Application of Group Method of Data Handling Methodology for the Future Prediction of Birth Rate: A Case Study of India

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Abstract: The present study aimed at comparing the group method of data handling type artificial neural network (GMDH-ANN) model with several time series models, namely the moving average (MA), double exponential smoothing (DES), autoregressive (AR) and autoregressive integrated moving average (ARIMA) for the future prediction of India's birth rate. In this study, time series data concerning the birth rate was collected from the period 1995–96 to 2019–2020. The coefficient of determination (R^2), mean absolute error (MAE), mean absolute percentage error (MAPE), mean squared error (MSE) and root mean square error (RMSE) have been used to compare the performance of various considered models. Based on these criteria's GMDH-type ANN model performs better than other conventional statistical models. Thus, the GMDH-type ANN model was used for the future prediction of the birth rate over the next 20 years. The work done in this study will be immensely useful for the government in allocating resources and planning the future for children's services that are based on the expected fertility rate.

Keywords: *Birth Rate, MA, Holt's model, AR, ARIMA, GMDH-type ANN.*

1. Introduction

Fertility is one of the most important demographic characteristics of population growth [8, 39, 41]. The fertility rate is used to estimate the birth rate in a population. They are essential in determining a population's growth rate as well as its age structure [45]. The fertility rate is a quantitative figure based on demographic characteristics that indicate how many children a woman will have in her lifetime. Academics, legislators, national international organizations and demographers have been modeling and predicting demographic factors to execute proactive policy and planning. Population predictions are very useful to governments for predicting population variation over time [43].

Population fertility (measured in terms of birth rate) introduces analytical techniques for evaluating changes in the birth rate in India between 1985 and 2007 using data from the Sample Registration System. Which shows a nearly monotonic decline in the birth rate in India [10]. Thus, he calculated that the birth rate in the Indian states decreases as socio-economic development increases. The higher birth rates throughout the states have a negative impact on overall socio-economic development like modernization, health, education, and family planning [7].

A significant expenditure is necessary in India to fully prepare for child survival programs. Given India's present economic position, reliable future prediction of childhood fertility will guide the appropriate use of the country's scarce health resources [40]. On that basis, India needs an accurate modeling technique to improve childhood fertility estimates [6]. Given the applicability of classic time series models for future prediction birth rates, the research will be very valuable for the government in allocating resources and planning future children's services based on the predicted fertility rate. The argument is that it is difficult for researchers to choose adequate time series modeling techniques that can discover non-linear trends in fertility rates [2, 15, 29]. The moving averages methodology was used to analyze the monthly district birth patterns of many regions. These moving averages were specified in cycles of 12 months [37]. Predictions of the total and birth prevalence are calculated yearly and presented as 3-year moving averages. Using Holt's method, the life expectancy at birth can be predicted for 2021 to 2030 [4]. The outcomes of Holt's method of predicting future trends in adolescent fertility in Sri Lanka will probably reveal the future burden of adolescent births [35]. An analysis and prediction of a time-dependent variable's behavior using a time series model known as an AR model is a frequent research technique. An AR model might be used in the study of birth rates to analyze historical patterns and predict future trends [12]. For the future prediction of birth rates, several researchers examined different ARIMA models [19, 20, 21].

A series of studies have explored the application and enhancement of the GMDH-type ANN [27] introduced a dynamic GMDH-type ANN for system identification, while [18] proposed a revised GMDH-type ANN using principal component-regression analysis for nonlinear system identification. [28] demonstrated the effectiveness of GMDH in software reliability prediction, outperforming other techniques. [3] further improved the GMDH technique by replacing basic polynomial functions with popular machine learning models, such as support vector regression and random forest, to enhance its complexity modeling ability. These studies collectively highlight the versatility and potential of GMDH-type ANN in various applications. Since it is important to provide an accurate under-five mortality rate (U5MR) for Nigeria in the Sustainable Development Goal (SDG), the goal of this work was to model long-term U5MR using GMDH-type ANN and compare the predictions with ARIMA and Holt's models, two of the most widely used conventional statistical methods [1]. Some researchers compared the ARIMA and Holt's approaches with the GMDH-type ANN along with additional techniques [25] and they concluded that the predictive power of the GMDH-type ANN technique is greater than that of other predictive models.

