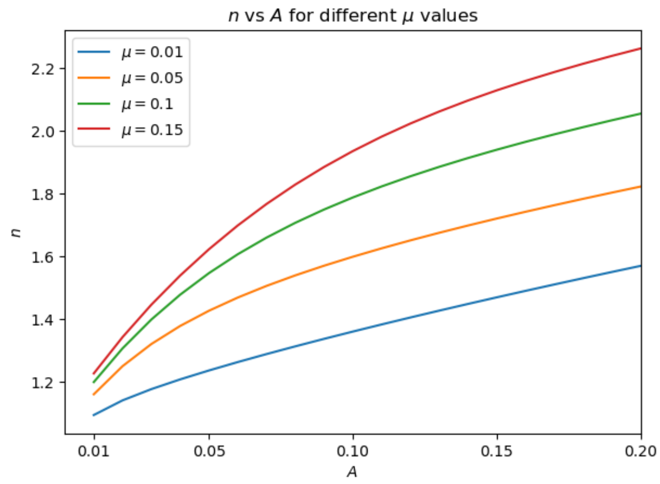

Fig-2 Graph of A **vs** n **for different values** μ

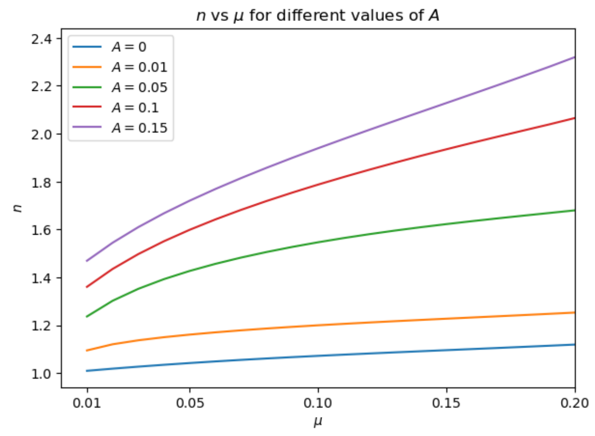

Fig-3 Graph of μ **vs** n **for different values of** A

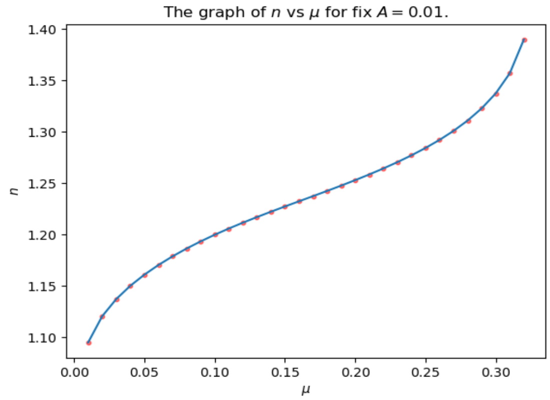

Fig-4 (Graph of μ vs n for a fixed value of A)

9.Summary

In the theorem of section 2.1 two masses situated on the opposite sides of the axis of symmetry are shown to be equal and supposed to be the mass ratio μ of either of the equal masses to the total mass $M = m_1 + m_2 + m_3 + m_4$ of kite configuration .Later on other two unequal masses lying on the axis of symmetry have been represented explicitly . Not only that ,in section 2.2 ,it has been proved independently that for a cyclic kite configuration $P_1P_2P_3P_4$ (in figure-1) the diagonals P_1OP_3 and P_2P_4 are perpendicular to each other and at least one of them is the axis of symmetry of the kite. In our work; the diagonal P_1OP_3 is supposed to be the axis of symmetry and masses m_2 & m_4 lying on the opposite side of the axis of symmetry to be equal that is $m_2 = m_4 = M\mu$, $m_1 = M(1-\mu)/2$, $m_3 = M(1-3\mu)/2$ .In section 3 the equations of motion of each mass of the kite relative to the other three bodies have been derived ,then by eliminating the position vectors of the four bodies , the equations for the mean motion was established in equation (15). By using Mathematica 11the mean motion n has been expressed in terms of μ & A i.e.; $n = f(\mu, A)$..In section 3.1 we have analytically defined the exact domain of the mass parameter μ as $(0, 3^{-1})$ in classical case .In section -4 we have tabulated the data for μ, n, A for graphical analysis ..In figure -2 we have shown that mean motion n increases with the increase of mass parameter μ when $A = 0$ .In figures -3 & 4 keeping μ fixed, it has been shown that n increases with the increase of A but n becomes imaginary if $\mu \geq 0.2$ whereas as analytically $0 < \mu < 1/3$. When $\mu > 0.2$ is taken ,the graph of n versus μ becomes discontinuous .Thus in the present paper the effect of oblateness have been discussed analytically and graphically both on the mean motion of the rotating frame .For further study of the satellite motion in the gravitational field of the kite configuration it is necessary to find n .

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