Mathematics Foundation Moderates Link between Cognitive Failure and Electrical and Electronic Engineering Education Achievements

Theodore Oduro-Okyireh[1], Budi Mulyanti[2], Dedi Rohendi[3], Alice Constance Mensah[4], George Oduro-Okyireh[5], Kennedy Acheampong[6]

[1,2,3]*Technology and Vocational Education Study Program, Universitas Pendidikan Indonesia,Bandung, Indonesia*

[4]*Department of Mathematics and Statistics, Accra Technical University, Ghana*

[5]*Department of Interdisciplinary Studies, Akenten Appiah-Minka University of Skills Training and Entrepreneurial Development, Ghana*

[6]*Department of Economics Education, Universitas Pendidikan Indonesia, Bandung, Indonesia*

Abstract:-This study explores the intertwining of mathematics and electrical and electronic engineering education, aiming to reveal how crucial mathematics problem-solving skills and cognitive development are to the field. The research explores the mediating role of students' cognitive failure in the relationship between mathematics failure and achievement in electrical and electronic engineering education. Additionally, it investigates the moderating effect of students' mathematics background on this relationship. The study adopted the quantitative research design where random cluster sampling was used to select a total of 488 final year students from four technical universities in Ghana. Data consisted of mathematics achievement test scores and students' examination results in ten core courses. The findings reveal that the adverse effect of students' failure in specific mathematics topics on achievements in electrical and electronic engineering education is mediated by cognitive failure. Notably, this mediation is moderated by the students' pre-university foundation in mathematics. The study makes recommendations for engineering mathematics curriculum development, emphasizing more practical applications, especially in Algebra and Probability, to prevent mathematics and cognitive failures. Intervention for applicants with grade 4 or worse is recommended for successful completion of the program.

Keywords: *Achievement, Cognitive failure, Foundation, Mathematics, Problem-solving.*

1. Introduction

Mathematics is considered the language of science, technology, and engineering. It contributes to the core of science and engineering and serves as a source of knowledge for engineering students [1]. Despite the use of technology, mathematics remains a vital tool for solving engineering problems. Electrical and electronic engineering rely heavily on mathematics. Calculus appears to be the most important knowledge in electrical engineering, while linear algebra is widely used in digital signal processing, communication systems, error control coding, circuit analysis, robotics circuits, and power systems. Fourier Transforms, Vector Algebra, and Probability and Statistics are also used in different areas of electrical and electronic engineering [2].

According to cognitive learning theory, learning depends on how people mentally process stimuli they encounter [3]. In response to cognitive issues, Benjamin Bloom developed a taxonomy of cognitive skills in

1956 that included knowledge, comprehension, application, analysis, synthesis, and evaluation [4], [5]. Bloom thought teachers should support students in developing these skills so they may use them to solve problems. Cognitive skills involve the ability to understand complex ideas, such as thinking, reasoning, and remembering that, in some combination, affect learning in many academic domains, including mathematics, and electrical and electronic engineering courses [6], [7]. Most prior research followed this framework by including academic skills, mostly mathematics skills, and cognitive skills to predict subsequent mathematics and engineering education outcomes [8]–[11].

A pertinent issue among electrical and electronic students in Ghana is that students with weak Senior High School (SHS) mathematics background do not generally perform well in electrical and electronic engineering courses. Over a decade experience gained by the first author of this article as an engineering mathematics lecturer in the department of electrical and electronic engineering in Cape Coast Technical University (CCTU) in Ghana, is that students who did not perform well in the Senior High School mathematics mostly do not perform well in the engineering mathematics and also in the electrical and electronic engineering courses. This pattern justifies the use of a grade 6 cutoff for admissions into Higher National Diploma (HND) and undergraduate programs in electrical and electronic engineering. However, a critical question is posed: Is grade 6 the optimal cutoff for ensuring success in the electrical and electronic engineering program?

The main objective of this research was therefore to examine the mediating role of students’ cognitive failure (CF) on the relationship between mathematics failure (MF) and achievement in electrical and electronic engineering education (ENG), as well as the causal moderating role of pre-university mathematics achievement (PMA) on this effect and the relationship. This would help us to investigate into the Ghanaian Technical University issue, and its related occurrences in other places in the world. It was also to propose a model that explains the relationship between the four interrelating variables, MF, CF, ENG and PMA, which cannot be found in a single structure, according to literature.

In the realm of electrical and electronic engineering education, the role of mathematics cannot be overstated. It serves as the backbone, shaping students' understanding and problem-solving skills. However, the impact of mathematics failure on students' achievement in this field is a complex interplay influenced by cognitive factors and the strength of their mathematical foundations [12].

1.1 Mathematics Failure and Academic Performance

Numerous studies have explored the correlation between mathematics failure and academic performance in various disciplines, emphasizing its heightened significance in technical fields such as electrical and electronic engineering. Students struggling with mathematics often face challenges in grasping the intricacies of engineering concepts, hindering their overall achievement [13].

1.2 Cognitive Mediation

Cognitive processes play a pivotal role in shaping how students approach and engage with mathematical and engineering tasks [14], [15]. When mathematics failure occurs, it is essential to understand the cognitive mechanisms involved. Cognitive mediation acts as a lens through which students process information, highlighting the cognitive hurdles that may intensify the effect of mathematics failure on academic achievement [16].

1.3 Mathematics Foundation as a Moderator

The strength of students' mathematical foundations emerges as a critical moderator in the influence of cognitive failure in mathematics problem-solving on academic achievement in engineering [17]. A robust mathematics foundation may buffer the negative effects of mathematics failure, providing students with a resilient base upon which to build their engineering knowledge. Conversely, a weak foundation could amplify the challenges associated with mathematics failure, causing a negative effect on academic achievement.

In navigating the complex landscape of electrical and electronic engineering education, addressing mathematics failure requires an understanding of the cognitive processes at play. By considering the moderating influence of mathematics foundation, educators and policymakers can develop targeted interventions to support students in overcoming mathematical challenges and fostering a robust academic journey in the field. This exploration aims to contribute to the ongoing discourse on enhancing educational strategies in technical disciplines.