This research shows that for future prediction of the birth rate in India, the GMDH-type ANN technique is more appropriate than the MA, Holt’s, AR and ARIMA models. The declining birth rate is a result of cultural changes, such as a higher standard of living. Therefore, this study can help decision-makers to make better decisions for more effective, advantageous planning and resource allocation within a certain region, which may help to build new schools, colleges, hospitals and parks or provide access to quality child care.

2. Materials and Methods

The data for the period 1995 to 2020 were collected from the source ‘‘indiastat.com”. MS-Excel, R and GMDH-NN shell Software were used for the data analysis.

2.1 Moving Average (MA) Model

[38] developed the q-term moving average (MA) is often used to smooth nonseasonal variation or irregular fluctuations in time series data, allowing the analyst to more quickly spot structural trends. It is also used to determine time series' short- and long-term future forecasts. A technique that is frequently used in time series analysis for modeling univariate time series is the MA model, commonly referred to as the moving-average process. According to the MA model, the output variable is cross-correlated with an independent random variable. The MA model of order q is indicated by the notation MA(q):

$$Y_t = \mu + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \ldots \tag{1}$$

Where μ is mean of the series the $\theta_1, \ldots, \theta_q$ *are* the model's parameters and the $\varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_{t-q}$ are white noise error terms. The MA model's order is denoted by the value of q. In its most basic form, the MA model is a linear regression of the series' most recent value against both prior and present white noise error terms or random shocks. The random shocks are considered independent of one another and come from the same distribution at each point, which is commonly a normal distribution with origin at zero and scale constant.

Following the trend of the presented time series data is the aim of the moving average technique, which has many different models. Simple Moving Average (SMA) is the sort of moving average that is the simplest to calculate. Each time series data point in SMA is given the same weight regardless of where it falls in the sequence [30]. A weighting factor is provided for each point in time series data by the Weighted Moving Average (WMA), an enhancement of the SMA, whereas the Exponential Moving Average (EMA) is a variant of the WMA that uses exponential numbers as the foundation for calculating weighting factors in time series analysis [23].

2.2 Double Exponential Smoothing (Holt’s) Model

[16] developed double exponential smoothing that can be used to forecast a time series with a linear trend. Holt’s linear exponential smoothing is also known as double exponential smoothing [34]. Forecasts for Holt’s linear exponential smoothing model are obtained using two smoothing parameters (α and β) and the three equations are given below:

Level smoothing equation: $L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + b_{t-1})$ (2)

Trend smoothing equation: $b_t = \beta(L_t - L_{t-1}) + (1-\beta) b_{t-1}$ (3)

Forecasting equation: $F_{t+m} = L_t + m b_t$ (4)

α=smoothing constant level component (0<α<1)
β= smoothing constant trend component (0<β<1)
m=number of periods ahead to be forecast

One of the members of the exponential smoothing family is the double exponential smoothing, often known as Holt's model [33, 36]. It is a useful technique for studying and predicting time series data with trend features [32, 46]. One of the main differences between Holt's method and other exponential smoothing is that it smooths out the trend by using two different smoothing parameters which are α and β which later will be used to forecast the future time series without having to use further exponential smoothing [24, 31].

2. 3 Autoregressive (AR) Model

The autoregressive (AR) model is a popular modeling approach for time-dependent interactions and problem-solving in many disciplines, including natural science, economics, and finance. Along with offering statistical diagnostic tools and guidance on how to apply the model, the objective is to comprehend the basic dynamics that underpin the AR model [14, 20]. The autoregressive model, in general, describes a system whose status (dependent variable) depends linearly on its own condition in the past. The system may be theoretically described using a stochastic difference equation like the one shown below.

$$X_t = \sum_{i=1}^{p} \beta_i X_{t-i} + \varepsilon_t \ldots \quad (5)$$

Here, the βs express how much the system's condition from i steps ago will influence the present values. Normally, one would anticipate that s would decline as i increased, indicating that events that occurred further in the past would have less of an influence on present events. Frequency noise uses the AR time series model to develop predictions, and the trend frequency noise term's prediction outcomes are added jointly to get the overall time series predicted value [11]. Use t to denote the initial period for which there is no data yet; then, in the autoregressive equation, replace i=1,…, p with the known previous values X_{t-i} while setting the error term ε_t equal to zero (As a result of our prediction that X_t will match its expected value and the unobserved error term will have a value of zero).

