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Channel Replacement Policy Using Subsethood and Supersethood Concepts under Type-2 Fuzzy Environment

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Abstract:- Decision theory is a branch of operations research which is used to take optimal decisions and so to arrive at optimal solution for that particular problem. Replacement theory deals with choosing optimal replacement policy for any equipment, in particular, in this paper, optimal replacement policy for a communication channel using subsethood and supersethood entropy concepts is discussed and is illustrated with a numerical example.

Keywords*: CHANNEL REPLACEMENT, FUZZY ENVIRONMENT, optimal replacement, operation Research*.

1. Introduction

The term "operation Research" may be defined as the application of scientific methods to solve problems that arise from operations that involve men, machines, materials, etc,[7]. In general, Operations Research utilizes the knowledge and skills of inter-disciplinary fields to obtain an optimal solution for the problem under consideration. But, in most of the cases, the knowledge about the data of the problem under consideration is uncertain.

Decision theory is a branch of operations research that helps the decision maker to choose an optimal decision-making process and hence to arrive at the optimal solution for the problem condition of uncertainty. The uncertainty in the data can be effectively handled by fuzzy theory. Fuzzy set theory is a research approach that could deal effectively with problems whose data are ambiguous or subjective [10].

Replacement theory deals with the problem of replacement of machines, men, materials, spare parts, etc. the replacement problem can be broadly classified into the following four categories: (i) Replacement when the item fails suddenly, (ii) Replacement when the items deteriorate gradually, (iii)Replacement when the items become outdated after some period of time, (iv) replacement when the maintenance cost of the items increases after some period of time. There are two types of replacement policies that can be adopted depending upon the situation under consideration, say, Individual replacement and group replacement.

In set theory, a subset of a set S is any set R such that every element of R is also an element of A. A Superset of a set R is a set S such that there are few elements in S which are not in R and also all the elements in R would be in S. For type -2 fuzzy sets, the subsethood and supersethood are defined based on the membership function of the fuzzy sets, subsethood and supersethood concepts can be applied to obtain the optimal replacement policy for subchannel stop working in a communication system.

In information theory, the information that has to be transmitted from one place to another is passed through the transmission channel. This transmission channel can be a single channel or a series of channels. Here, five different channels are considered which are fitted in a hypercube in a suitable manner such that the received information/ messages is/are transmitted through the channel successfully.

It is to be analyzed whether the individual replacement policy or group replacement is to be adopted in a particular situation where the middlemost channel has stopped working.

2. Preliminaries

2.1. Fuzzy Set

If R is a collection of objects denoted generically by r, then a fuzzy set $\tilde{s}$ in $\mathbb{R}$ is a set of ordered pair,

$\tilde{s} = \{(r, \mu_{\tilde{s}}(r) \mid r \in \mathbb{R})\}$

Where $\mu\tilde{s}(r)$ is called the membership function or grade of membership or degree of compatibility or degree of truth of r in $\tilde{s}$ that maps $\mathbb{R}$ to the membership space M [12].

2.2. Type-2 Fuzzy Set

A *type-2* fuzzy set [12] is a fuzzy set where membership values are *type-1* fuzzy sets on the closed interval [0,1].

2.3. Type-m Fuzzy Set

A *type-m* fuzzy set [12] is a fuzzy set whose membership values are *type-m*-1, m>1 fuzzy sets on [0, 1].

2.4. Replacement Policy

The replacement problem is to find when an item should be replaced by another one. There are two types of replacement policies. [6]

2.4.1. Individual Replacement Policy

In individual replacement policy, an item is replaced when it fails or when the maintenance cost is very high. [4]

2.4.2. Group Replacement Policy

In group replacement policy, the items are replaced in the group periodically irrespective of whether the items have failed or not. [4]

2.5. Sub-Set

"R" is a subset of "S", denoted by $R \subset S$, if and only if every element in "R" is an element of "S". The power set 2^s contains all the subsets of "S". Hence, "R" Is a subset of "S" if and only if "R" belongs to the power set of "S", (i.e), $R \subset S$ if and only if $R \in S$

2.6. Membership Function of Fuzzy Sub-Set

Let μR (x) and $\mu\tilde{S}$ (x) be the membership functions of the two fuzzy subsets "R" and "S" respectively, where μR (x): $X \rightarrow [0,1]$ and $\mu\tilde{S}$ (x): $X \rightarrow [0,1]$. "R" is a fuzzy-subset of "S" if and only if there is no element x that belongs to "R" but not to "S". The membership function is defined as $R \subset S$ if and only if μR $(x) \leq \mu\tilde{S}$ (x), for all x.