1.4 Framework and Research Hypotheses

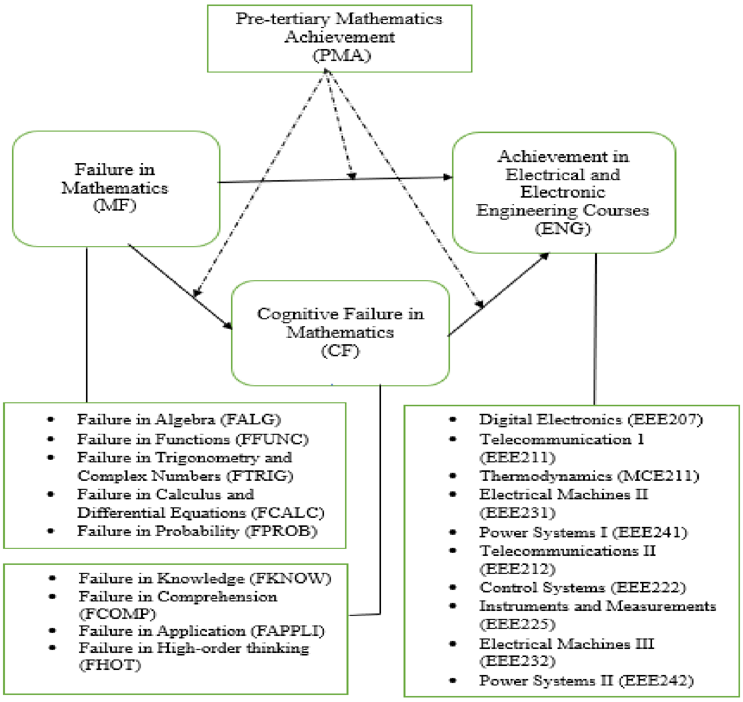

Fig. 1 The moderated mediation model

The framework for this study confidently followed the reversibility of the relationship between mathematics and cognitive abilities (working memory, reasoning, and executive function) as predictors of each other's development [18]. It also considered existing research works, including those on cognitive ability [19]–[23]. Also, studies on mathematics problem solving were included [24]. Again, research work that concluded on the influence of students' foundation in mathematics at the pre-university level was considered [25]. Thus, we proposed a model as shown in Fig. 1 to include students' foundation in mathematics (PMA) to interact with failure in mathematics (MF), as well as failure in cognition (CF).

The study therefore hypothesized that:

H_{11}: Mathematics failure has a negative effect on achievement in electrical and electronic engineering education

H_{12}: Mathematics failure has a positive effect on Cognitive failure in mathematics

H_{13}: Cognitive failure in mathematics has a negative effect on achievement in electrical and electronic engineering education

H_{14}: Pre-tertiary mathematics moderates the relationship between mathematics failure and achievement in engineering education

2 Method

2.1 Research Design

The causal comparative design was used with the relational survey model. The relational survey model aims to measure the presence and degree of variation between two or more variables [26]. We aimed to describe the effects that students' drawbacks in mathematics have on their main electrical and electronic engineering subjects without any attempt to change or influence it.

2.2 Participants

The subjects of the study are the 2021/2022 final year HND electrical and electronic engineering students in the ten technical universities (TUs) in Ghana who have already completed their engineering mathematics courses over their first four semesters. These students are the research population of interest in this study. This population is made up of students who previously studied in Senior High Schools, and also those from the pure Technical Schools.

Four TUs were randomly selected from the ten. Second-year HND electrical and electronic students at Cape Coast Technical University (CCTU) who had just completed their fourth semester were selected for testing the MAT instrument. We anticipated that the data from the four technical universities will be homogeneous. This is as a result of a common entry requirement and syllabus for the HND electrical and electronic engineering program. Within the TUs that were randomly sampled, a cluster sample total of n = 488 students was obtained.

2.3 Research Instrument

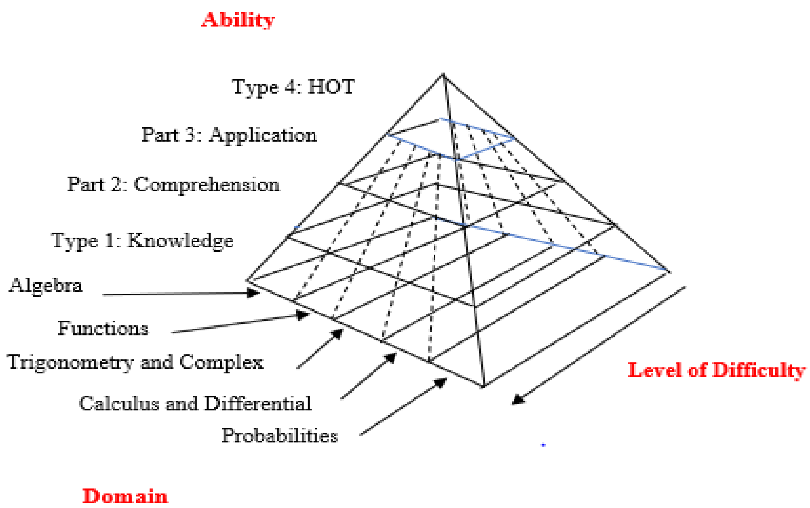

Fig. 2 De Lange's assessment pyramid for curriculum mathematics

De Lange's pyramid is redesigned to measure curriculum mathematics in three dimensions as shown in Fig. 2: domain of knowledge (algebra, functions, trigonometry, calculus, and probability); levels of mathematics difficulty (low to high); and cognitive level (knowledge, comprehension, application, and HOT) [27]. The five mathematics content areas (domain of knowledge) were purposively selected from the HND electrical and electronic engineering curriculum in Ghanaian TUs. The test items for the MAT were carefully planned to ensure that the level of difficulty is not above that of the content of the HND syllabus.

The Mathematics Achievement Test (MAT) used for this research were two types: the subjective type (MAT I) and objective type (MAT II). Both MAT I and MAT II were made up of five sections, A to E, covering the areas of Algebra, Functions, Trigonometry and Complex Numbers, Calculus and Differential Equations, and Probability, respectively. Both consisted of test items in each section that measured students' failure in getting the concepts in these areas of mathematics, as well as their cognitive failure according to the Bloom's taxonomy (that is, failure in knowledge, comprehension, applications, analysis, evaluation, and creativity) [5]. While MAT I consisted of five subjective test items that measured the aforementioned variables, MAT II consisted of twenty (20) objective test items in each section, making a total of 100 items, all within the scope of the engineering mathematics curriculum. Because of the volume of the test, MAT II was administered on three different occasions under standard examination conditions. The large number of items in the MAT II ensured repeated measurements in the cognitive domain and thus reducing the effect of using multiple choices in the measurements. We were also motivated by the positive impact multiple-choice question authoring and regular participation have on students' learning [28]. Table 1 shows the test item specifications for MAT II.

Table 1 Achievement test item specification in MAT II

Domain of Knowledge	Cognitive Level				Total
	Knowledge	Comprehension	Applications	HOT	
Algebra (ALG)	5	5	5	5	20
Functions (FUN	5	5	5	5	20
Trigonometry and Complex Numbers (TRIG)	5	5	5	5	20
Calculus and Differential Equations (CALC)	5	5	5	5	20
Probability (PROB)	5	5	5	5	20
Total	25	25	25	25	100

Authors' construct, 2023

A secondary data consisting of each students' results in the third and fourth semester of their studies in the universities were collected from the examination units of the universities. These are the raw scores obtained from students' core engineering subjects. The accuracy of students' self-reported pre-tertiary mathematics achievement (grades) was verified from records in their respective academic departments. The data were entered into SPSS version 26.0, and Analysis of Moment Structures (AMOS) was employed to construct a structural equation model, facilitating simultaneous testing of hypothesized relationships. AMOS, an extension of the Generalized Linear Model (GLM), allows researchers to test multiple regression equations concurrently and incorporates latent variables, considering measurement errors[29]. The Hayes Process was applied to assess the moderated mediation effects in the model[30]. Mediator and moderator variables were critical in this survey-based research, and the SPSS AMOS program package simplified the complex analysis, enhancing efficiency.