2.4 Autoregressive Integrated Moving Average (ARIMA)

[22] developed an ARIMA model to predict future values of time series that, like statistical models, uses past data to produce forecasted values of variables. AR and MA, the two components that make up an autoregressive moving average **(**ARMA) model, are to be modified by the ARIMA model. An ARMA model expressed the conditional average of Y_t as a function of both previous observations $Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}$, and previous innovations, $\varepsilon_{t-1}, \varepsilon_{t-q}$. The number of previous observations that Y_t depends on, p, is the AR degree. The number of previous innovations that Y_t depends q, is the MA degree. In general, these methodologies are denoted by ARMA (p, q). The form of the ARMA (p, q) approach is

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \ldots + \beta_p Y_{t-p} + \varepsilon_t + \varphi_1 \varepsilon_{t-1} + \varphi_2 \varepsilon_{t-2} + \ldots. + \varphi_q \varepsilon_{t-q} \quad (6)$$

where α is a constant term, $\beta_1, \ldots, \beta_p$ autoregressive (AR) coefficients, φ_1 moving average (MA) coefficients, $Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}$, AR lags corresponding to non-zero, $\varepsilon_{t-1}, \ldots, \varepsilon_{t-q}$ MA lags corresponding to nonzero, MA coefficients, and degree of differencing D, if D has value 0 which means no integration. $\beta(B)Y_t = \varphi(B)\varepsilon_t$ where B is the backward shift operator, an estimate of the ARIMA model

Time series forecasting often uses the ARIMA model, which was developed by Box and Jenkins [44]. Identification of the time series stochastic process and accurate future value prediction are the two main objectives of fitting the ARIMA model [5, 26]. When there is a signal of non-stationarity in the data, ARIMA may be used.

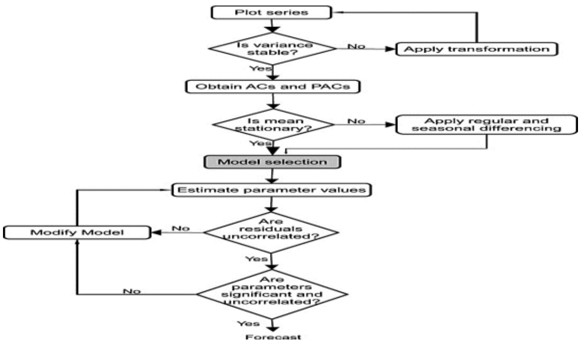

Figure 1. Structure of the Box-Jenkins methodology

The ARIMA model consists of "autoregressive" (p), "integrated" (d), and "moving average" (q) forms of stationary series, respectively. If a time series has to be differentiated in order to become stationary, it is referred to as such. The data differences for the stabilization process must be subtracted from the d degree before adding the ARMA (p, q) model in order to produce ARIMA (p, d, q) models. In the ARIMA (p, d, q) models, the degree of the AR model is denoted by p, the degree of the MA model by q, and the number of differences required to stabilize the data is denoted by d. [47].

2.5 Group Method of Data Handling type Artificial Neural Network (GMDH-type ANN)

[17] used the GMDH method for the first time to analyze complex systems that consisted of a collection of data with several inputs and a single output. Building a feed-forward network function utilizing a second-degree transfer function is the main goal of the GMDH network. With the help of the input variables, hidden layer neurons, and layer count, the GMDH algorithm automatically chooses the best model structure.

$$\hat{y} = a_0 + \sum_{i=1}^{m} a_i x_i + \sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij} x_i x_j + \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} a_{ijk} x_i x_j x_k + \ldots\ldots \quad (7)$$

where x represents the input variable, a_i represents coefficients, y represents the output variable and m represents the number of observations of input variable.

$$G(x_i) = a_0 + a_1 x_i + a_2 x_i^2 \quad (8)$$

The aim of the GMDH algorithm is to find the a_i unknown coefficients. The a_i coefficients are solved with regression methods for x_i input variable [13]. A schematic representation of the GMDH-NN model development process is depicted in figure 2.