This relation for fuzzy-set containment is called as dominated membership function relationship.

3. Illustrations

Illustration 3.1:

Let $A = \{(a1/0.3,\ 0.3/0.3),\ (a2/0.5,\ 0.5/0.6)\}$ and $B = \{(b1/0.4,\ 0.4/0.5),\ (b2/0.6,\ 0.6/0.7)\}$ be two fuzzy type-2 sub-sets. Then clearly, A is a of sub-set (or) B is a super-set of A. (i.e) $A \subset B$.

Illustration 3.2:

Let $C = \{(c1/0.2, 0.2/0.8), (c2/0.4, 0.4/0.2)\}$ and $D = \{(d1/0.7, 0.7/0.2), (d2/0.2, 0.2/0.8)\}$ be two fuzzy type-2 sub-sets. Then clearly, neither C is a sub-set of D nor D is a sub-set of C. Here, the dominated membership function relationship does not hold.

4. Channel Replacement using Subsethood and Supersethood of Unit Hypercube

4.1 Subsethood and Supersethood of Type-2 Fuzzy Set

In set theory, the set "R" is said to be a subset of a set "S" if every element present in "R" is also present in "S". Moreover, if there are a few other elements that are present in "S" but not in "R", then "R" is a strict subset of "S", or, "S" is a strict super set of" R".

In fuzzy set theory, the subset and superset are defined based on the membership function of the sets which are under consideration. For instance, consider three sets L, M, N with membership functions $\tilde{\mu}L(x)$, $\tilde{\mu}M(x)$, $\tilde{\mu}N(x)$ whose values always lies in the closed interval [0,1].

Illustration 3.1.1: If $(L/0.3)$, $(M/0.6)$, $(N/0.7)$, then clearly, L is a subset of M and L is a subset of N. M is a subset of N but M is a superset of L. N is a superset of L and also N is a superset of L. Diagrammatically,

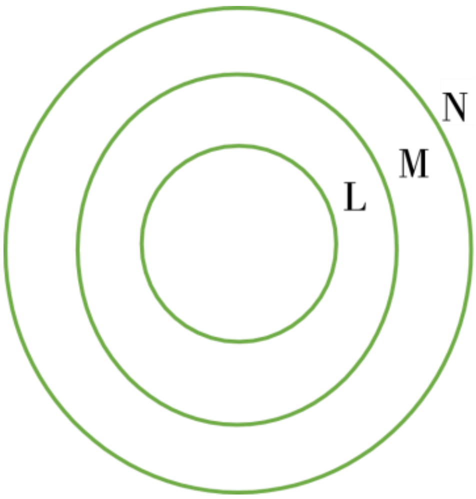

Figure. 1 Superset

We know that, A type-2 fuzzy set is an extension of fuzzy set. To be more precise, A type-2 fuzzy set is a Fuzzy set whose membership values are fuzzy set (type-1 fuzzy set) on the closed interval [0,1].

Definition 3.1.1: Dominated Membership Function Relationship

$$R = \left(\frac{\quad}{\mu_R(x)}, \frac{\mu_R(x)}{\mu_R(x)}\right) \qquad S = \left(\frac{\quad}{\mu_S(x)}, \frac{\mu_{\tilde{S}}(x)}{\mu_S(x)}\right)$$

Consider two fuzzy type-2 sets R = and , then R is called subset of S if $\mu_{\mathbb{R}}(x) \leq \mu\tilde{S}(x)$ for all "x". Also, here S is called Superset of R.

5. Unit Hypercube

In geometry, a hypercube [2] is defined as an n-dimensional analogue of a square and a cube. In particular, a unit hypercube is a hypercube whose side has length one unit. In fuzzy theory, theset of all fuzzy subsets equals to the unit hypercube, which is defined by I^n In and is defined as $I^n = ([0, 1])$.

Clearly, the vertices of the unit hypercube is non fuzzy in nature, (i.e), non- fuzzy sets. Also, the midpoint of the unit hypercube I^n is maximally fuzzy as all the membership values of the midpoint of the unit hypercube I^n is exactly equal to ½. The unit hypercube In could be sliced into 2^n hyper-squares by extending the slides of the hyperplanes from the midpoint to the edges of the unit hypercube.