2.4 Data Analysis

Analytical Approach: The research employed AMOS version 26 and covariance-based structural equation modelling (CB-SEM) to evaluate hypotheses regarding the effect of students' grasp of engineering mathematics on their understanding of electrical and electronic engineering courses. CB-SEM, guided by established theories, allows for testing and validation of theoretical constructs with empirical data [31], [32]. The study focused on the interplay between failure to comprehend engineering mathematics concepts and the subsequent effects on learning electrical and electronic engineering. Cognitive training, specifically in creative thinking, was considered crucial for student success. The maximum likelihood estimation technique was chosen for its robustness, even when variables deviated from multivariate normality[33], [34].

The analytical process comprised three stages: a confirmatory factor analysis (CFA) to assess the measurement model, a mediation analysis using bootstrapping for testing the hypotheses, H_{12} and H_{13}, and a moderated mediation analysis was made to assess research hypothesis H_{14}. Effect sizes, Pearson's correlation coefficient (r), and the coefficient of multiple determination (R^2) were employed to quantify relationships between constructs [35], [36]. The use of 5000 bootstrap samples enhanced the robustness of the analyses, and the subgroup approach (-1 SD and +1SD) provided additional insights[31]. This comprehensive method not only contributes to understanding the dynamics of mathematics failure in electrical and electronic engineering education but also establishes a foundation for future theoretical development and empirical research in the field [37].

Mediation Analysis: Fig 3 illustrates a mediation model in the study, showing path coefficients *a*, *b*, and *c*' denoting relationships between MF and CF, CF, and ENG, and MF and ENG respectively. The model assesses the direct effect (c') of MF on ENG, with a specific indirect effect (SIE) through CF (*a* and*b*). The total effect is expressed as SIE + c'. Mediation studies offer insight into how a predictive variable influences an outcome variable [38]. Understanding mediation effects is necessary for a comprehensive grasp of an influence of a variable on an outcome variable in a study [37].

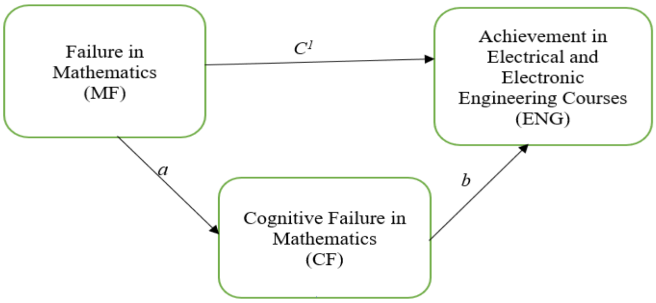

Fig. 3 Mediation model

In estimating indirect effects, the widely used causal steps approach is that of Baron and Kenny(1986)[39], which unfortunately, faces criticism for its low statistical power and therefore applicable to only simple mediation models[31], [40]. To address this, researchers often supplement it with the Sobel test[31]. However, both methods rely on the challenging assumption that the product of path coefficients *a* and *b* follows a normal distribution which is difficult to achieve[41][38]. Recognizing this limitation, the study opted for the Bollen-Stine bootstrapping techniques to mitigate nonnormality effects[41][42][31][38]. Both Bollen-Stine and Naive bootstrapping methods were utilized, with Bollen-Stine modifying the enlarged χ^2 and naive bootstrapping applied in the mediation analysis [43], [44]. This comprehensive approach enhances the reliability of indirect effect estimations, providing a more robust foundation for drawing conclusions in the study.

Moderated Mediation Analysis: The influence of an independent variable on a dependent variable can vary based on the levels of another variable, *M*, termed a moderator [45]. This study delves into conditional indirect effects or moderated mediation, exploring the circumstances under which an indirect effect may occur. Investigating conditional indirect effects allows researchers to understand the interactive influence of one variable on another [38], [46]. The common technique for examining moderated mediation that involves assessing the mediation effect at each level of the moderator is known as the subgroup approach [45], [47] Following Preacher et al.'s (2007) suggestion, mediation effects were estimated within subgroups, one standard deviation below and one standard deviation above the mean, using bootstrapping with the PROCESS micro. This approach enhanced the understanding of variable interactions and their effects [47].

3 Findings

3.1 Data Assessment

In the Confirmatory Factor Analysis process, first, researchers ensured result credibility by examining outliers and distribution assumptions [48]. Although, few outliers were identified in the MAT data, caution prevailed against deletion of these data points due to the realization that these were scored by outstanding students who had correspondingly scored high or low marks in the electrical and electronic engineering courses. Thus, those observations (outliers) were seen as part of the population and were therefore not deleted [49]. Multivariate normality assessment revealed nonnormality, particularly in the kurtosis value which was greater than 5.00 (*c.r.* = 10.825), indicating multivariate nonnormality[50]. This could lead to rejection of the appropriate model [32], [51]. To address this, the Bollen Stine bootstrapping techniques were applied for re-estimating parameters and standard errors. This approach enhances understanding of test statistic behavior in a nonnormal data, aligning with methodological recommendations [43].

3.2 Evaluation of the Measurement Model

Before the comprehensive evaluation, a preliminary CFA diagnosed potential issues in the model, prompting slight modifications for enhanced quality [52]. Troublesome indicators indicating cross-loadings and correlations in error terms were addressed through expert consultations and curriculum analysis, leading to the exclusion of one indicator (EEE241). Measurement error concerns in moderated-mediation analysis were acknowledged due to potential relationship attenuation [53], [54]. We now discuss the development of the measurement model, including all the tests of the outer loading reliability, internal consistency, convergent validity, discriminant validity, and structural model measurement techniques that are important in the analysis. Table 2 is a set of model fit indices[48][55].

Table 2 Model fit indices for the hypothesized model

Model Fit Index	Value	*Recommended Guideline
χ^2	539.168, p=0.00	Not Significant
χ^2/df	3.685	<5
TLI	.921	≥.90
CFI	.932	≥.90
IFI	.932	≥.90
RMSEA	.074	<.08 (acceptable fit)
SRMR	.043	<.05

*[56][48]

The results in Table 2, aligned with recommended guidelines, strongly support the well-fitted factor structure to the sample data [56]. Construct validity, the primary focus of CFA, was thoroughly assessed in Table 3[57]. All observed variables exhibited excellent convergence on their respective latent constructs, as indicated by standardized factor loadings exceeding .50 and being statistically significant (p < .001) [58]. Additionally, AVE values surpassing .50 depicted substantial convergence, while SMC values above .36 affirmed the reliability of each item[59][57]. With composite reliability exceeding .70, there is ample evidence for good reliability and internal consistency across constructs [58]. Discriminant validity, evaluated in Table 4, confirmed the uniqueness of each construct, as square roots of AVEs exceeded their corresponding Pearson correlation coefficients[57]. In conclusion, the CFA results affirm the well-fitted measurement model, laying a crucial foundation for subsequent structural model analyses [57].