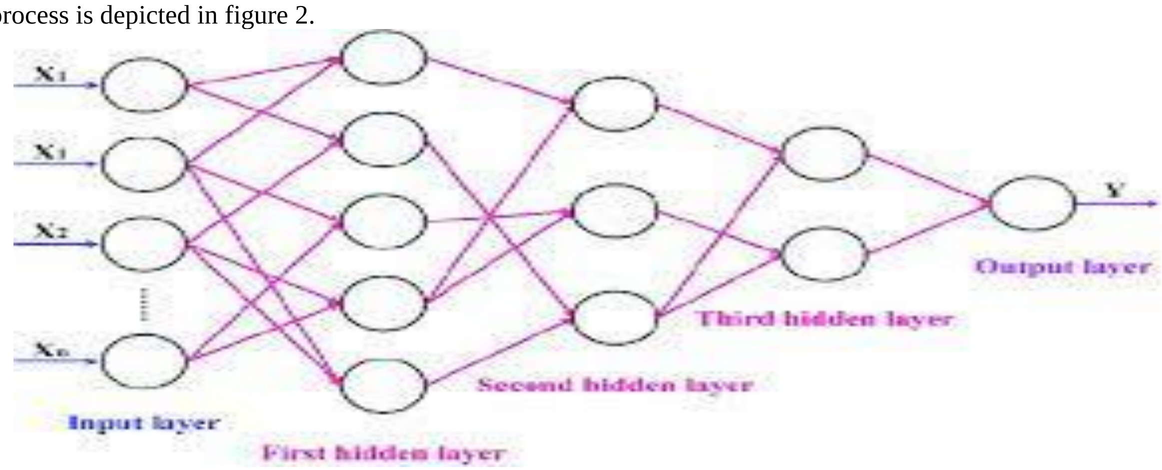

Figure 2. GMDH- type ANN model development process

2.6 The Prediction Models' Accuracy Measures

Model selection criteria are the guidelines used to determine a model's performance for the data under investigation to determine if there is a consistent pattern in the performance of the models under consideration [42, 9, 20]. The performance of the models under consideration has been compared using a number of comparison metrics, including coefficient of determination (R^2), mean absolute error (MAE), mean absolute percentage error (MAPE), mean square error (MSE), and root mean square error (RMSE) with a higher value of R^2 and smaller values of MAE, MAPE, MSE, and RMSE the model was more accurate.

MAE is the average value of the absolute difference between the actual value and the prediction of the data set is represented by the average absolute error. The residuals of the data set are averaged out in this measurement.

$$MAE = \frac{1}{N}\sum_{i=1}^{N} |y_i - \hat{y}_i|$$

Where

N= is the number of observations

$\hat{y}_i$ = Predicted value of y

y_i = The actual value of y

The most popular method for predicting error is called the mean absolute percentage error (MAPE), perhaps because the units of the variable are scaled to percentages, which makes them simpler to comprehend. If the data do not have any extremes, it works best (and without zeros). In the evaluation of models and regression analyses, it is commonly utilized as a loss function.

$$MAPE = \frac{1}{N}\sum_{i=1}^{N} \left| \frac{y_i - \widehat{y_i}}{y_i} \right|$$

The squared difference between the data set's original and forecasted values is averaged out to get the term MSE. It computes a residual's variance.

$$MSE = \frac{1}{N}\sum_{i=1}^{N} (y_i - \widehat{y_i})^2$$

The RMSE is equal to its square root of the MSE. This parameter calculates the standard deviation of the residuals.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (y_i - \widehat{y_i})^2}$$

R-squared, also known as the coefficient of determination, measures how much of the variation in the dependent variable is explained by the linear regression model. It is a scale-free score; therefore, regardless of whether the values are small or large, the square value R will be below one.

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - \widehat{y_i})^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2}$$

Where

$\widehat{y_i}$ = Predicted value of y
y_i = The actual value of y
$\bar{y}$ = mean value of y

3. Results and Discussion

The predictive value of the GMDH-type ANN model was compared with the MA, Holt's, AR and ARIMA models has been compared in this section using a variety of model selection criteria. The simplest techniques to choose the best prediction are based on the higher value of R^2 and its lowest values of predicted errors, particularly MAE, MAPE, MSE and RMSE. The results showed that the GMDH-type ANN technique is much superior to the other time series models (Table 1).