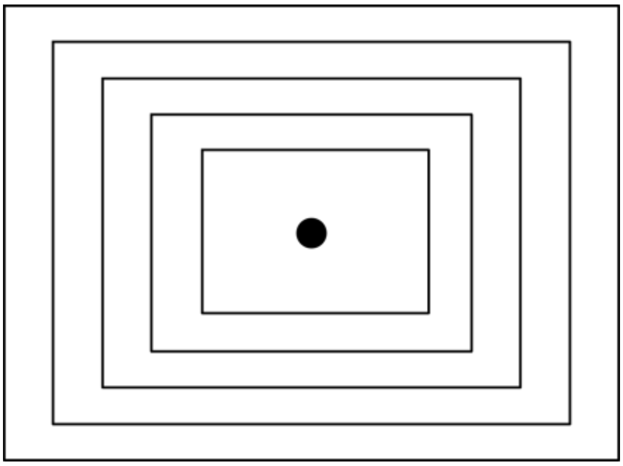

Figure. 2 Hyper-squares

The hyper-square interiors correspond to the 2^n cases where $\mu\tilde{R}\ (xi) > \mu\tilde{S}\ (xi)$ for fixed S and arbitrary R. The 2^n hyper-squares are classified as pure membership domination or mixed membership domination. In the case of pure membership domination, either $\mu\tilde{R}\ (xi) < \mu\tilde{S}\ (xi)$ or $\mu\tilde{R}\ (xi) > \mu\tilde{S}\ (xi)$ holds in the hyper-square interior for all x. In case of the mixed membership domination, $\mu\tilde{R}\ (xi) < \mu\tilde{S}\ (xi)$ holds for some of the coordinates xi and $\mu R\ (xi) > \mu\tilde{S}\ (xi)$ holds for the remaining coordinates xj in the interior of the hyper-squares for xi and xj.

5.1. Transmission Of Messages Through Channel

When information or message has to be sent from one place to another place, the information or message is passed through a communication channel. Here, the channel which is used to transmit the information would be considered as a unit hypercube.

The communication channel "C" which is considered a unit hypercube could be sliced into many hyper square. In this paper, we shall slice the unit hypercube as five fuzzy subsets namely, P, Q, R, S,

T. these five fuzzy subchannels P, Q, R, S, and T have their membership values in the closed interval [0,1.9], [2.0,3.9], [4.0,5.9], [6.0,7.9], [0.8,1.0] respectively. The particular message (or) information passed through the communication channel would have some specific membership value. Depending on the membership value of the message transmitted, the message would be transmitted through P or Q or R or S or T. For instance, if the membership value if the message which is to be transmitted us 2.33, then the message would be transmitted through "Q."

The expected value of information can also be interpreted as the expected amount of information needed to determine which event of the set under consideration has occurred.

Definition 5.1.1: Entropy Function Fro Type-2 Fuzzy Set

For a *type-2* fuzzy set whose membership function is *type-1* fuzzy set, whose membership function is denoted by μri for every $ri \in R$ is defined as:

$$H(R) = -\sum_{i=1}^{n}(\mu_{(r_i)}).(\log(\mu_{r_i}))\text{ , where } r_i \in R$$

5.2 Replacement Policy for the Channel:

The channel under consideration "C" has five different fuzzy subchannels P, Q, R, S, T such that P$\subset Q \subset R \subset S \subset T$. The replacement problem deals with the replacement of part/ parts/ whole of the channel that is used to transmit information. There are two replacement policies available: individual part replacement policy or group replacement policy. Either of the abovementioned two replacement policies could be adopted depending upon the situation.

Individual part replacement policy could be adopted when one among P, Q, R, S, T Stops working suddenly but the information transmitted through the channel "C" is not much affected. A group replacement policy could be adopted when one or more among P, Q, R, S, T stops working suddenly or the efficiency the, deteriorates rapidly as time period increases and the information transmitted through channel "C" is not properly transmitted.

6. Channel Replacement using Subsethood and Supersethood Concept

6.1. Illustration

Let us consider a channel "C" which is made up of series of five sub-channels, say, P, Q, R, S, T, whose membership values are interval-valued: 0.0 to 0.19, 0.20 to0.39, 0.40 to 0.59, 0.60 to 0.79, 0.80 to1.0 respectively. The middle-most sub-channel R whose membership value [0.40,0.59] has stopped working. Assume that the message transmitted through the channel "C" has a membership value of 0.5.six messages, say, $m1$, $m2$, $m3$, $m4$, $m5$, $m6$ was observed to show the following data,

{($m1/P$, P/0.19), ($m2/S$, S/0.72), ($m3/T$, T/0.9) ($m4/Q$, Q/0.23), ($m5/P$, P/0.11), ($m6/Q$, Q/0.21)}

Check whether the messages was transmitted without channel R as the message has a membership value 0.5. which type of replacement policy is suggested to transmit the message the channel C.