Table 3 The result of construct validity

Construct		Obv. Var	Test of Significance				Factor Loading (Std)	SMC	CR	AVE
			Unstd.	SE	t-Value	p				
MF	CF		1.192	.088	13.588	***	.765	.595		
MF	ENG		-1.805	.332	-5.438	***	-.721	.520		
CF	ENG		-1.391	.215	-6.478	***	-.451	.545		
MF		FALG	1.08	.062	17.504	***	.801	.642	.910	.670
		FFUNC	.936	.055	17.111	***	.785	.616		
		FTRIG	.863	.051	16.931	***	.780	.609		
		FCALC	.840	.053	15.702	***	.714	.510		
		FPROB	1.000				.750	.563		

Construct		Obv. Var	Test of Significance				Factor Loading (Std)	SMC	CR	AVE
			Unstd.	SE	*t*-Value	*p*				
ENG		EEE207	1.000				.693	.480	.930	.580
		EEE211	.929	.060	15.573	***	.773	.598		
		MCE211	.945	.065	14.462	***	.693	.480		
		EEE212	.762	.051	14.966	***	.736	.541		
		EEE222	1.172	.080	14.689	***	.710	.504		
		EEE225	.937	.057	16.454	***	.811	.658		
		EEE231	.878	.059	14.847	***	.733	.537		
		EEE232	.755	.050	15.193	***	.745	.555		
		EEE242	.782	.056	13.921	***	.668	.446		
CF		FKNOW	.714	.044	16.391	***	.742	.551	.910	.710
		FCOMP	.681	.039	17,338	***	.786	.618		
		FAPPLI	.765	.040	19,320	***	.814	.662		
		FHOT	1.000				.793	.629		

Note: N=488. *MF*, mathematics failure*; CF*, cognitive failure*; ENG*, Achievement in Electrical and Electronic Engineering Education*, AVE*, average variance extracted, *SMC*, squared multiple correlation*; Std*, standardized*; Unst*, unstandardized*; SE*, standard error.

***p<.00

Table 4 Inter-construct correlation and square root of average variance extracted

Construct	ENG	MF	CF
ENG	**-0.761**		
MF	-.721**	**-0.819**	
CF	-.738**	.765**	**-0.843**

Note: N=488. The diagonal elements in parenthesis are the square roots of the average variance extracted. *MF*, mathematics failure*; CF*, cognitive failure*; ENG*, Achievement in Electrical and Electronic Engineering Education,

**p<.01 (significant at .01 level)

3.2 Evaluation of Structural Model

In the second stage of SEM analysis, structural equation modeling (SEM) is employed to establish connections between constructs, offering precise information about variables. If the measurement model is violated, adjustments can be made. The structural model, examined in this stage, gives the detail of the relationships between independent (exogenous) and dependent (endogenous) variables [60], [61].

The results of testing the hypothesized structural theory, which concentrated on evaluating the overall structural model fit and the hypothesized structural connections between constructs, are consequently summarized in this section. The structural model generated a χ^2 of 486.410 with 132 degrees of freedom, which gave out a relative chi-square of 3.685, an RMSEA of .074, a TLI of .921, and a CFI of .932, indicating an adequate fit of the hypothesized structure for the observed covariance matrix.

3.3 Additional Analysis

To overcome the limitations in evaluating moderated mediation effects, the study employed the PROCESS macro for conducting simultaneous tests of hypothesized relations[38]. This macro utilizes a path analysis approach described by Preacher et al. (2007) and allows bootstrap testing of indirect effects at different moderator levels[62]. Bootstrap sampling, with 5000 samples, generated bias-corrected confidence intervals for the indirect effect conditioned by the moderator (PMA). This approach compensates for nonnormality, providing empirical approximations of sampling distributions. The analysis, incorporating standardized and control variables, was conducted using PROCESS model 59, offering insights into both direct and indirect effects conditioned by students' pre-tertiary mathematic (PMA, the moderator).

3.4 Mediation Analysis

Fig. 4 shows the standardized path estimates and $R^2$of the hypothesized model. It can be seen that every path coefficient was statistically significant ($p< .05$) which did not give us any evidence to reject the hypothesized relationships between the constructs. The R^2 for ENG was .60, indicating that the structural model explained 60% of the variance of ENG. In other words, the two independent variables, MF andCF accounted for 60% of the variation in ENG. 5000 bootstrapping with 95% confidence also revealed that the direct path from MF to ENG was statistically significant. Specifically, from IBM AMOS 26, it was revealed that there is a significant direct effect [c^1 path: t = -6.6818, p = .0000] as well as a significant indirecteffect [*ab* path: t = -9.2328, p = .0000] of MF on ENG.The Conditional Direct effect of MF on ENG and Conditional Indirect Effects of MF on ENG are all significant. The zero does not fall between the lower and upper limits of the 95% confidence interval for the independent variable MF [-.4132, -.2254] and at each level of the mediating variable, CF [-.4973, -.3228]. Therefore, there is a statistically significant effect of MF on CF, and CF on ENG. The results therefore show a partial mediating effect of CF on ENG.

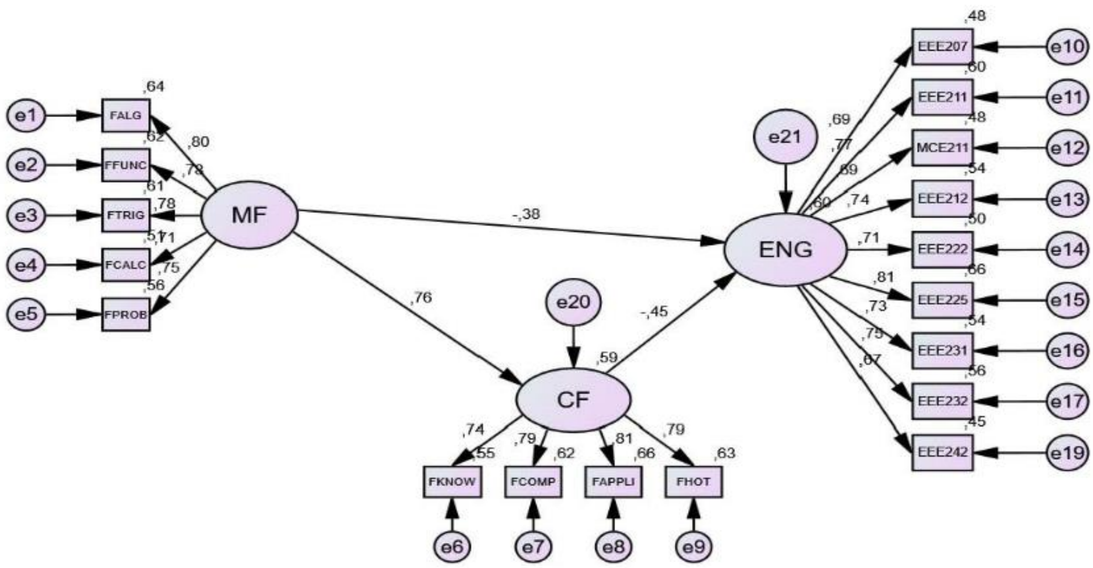

Fig. 4 Structural equation modelling of the hypothesized model with standardized coefficients and R^2