Table 1. The accurate measurements for MA, Holt's method, AR, ARIMA and GMDH-type ANN models.

Models	**GMDH-type ANN**	**ARIMA**	**Holt's method**	**MA**	**AR**
Best parameters	**Training set=80%, Testing set= 20%**	**p = 5, d = 1, q = 1**	**α =0.99, β =0.10**		
MSE	**0.0019**	0.0062	0.0065	0.2179	0.2888
RMSE	**0.0443**	0.0789	0.0792	0.4668	0.5374
MAE	**0.0333**	0.0364	0.0378	0.4378	0.4392
MAPE	**0.1538**	0.1717	0.1812	1.8936	2.1732
R^2	**0.9998**	0.9995	0.9995	0.9838	0.9785

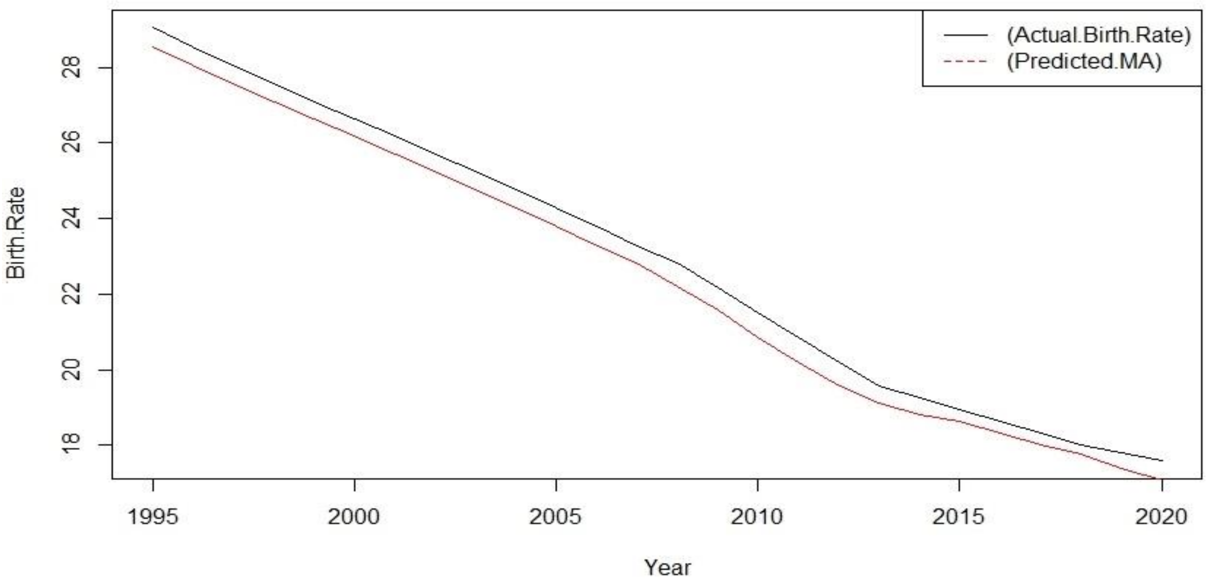

Figure 3. The graphic shows the birth rate data for India as both observed and predicted by the MA Model (1995-2020).

The predicted birth rate and the actual data vary by many different variables, according to the AR model. Between 1995-2020, the birth rate declined over time.