Solution:

Given that the most appropriate membership value for perfect transmission of the messages 0.5 and also given that the sub-channel R has stopped working. Clearly, the membership value 0.5 which belongs to the sub-channel R. But channel R cannot transmit the message and so the message has been transmitted in the following manner:

The messages $m1$ and $m5$ were transmitted by the sub-channel "P". The messages $m4$ *and* $m6$ were transmitted by the sub-channel "Q". The message $m2$ was transmitted by the sub-channel "S"

The message $m3$ was transmitted by the sub-channel "T"

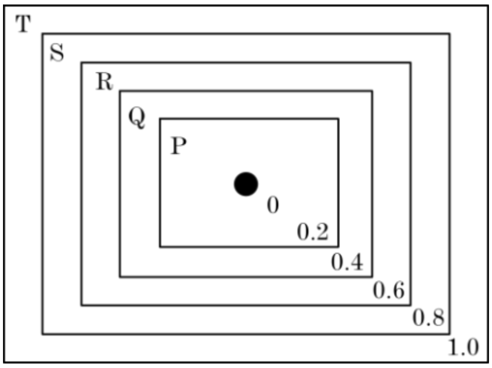

Figure. 3 Channel "C"

Entropy for Each Sub-Channels:

$$(P, \mu_{\tilde{p}}(x)) = (P, \sum_{i=1}^{2} \mu_{\tilde{p}}(x_i))$$
$$= (P, (0.19 + 0.11))$$
$$\Longrightarrow (p, \mu_{\tilde{p}}(x)) = (P, 0.30)$$
$$\therefore E(P) = \mu_{\tilde{p}}(x) * \log(\mu_{\tilde{p}}(x))$$
$$= 0.30 * \log(0.30)$$
$$= 0.30 * 0.09$$
$$\Longrightarrow E(P) = 0.027$$

$$(Q, \mu_Q(x)) = (Q, \sum_{i=1}^{2} \mu_Q(x_i))$$
$$= (Q, (0.21 + 0.23))$$
$$\Longrightarrow (Q, \mu_Q(x)) = (Q,\ 0.44)$$
$$\therefore E(Q) = \mu_Q(x) * \log(\mu_Q(x))$$
$$= 0.44 * \log(0.44)$$
$$= 0.44 * 0.132$$
$$\Longrightarrow E(Q) = 0.0580$$

$$(S, \mu_S(x)) = (S, \sum_{i=1}^{2} \mu_{QS}(x_i))$$
$$\Longrightarrow (S, \mu_S(x)) = (S,\ 0.72)$$
$$\therefore E(S) = \mu_S(x) * \log(\mu_S(x))$$
$$= 0.72 * \log(0.72)$$
$$= 0.72 * 0.3168$$
$$\Longrightarrow E(Q) = 0.2280$$

$$(T, \mu_T(x)) = (T, \sum_{i=1}^{2} \mu_T(x_i))$$
$$\Longrightarrow (T, \mu_T(x)) = (T, 0.9)$$
$$\therefore (T) = \mu_T(x) * \log(\mu_T(x))$$
$$= 0.9 * \log(0.9)$$
$$= 0.9 * 0.2851$$
$$\Longrightarrow E(T) = 0.2566$$

The entropy of the channel C using the sub-channel P, Q, S, T, is,

$E(C) = E(P) + E(Q) + E(S) + E(T)$

$= 0.027 + 0.0580 + 0.2280 + 0.2566$

Clearly, $(P) < (Q) < E(S) < E(T)$.

$\Longrightarrow \mu\tilde{P}(x) < \mu\tilde{Q}\ (x) < \mu\tilde{R}\ (x) < \mu\tilde{T}\ (x)$

Hence, $P \subset Q \subset S \subset T$. Also, $\mathrm{E}(C) = 0.5696 \cong 0.5$

Hence, the message has been transmitted with very small error.

We know that "It is possible to transmit information through a channel at any rate less than the channel capacity with an arbitrarily small probability of error."

For further transmission of messages through the channel C, it is suggested to adopt an individual replacement policy. Hence, it is suggested to replace the sub-channel R alone.

7. Conclusion

A communication channel "C" is considered as a unit hypercube which is sliced into five sub channels, wherein one among them stops working suddenly in an unexpected manner. It has been inspected whether the information that was passed through the communication channel "C" was successfully transmitted through the neighboring sub-channels using subsethood and supersethood concepts even though the middlemost sub-channel stopped working. The inspection can be extended to any one of the sub channels using subsethood and supersethood concepts. Communication helps to increase workplace productivity and creativity; thus the above-mentioned communication channel model would be applied to many mediums of communication channels, namely, telephone, video, and radio communication.

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