Table 5 shows CIs for bootstrap tests at different levels of PMA: (1) one SD below mean, (2) the mean, and (3) one SD above the mean. Statistically, the CIs considered significant if the values between the lower and upper confidence limits do not include zero[63]. From Table 5, one of the bootstrap CIs for PMA levels is one SD below the mean (-.4408, -.2317) which does not include zero, indicating that there is significant indirect effect under this condition. And the others are, similarly, under conditions of the mean and one SD above the mean [95% CI with the intervals (-.2875, -.1575) and (-.1977, -.0742) respectively, which do not include zero]. All these intervals indicate that there are significant indirect (mediated) negative effects of MF on ENG through CF.

Table 5 Bootstrap results for the conditional indirect effects

PMA	Boot Indirect Effect	BootSE	BootLLCI	BootULCI
-1 SD (1.415)	-0.3214	0.53	-0.4408	-0.2317
Mean (0)	-0.2153	0.334	-0.2875	-0.1575
+1 SD (1.415)	-0.1303	0.312	-0.1977	-0.0742

Note: Bootstrap sample size = 5000

3.5 Moderated Mediation Analysis

In moderated mediation analysis, the index of moderated mediation is available when the indirect effect is a linear function of a single moderator. In some models such as 59, the indirect effect is a nonlinear function of the moderator, and so no index of moderated mediation is provided. However, the Bootstrap results for the regression model parameters table (Table 6) provides the CI for the *a*, *b* and c^lpaths. The summary in Table 6 shows a significant moderating effect [p < 0.05, (.0679, .1877)], and a significant interaction of PMA on the association between MF and CF [*p*< 0.05, CI= (-0,1209, -0,037)]. In the presence of the moderating variable PMA, the multiple regression equation from Hayes Process connecting MF, and the interaction between the two, Int_1 to CF is given by the equation,

$$CF = 0.5250MF + 0.1278PMA - 0.0790Int_1 \quad (1)$$

Table 6 Regression of *CF on MF, PMA* and *MF×PMA*

	Model 1: CF					
	Coeff	SE	*t*-Value	*p*	LLCI	ULCI
constant	.0710	.0377	1.8824	.0604	-.0031	.1450
MF	.5250	.0421	12.4598	.0000	.4422	.6078
PMA	.1278	.0305	4.1911	.0000	.0679	.1877
MF×PMA	-.0790	.0214	-3.6978	.0000	-.1209	-.0370

Outcome: CF, R^2 = 0.4895, *F* = 1,547,018, *p* = 0.000

We did not include the constant term because it is not significant at 95% confidence level [*p*> 0.05, CI= (-.0031, .1450)]. Thus, there was a significant reduction in students' CF when

Table 7, the direct effects of MF and CF on ENG were also significant at 95% confidence the moderator is present in the model.

From level [p<.05, CL= (-.4132, -.2254) and (-.4973, -.3228)]. Again, the moderating effect of PMA was also significant [p < .05, CI = (-.1167, - -.0023)], as well as its interactions with MF [*p*< .05, CI = (-.1539, -,0362)] and with CF [*p*< .05, CI = (0,0098, -0,1241)]. In the presence of the moderating variable PMA, the multiple regression equation from Hayes Process connecting MF, CF, Int_2, the interaction between the moderator and MF, and Int_3, the interaction between the moderator and CF, to the response variable, ENG, is given by the equation:

$$ENG = -0.3193MF - 0.4100CF - 0.0572PMA - 0.0951Int_2 + 0.0669Int_3 \quad (2)$$

Table 7 Regression of ENG on MF, *PMA*, *MF×PMA*and C*F×PMA*

	Model 2: ENG					
	Coeff	SE	*t*-Value	*p*	LLCI	ULCI
constant	.0331	.0371	.8915	.3731	-.0398	.1060
MF	-.3193	.0478	-6.6818	.0000	-.4132	-.2254
CF	-.4100	.0444	-9.2328	.0000	-.4973	-.3228
PMA	-.0572	.0303	-1.8885	.0496	-.1167	-.0023
MF×PMA	-.0951	.0300	-3.1747	.0016	-.1539	-.0362
C*F×PMA*	.0669	.0291	2.3012	.0218	.0098	.1241

Outcome: CF, R^2 = 0.5160, F = 1,027,872, p = 0.000

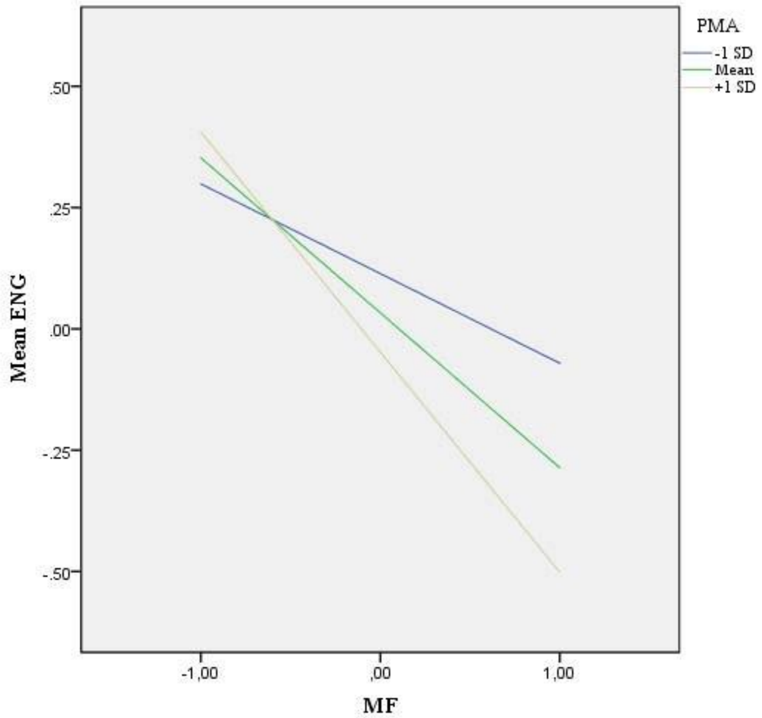

Fig. 5 Interaction effect ofPMA on the influence of MF on ENG

Again, we did not include the constant term because it is not significant at 95% confidence level [p> 0.05, CI=(-.00398, .1060)]. Thus, there is a significant reduction in ENG when PMA interacts with MF, but a significant increase ENG is realized when it interacts with CF.