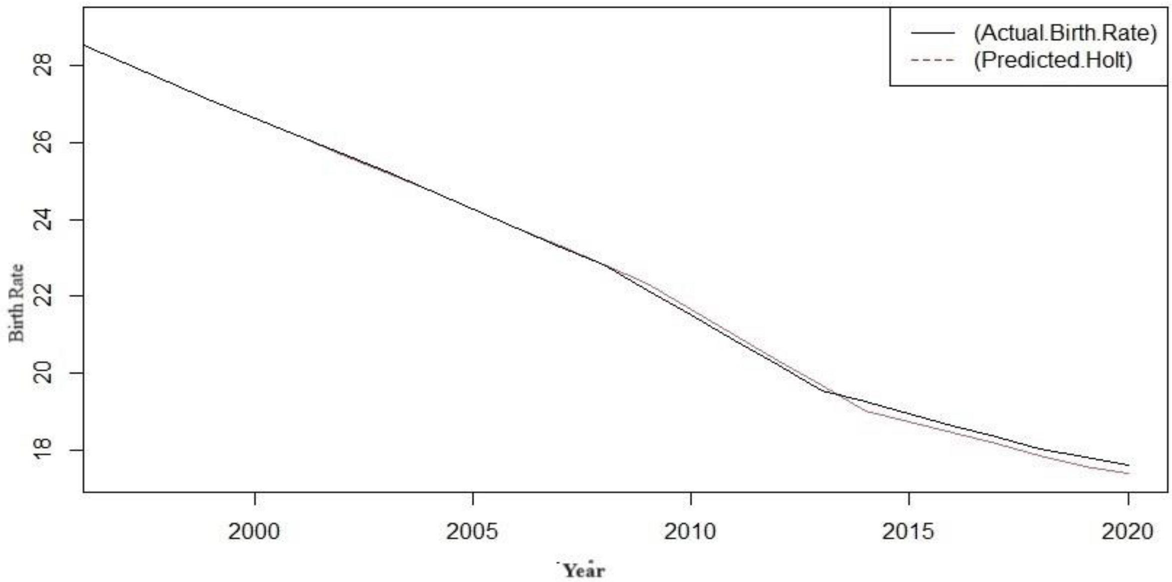

Figure 4. The figure depicts India's birth rate data, as predicted and as observed using Holt's method (1995-2020).

Using Holt's method, there is relatively little difference between the birth rate the predicted and actual data. The birth rate has been steadily falling from 1995–1996 to 2019–2020.

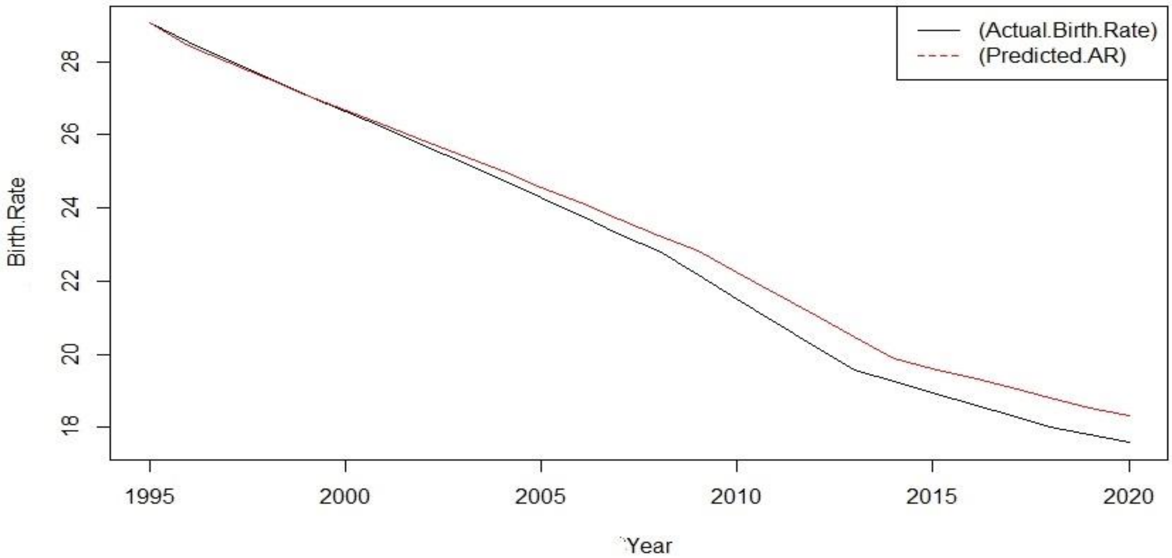

Figure 5. The actual and predicted plot of the Birth rate data for India using the AR model (1995-2020).

The AR model shows multiple differences between the predicted birth rate and the actual data. The birth rate steadily dropped between 1995 and 2020.