The research findings revealed that there is a significant moderating effect of *PMA* on the relationship between MF and ENG,MF and CF, and MF and ENG. Our interest is to briefly look at how PMA affects ENG for students with low PMA and also with high PMA. In Fig. 5, we generated a graph showing the Interaction effect of *PMA* on the influence of MF on ENG. From Fig. 5, we can see that there was generally, a negative linear relationship between MF and ENG. Students with higher *PMA* (+1 SD), that is, students with weaker foundation in mathematics (brown line) had higher negative slope than their counterparts with lower PMA (-1 SD) or stronger foundation in mathematics (blue line). The green line also shows that effort to reduce *PMA* (or strengthen mathematics foundation) reduced the slope for MF on ENG, an implication that a unit increase in *MF* would impact less negatively on ENG. Thus, the moderator, PMA reduced the high-effect variable and increased the low-effect variable. The next section discusses this in more detail by looking at the implication on electrical and electronic engineering education in Ghanaian TUs.

4 Discussions

The paper explores the mediating role of students' cognitive failure in the relationship between mathematics failure and achievement in electrical and electronic engineering education. The paper practically investigated this for Ghanaian Technical Universities' students' performance in nine electrical and electronic engineering core courses in relation to their struggle with understanding five selected curriculum mathematics topics and cognitive failure. The findings revealed significant negative effects of students' inability to grasp the concepts of Algebra, Functions, Trigonometry, Complex Numbers, Calculus and Differential Equations, and Probability on their achievements in engineering education. This also sheds light on the role of cognitive failure, aligned with Bloom's taxonomy, and emphasizes the importance of foundation knowledge in High School mathematics as a moderator[4], [5]. The study suggests implications for students, high school teachers, university lecturers, curriculum providers, and policymakers, emphasizing the interconnectedness of mathematics understanding and success in electrical and electronic engineering education

4.1 Theoretical Contributions

This study aimed to create a comprehensive conceptual framework by synthesizing existing research on cognitive ability in mathematics and engineering. Various studies formed the foundation, exploring the interplay between mathematics, cognitive abilities, and students' pre-university foundation in mathematics [19]–[25]. The reversible relationship between mathematics and cognitive abilities was considered [18].

The study's first theoretical contribution challenges the assertion that behavioral analysis alone is insufficient for understanding the impact of instruction in electrical and electronic engineering education [64]. It emphasizes the need for both behavioral and cognitive analyses, highlighting mathematics teaching as integral to problem-solving skills and linking to cognitive enhancement [14], [65]. The research supports the findings that cognitive ability is enhanced through mathematics problem-solving [18].

The study further underscores that mathematical problem-solving processes not only enhance knowledge acquisition but also elevate reasoning skills in electrical and electronic engineering students [66]. Effective problem-solving techniques are deemed critical for improving mathematics achievement and consequently, overall success in electrical and electronic engineering education. The study suggests that mathematics problem-solving training reduces cognitive failure, contributing positively to academic achievement in this field, while inadequate techniques have adverse effects. Overall, the research emphasizes the pivotal role of mathematics instruction and problem-solving techniques in shaping cognitive abilities and, consequently, success in engineering education [67]. Thus, positive problem solving techniques in mathematics reduce cognitive failure annd consequently have positive effects on students’ academic achievement in electrical and electronic engineering.

4.2 Practical Implications

This research highlights the significance of mathematics proficiency in the success of technical students, particularly in electrical and electronic engineering programs. The implications extend beyond Ghana, emphasizing the universal importance of mathematical understanding in advancing through engineering education. Proficiency in Algebra, Functions, Trigonometry and Complex Numbers, Calculus and Probability merges as very essential for success in electrical and electronic engineering courses.

The study identifies cognitive failure and students' weak foundation in mathematics as key factors hindering success in electrical and electronic engineering education. Cognitive failure mediates the impact, underlining its disruptive role in electrical and electronic engineering students' achievements. The research recommends diverse student groups, combining strong mathematical and cognitive abilities for collaborative problem-solving.

Additionally, the study validates the effectiveness of West African Secondary School Certificate Examination (WASSCE) results in assessing students' pre-university mathematical background necessary for electrical and electronic engineering education. It suggests the need for targeted intervention strategies to enhance mathematical and cognitive abilities, particularly for students with lower mathematics background, aiming to improve overall achievement in this field. Ultimately, the research provides practical insights for educational institutions, suggesting the consideration of both cognitive and mathematical factors in student assessments and interventions to enhance the learning experience in electrical and electronic engineering education.

5 Conclusion

The research findings underscore a collective negative effect of students' failure in five key mathematics topics on their performance in electrical and electronic engineering education. This effect is partially mediated by cognitive failure, indicating that a lack of understanding in Algebra, Functions, Trigonometry and Complex Numbers, Calculus, and Differential Equations, and Probability negatively influences achievement in electrical and electronic engineering education, particularly in Algebra. Notably, Algebra emerges as the most influential area among these topics in the Ghanaian HND electrical and electronic engineering curriculum, followed by Probability. The most significantly affected course is EEE222 (Control Systems).

The negative effect of mathematics failure on achievement implies potential barriers for students with weak mathematical foundations. Academic interventions, such as mathematics support and tutorials, should be implemented for identified weak students, particularly those with grades worse than +1 SD of the mean. We estimated this from the Ghanaian TUs data as grade 4 or worse.

Cognitive failure in mathematics problem-solving should be prevented for a successful electrical and electronic engineering education, as it strongly mediates the relationship between mathematics failure and achievement in electrical and electronic engineering education. To do this, profound understanding of mathematics concepts, especially Algebra and Probability, is crucial. Continuous revision and problem-solving, focusing on these topics from primary to tertiary levels are recommended. Teachers should employ effective teaching methods, such as the Realistic Mathematics Education (RME) approach, to foster critical and creative thinking. Emphasizing the practical applications of engineering mathematics in everyday life can make the subject more relevant and engaging.

Despite the study's valuable insights, it only considered five mathematics indicators, explaining 51.6% of the variation in students' achievements. Future research should explore additional engineering mathematics areas and factors such as the high school attended, student gender, admission mode, and attitude toward the program of study. This holistic approach may uncover further factors influencing students' achievement in electrical and electronic engineering education, contributing to more comprehensive educational strategies.

References

[1] R. Lehrer and A. S. Palincsar, “I,” in *Investigating Participant Structures in the Context of Science Instruction*, Routledge, 2014, pp. 389–392.

[2] P. Dash, F. T. Zohora, M. Rahaman, M. M. Hasan, and M. Arifuzzaman, “Usage of Mathematics Tools with Example in Electrical and Electronic Engineering,” *Am. Acad. Sci. Res. J. Eng. Technol. Sci.*, vol. 46, no. 1, pp. 178–188, 2018.

[3] D. H. Schunk, *Learning theories an educational perspective*. Pearson Education, Inc, 2012.

[4] B. Bloom, “Bloom’s taxonomy.” 1956.

[5] M. Kartikasari, T. A. Kusmayadi, and B. Usodo, “ASSESSMENT OF EXAM QUESTIONS QUALITY ACCORDING TO COGNITIVE DOMAIN OF BLOOM’S TAXONOMY,” in *Proceedings Education and Language International Conference*, 2017, vol. 1, no. 1.

[6] S. M. Jaeggi, M. Buschkuehl, J. Jonides, and W. J. Perrig, “Improving fluid intelligence with training on working memory,” *Proc. Natl. Acad. Sci.*, vol. 105, no. 19, pp. 6829–6833, 2008.

[7] N. Cowan, “Working memory underpins cognitive development, learning, and education,” *Educ. Psychol. Rev.*, vol. 26, pp. 197–223, 2014.

[8] L. S. Fuchs *et al.*, “versus prealgebraic knowledge.,” *Dev. Psychol.*, vol. 52, no. 12, p. 2085, 2016.

[9] D. C. Geary, “Cognitive predictors of achievement growth in mathematics: a 5-year longitudinal study.,” *Dev. Psychol.*, vol. 47, no. 6, p. 1539, 2011.

[10] D. C. Geary, A. Nicholas, Y. Li, and J. Sun, “Developmental change in the influence of domain-general abilities and domain-specific knowledge on mathematics achievement: An eight-year longitudinal study.,” *J. Educ. Psychol.*, vol. 109, no. 5, p. 680, 2017.

[11] J. LeFevre*et al.*, “Pathways to mathematics: Longitudinal predictors of performance,” *Child Dev.*, vol. 81, no. 6, pp. 1753–1767, 2010.

[12] M.-T. Wang and J. L. Degol, “ions,” *Educ. Psychol. Rev.*, vol. 29, pp. 119–140, 2017.

[13] H. Bhasin, “The Significance of Advanced Mathematics in Secondary and Higher Education,” *Eur. Econ. Lett.*, vol. 13, no. 3, pp. 1694–1698, 2023.

[14] W. N. E. Saputra, S. Alhadi, A. Supriyanto, and S. Adiputra, “The development of creative cognitive-behaviorcounseling model as a strategy to improve self-regulated learning of student,” *Int. J. Instr.*, vol. 14, no. 2, pp. 627–646, 2021.

[15] R. Lamb, T. Akmal, and K. Petrie, “Development of a cognition-priming model describing learning in a STEM classroom,” *J. Res. Sci. Teach.*, vol. 52, no. 3, pp. 410–437, 2015.

[16] J. Irvine, “A Framework for Comparing Theories Related to Motivation in Education.,” *Res. High. Educ. J.*, vol. 35, 2018.

[17] M. S. Hannula*et al.*, *Attitudes, beliefs, motivation and identity in mathematics education: An overview of the field and future directions*. Springer Nature, 2016.

[18] P. Peng and R. A. Kievit, “The development of academic achievement and cognitive abilities: A bidirectional perspective,” *Child Dev. Perspect.*, vol. 14, no. 1, pp. 15–20, 2020.

[19] A. Efklides, M. Papadaki, G. Papantoniou, and G. Kiosseoglou, “Effects of cognitive ability and affect on school mathematics performance and feelings of difficulty,” *Am. J. Psychol.*, vol. 110, no. 2, pp. 225–258, 1997.

[20] I. Deary, S. Strand, P. Smith, and C. Fernandes, “Intelligence and educational experience,” *Intelligence*, vol. 35, pp. 12–21, 2007.

[21] G. E. Taub, T. Z. Keith, R. G. Floyd, and K. S. McGrew, “Effects of general and broad cognitive abilities on mathematics achievement.,” *Sch. Psychol. Q.*, vol. 23, no. 2, p. 187, 2008.

[22] A. Eichler and J. Gradwohl, “Investigating motivational and cognitive factors which impact the success of engineering students,” *Int. J. Res. Undergrad. Math. Educ.*, pp. 1–21, 2021.

[23] M. Barak, “Fostering learning in the engineering and technology class: From content-oriented instruction toward a focus on cognition, metacognition and motivation,” in *Fostering human development through engineering and technology education*, Brill, 2011, pp. 35–53.

[24] A. H. Schoenfeld, “Pólya, problem solving, and education,” *Math. Mag.*, vol. 60, no. 5, pp. 283–291, 1987.

[25] G. T. G. Shim, A. M. H. A. Shakawi, and F. L. Azizan, “Relationship between Students’ Diagnostic Assessment and Achievement in a Pre-University Mathematics Course.,” *J. Educ. Learn.*, vol. 6, no. 4, pp. 364–371, 2017.

[26] N. Karasar, “Bilimselaraştırmayöntemi.[Scientific research method] Ankara.” Nobel Press, 2008.

[27] J. De Lange, “Large-scale assessment and mathematics education,” *Second Handb. Res. Math. Teach. Learn.*, vol. 2, pp. 1111–1144, 2007.

[28] C. D. Riggs, S. Kang, and O. Rennie, “Positive impact of multiple-choice question authoring and regular quiz participation on student learning,” *CBE—Life Sci. Educ.*, vol. 19, no. 2, p. ar16, 2020.

[29] J. F. Hair, R. E. Anderson, R. L. Tatham, and W. C. Black, “Multivariate Data Analysis. 5th Edn Prentice Hall International,” *Up. Saddle River, NJ*, 1998.

[30] A. F. Hayes, “An index and test of linear moderated mediation,” *Multivariate Behav. Res.*, vol. 50, no. 1, pp. 1–22, 2015.

[31] A. F. Hayes, “Beyond Baron and Kenny: Statistical mediation analysis in the new millennium,” *Commun. Monogr.*, vol. 76, no. 4, pp. 408–420, 2009.

[32] P. Lei and Q. Wu, “Introduction to structural equation modeling: Issues and practical considerations,” *Educ. Meas. issues Pract.*, vol. 26, no. 3, pp. 33–43, 2007.

[33] P. J. Curran, S. G. West, and J. F. Finch, “The robustness of test statistics to nonnormality and specification error in confirmatory factor analysis.,” *Psychol. Methods*, vol. 1, no. 1, p. 16, 1996.

[34] D. Iacobucci, “Structural equations modeling: Fit indices, sample size, and advanced topics,” *J. Consum. Psychol.*, vol. 20, no. 1, pp. 90–98, 2010.

[35] P. D. Ellis, *The essential guide to effect sizes: Statistical power, meta-analysis, and the interpretation of research results*. Cambridge university press, 2010.

[36] G. E. Gignac and E. T. Szodorai, “Effect size guidelines for individual differences researchers,” *Pers. Individ. Dif.*, vol. 102, pp. 74–78, 2016.

[37] A. F. Hayes and K. J. Preacher, “Quantifying and testing indirect effects in simple mediation models when the constituent paths are nonlinear,” *Multivariate Behav. Res.*, vol. 45, no. 4, pp. 627–660, 2010.