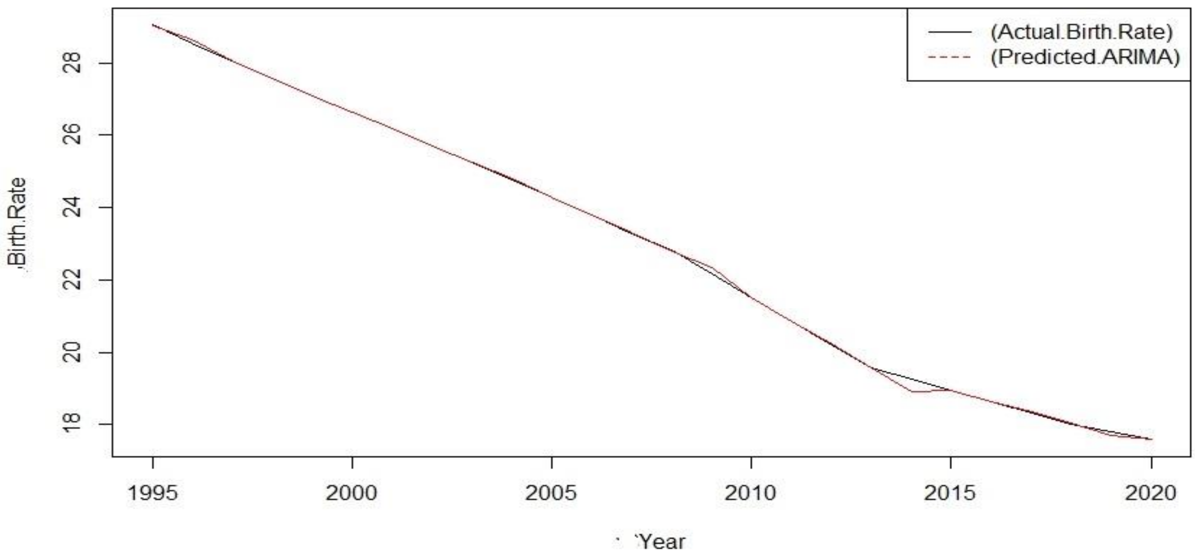

Figure 6. The plotting of India's birth rate data (1995–2020) using the ARIMA model, both actual and predicted.

There is relatively little difference between the Predicted data using the ARIMA model and the real data the birth rate. The birth rate did, however, gradually decrease between 1995-1996 and 2019-2020.

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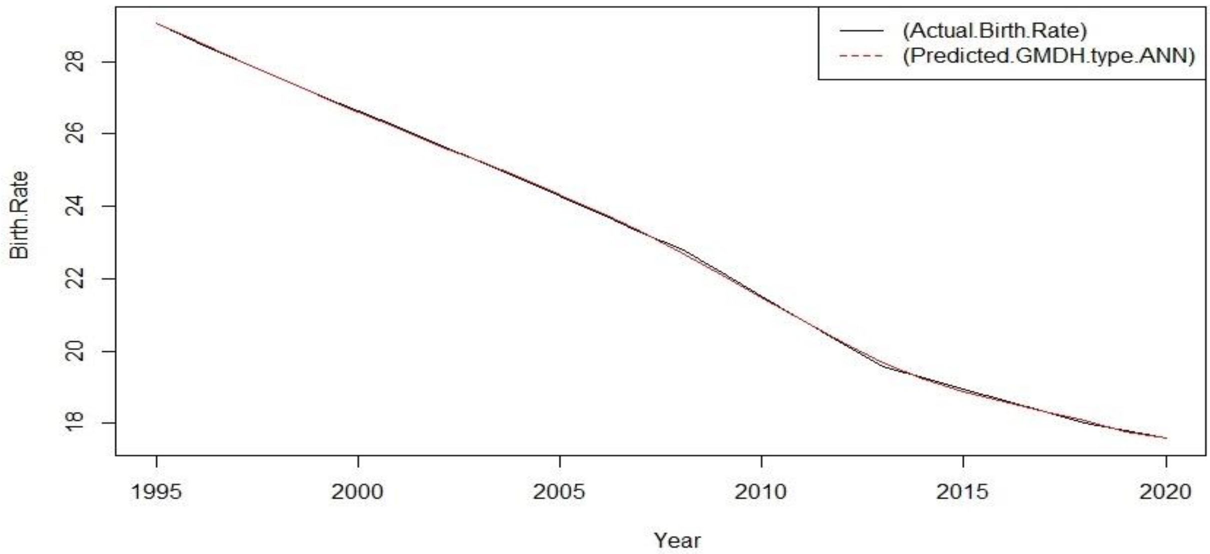

Figure 7. The graph displays India's birth rate data as both observed and predicted by the GMDH-type ANN (1995-2020).