[38] K. J. Preacher, D. D. Rucker, and A. F. Hayes, “Addressing moderated mediation hypotheses: Theory, methods, and prescriptions,” *Multivariate Behav. Res.*, vol. 42, no. 1, pp. 185–227, 2007.

[39] R. M. Baron and D. A. Kenny, “Baron and Kenny(1986),” *J. Pers. Soc. Psychol.*, vol. 51, no. 6, p. 1173, 1986.

[40] M. S. Fritz and D. P. MacKinnon, “Required sample size to detect the mediated effect,” *Psychol. Sci.*, vol. 18, no. 3, pp. 233–239, 2007.

[41] K. A. Bollen and R. Stine, “Direct and indirect effects: Classical and bootstrap estimates of variability,” *Sociol. Methodol.*, pp. 115–140, 1990.

[42] J. F. Hair Jr and L. P. Fávero, “Multilevel modeling for longitudinal data: concepts and applications,” *RAUSP Manag. J.*, vol. 54, pp. 459–489, 2019.

[43] K. A. Bollen and R. A. Stine, “Bootstrapping goodness-of-fit measures in structural equation models,” *Sociol. Methods Res.*, vol. 21, no. 2, pp. 205–229, 1992.

[44] Y.-F. Yung and P. M. Bentler, “Bootstrapping techniques in analysis of mean and covariance structures,” *Adv. Struct. Equ. Model. Issues Tech.*, pp. 195–226, 1996.

[45] J. R. Edwards and L. S. Lambert, “Methods for integrating moderation and mediation: a general analytical framework using moderated path analysis.,” *Psychol. Methods*, vol. 12, no. 1, p. 1, 2007.

[46] K. J. Preacher and K. Kelley, “Effect size measures for mediation models: quantitative strategies for communicating indirect effects.,” *Psychol. Methods*, vol. 16, no. 2, p. 93, 2011.

[47] L. R. Fabrigar and D. T. Wegener, “Exploring causal and noncausal hypotheses in nonexperimentaldata.,” 2014.

[48] D. L. Jackson, J. A. Gillaspy Jr, and R. Purc-Stephenson, "Reporting practices in confirmatory factor analysis: an overview and some recommendations.," *Psychol. Methods*, vol. 14, no. 1, p. 6, 2009.

[49] C. Leys, O. Klein, Y. Dominicy, and C. Ley, "Detecting multivariate outliers: Use a robust variant of the Mahalanobis distance," *J. Exp. Soc. Psychol.*, vol. 74, pp. 150–156, 2018.

[50] J. Nevitt and G. R. Hancock, "Performance of bootstrapping approaches to model test statistics and parameter standard error estimation in structural equation modeling," *Struct. Equ. Model.*, vol. 8, no. 3, pp. 353–377, 2001.

[51] X. Fan and L. Wang, "Effects of potential confounding factors on fit indices and parameter estimates for true and misspecified SEM models," *Educ. Psychol. Meas.*, vol. 58, no. 5, pp. 701–735, 1998.

[52] J. F. Hair Jr *et al.*, "An introduction to structural equation modeling," *Partial Least Squares Struct. Equ. Model. Using R A Workb.*, pp. 1–29, 2021.

[53] M. S. Fritz, D. A. Kenny, and D. P. MacKinnon, "The combined effects of measurement error and omitting confounders in the single-mediator model," *Multivariate Behav. Res.*, vol. 51, no. 5, pp. 681–697, 2016.

[54] R. Calantone, J. M. Whipple, J. Wang, H. Sardashti, and J. W. Miller, "A primer on moderated mediation analysis: Exploring logistics involvement in new product development," *J. Bus. Logist.*, vol. 38, no. 3, pp. 151–169, 2017.

[55] J. Hair, C. L. Hollingsworth, A. B. Randolph, and A. Y. L. Chong, "An updated and expanded assessment of PLS-SEM in information systems research," *Ind. Manag. data Syst.*, vol. 117, no. 3, pp. 442–458, 2017.

[56] J. F. Hair, G. T. M. Hult, C. M. Ringle, M. Sarstedt, and K. O. Thiele, "Mirror, mirror on the wall: a comparative evaluation of composite-based structural equation modeling methods," *J. Acad. Mark. Sci.*, vol. 45, pp. 616–632, 2017.

[57] G. T. M. Hult, J. F. Hair Jr, D. Proksch, M. Sarstedt, A. Pinkwart, and C. M. Ringle, "Addressing endogeneity in international marketing applications of partial least squares structural equation modeling," *J. Int. Mark.*, vol. 26, no. 3, pp. 1–21, 2018.

[58] J. F. H. Jr, W. C. Black, B. J. Babin, R. E. Anderson, W. C. Black, and R. E. Anderson, *Multivariate Data Analysis*. 2018. doi: 10.1002/9781119409137.ch4.

[59] C. Fornell and D. F. Larcker, "Evaluating structural equation models with unobservable variables and measurement error," *J. Mark. Res.*, vol. 18, no. 1, pp. 39–50, 1981.

[60] E. Hair, T. Halle, E. Terry-Humen, B. Lavelle, and J. Calkins, "Children's school readiness in the ECLS-K: Predictions to academic, health, and social outcomes in first grade," *Early Child. Res. Q.*, vol. 21, no. 4, pp. 431–454, 2006.

[61] R. Ho, *Handbook of univariate and multivariate data analysis and interpretation with SPSS*. CRC press, 2006.

[62] D. P. MacKinnon, C. M. Lockwood, and J. Williams, "Confidence limits for the indirect effect: Distribution of the product and resampling methods," *Multivariate Behav. Res.*, vol. 39, no. 1, pp. 99–128, 2004.

[63] A. F. Hayes and M. Scharkow, "The relative trustworthiness of inferential tests of the indirect effect in statistical mediation analysis: does method really matter?," *Psychol. Sci.*, vol. 24, no. 10, pp. 1918–1927, 2013.

[64] A. Lessani, A. S. Yunus, K. A. Bakar, and Z. Khameneh, "Comparison of learning theories in mathematics teaching methods," in *Fourth 21st CAF Conference in Harvard, Boston, Massachusetts, USA*, 2016, vol. 9, no. 1, p. 10.

[65] R. E. Simamora, D. R. Sidabutar, and E. Surya, “Improving learning activity and students’ problem solving skill through problem based learning (PBL) in junior high school,” *Int. J. Sci. Basic Appl. Res.*, vol. 33, no. 2, pp. 321–331, 2017.

[66] H. Innabi and O. El Sheikh, “The change in mathematics teachers’ perceptions of critical thinking after 15 years of educational reform in Jordan,” *Educ. Stud. Math.*, vol. 64, pp. 45–68, 2007.

[67] M. Salam, N. Ibrahim, and M. Sukardjo, “Effects of Instructional Models and Spatial Intelligence on the Mathematics Learning Outcomes after Controlling for Students’ Initial Competency.,” *Int. J. Instr.*, vol. 12, no. 3, pp. 699–716, 2019.