The GMDH-type ANN model predicts data that is fairly similar to the actual data birth rate. However, the birth rate did steadily decline from 1995-2020.

The Actual birth rate has been steadily declining from 1995 to 2020. From the prediction of the birth rate for 1995 to 2020 in MA, Holt's method, AR, ARIMA and the GMDH-type ANN were close to the observed post-rates for the five model's above the (figures 3, 4, 5, 6, 7).

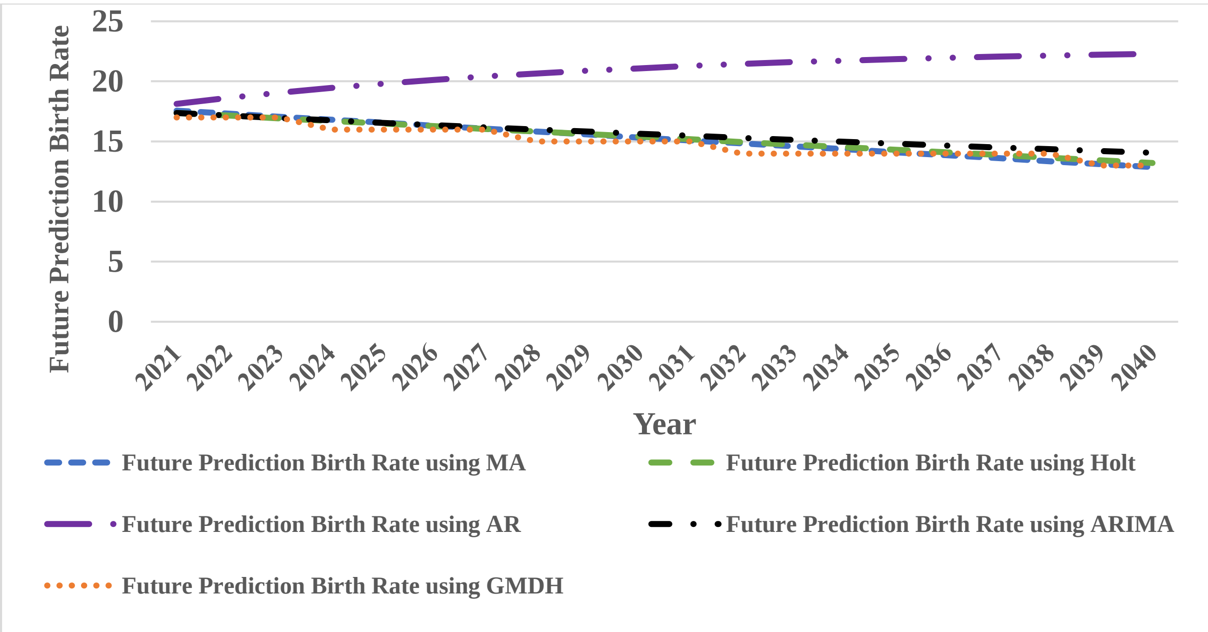

Figure 8. Comparison of the birth rate future prediction models MA, Holt, AR, ARIMA, and GMDH-NN.

MA, Holt's method, ARIMA and GMDH-type ANN models are very close for the out-of-sample future prediction from 2021 to 2040, however, AR is different. The GMDH-type ANN model's future prediction is better than others, coming in at 13 (95% prediction interval 13.545-12.454) per 1000 live births by 2040.

4. Conclusions

This research shows that for predicting the birth rate in India, the GMDH-type ANN technique is more appropriate than the MA, Holt's, AR and ARIMA models. It also does not require the application of complex assumptions, in contrast to traditional time-series models. The MA, Holt's, AR and ARIMA models may not be appropriate for long-term birth rate predictions in India. The GMDH neural network model may thus be more appropriate for non-linear or unexpected distribution data, such as fertility rates. The results of the research will be used to estimate future changes in fertility using GMDH-type ANN. The government will be able to allocate resources and make plans for children's services based on the expected fertility rate. The Indian government has begun to use a range of contraceptive techniques to reduce the nation's reproduction rate.

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