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Numerical Solution for a Slanted Crack Issues in Thermoelectric Bonded Materials under Diverse Mechanical Loading Conditions

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Abstract:- A new mathematical model is devised for the analytical examination of slanted crack issues in thermoelectric bonded materials (TEBMs) under diverse mechanical loading conditions, including shear, normal, tearing, and mixed stresses. To tackle these problems, we utilize the modified complex variable function (MCVF) method and introduce continuity conditions for both the resultant electric force and displacement electric function. This enables us to formulate hypersingular integral equations (HSIEs). The crack opening displacement (COD) function, electric current density, and energy flux load are transformed into a square root singularity function using the curved length coordinate method. Subsequently, we employ appropriate quadrature formulas to solve these equations numerically, with the traction along the crack serving as the right-hand term. The COD function obtained from our analysis is then used to compute dimensionless stress intensity factors (DSIFs). These DSIFs play a pivotal role in determining the stability behavior of TEBMs containing a slanted crack. Our numerical results for DSIFs at all crack tips are presented, and they are found to be in excellent agreement with previous research. It is important to note that the DSIFs at the crack tips are influenced by mechanical loading conditions, the ratio of elastic constants, crack geometries, and electric conductivity.

Keywords*: Bonded materials, Hypersingular integral equations, Slanted crack, Stress intensity factors, Thermoelectric.*

1. Introduction

Numerous scholars have explored and scrutinized the challenges associated with cracks in materials, be they infinite plane [1, 2], half plane [3, 4], or bonded plane [5, 6]. Nevertheless, all these investigations have exclusively focused on a singular type of mechanical loading, specifically shear or mixed stresses. These materials encompass various properties, such as elasticity [7, 8], thermoelasticity [9, 10], magnetoelasticity [11, 12], and electrical conductivity [13, 14]. Among the significant structures in manufacturing, thermoelectric materials hold particular importance.

The determination of dimensionless stress intensity factors (DSIFs) for fractures linked to a circumferential crack in a thermoelectric plate or shell exposed to thermal and electric shocks was accomplished through the application of a singular integral equation [13]. The variational principle is used to create connected equations that describe both the thermal and mechanical aspects of the system. Examining nano-sized cracks in thermoelectric material structures subjected to external stress, the finite element method was employed [14]. The study reveals insights into the influence of size effects on the variation of crack opening displacements (COD). The impact of dual collinear interface cracks on the electric potential and temperature in a thermoelectric bonded materials (TEBMs) system under electric and thermal loads was discussed [15]. The study highlights the significant contributions of crack length, crack spacing, and layer thickness ratio to the electric potential and temperature of TEBMs. This analysis is conducted through the application of Laplace equations to the DSIFs. The resulting integral-differential equations, describing the coupling of electric, thermal, and elastic effects, are formulated in terms of interface shear stress and tensile stress for both the thermoelectric film and the elastic substrate [16]. They model a thermoelectric thin film attached to a flexible infinite substrate, considering the film's bending stiffness. By considering the balance of stress and compatibility of strain between the film and the substrate, they derive a set of integral equations. The findings indicate that the tensile and shear stresses at the interface show singularity at the thin film's end. The singularity behavior of a thermoelectric thin film bonded to an elastic infinite substrate was characterized using integral equations, with the stress intensity factor serving as a descriptor [17]. The findings emphasize the importance of accounting for film stiffness in accurately assessing the thermal stress level within the thermoelectric film. The DSIFs at the tips of an interface crack in TEBMs subjected to a remote electric current, utilizing complex variable functions was examined [18]. Their results revealed that the influence of the electric current on DSIFs is contingent upon the parameters of the bonded material. Depending on these parameters, the electric current may either amplify or counteract the thermal stress intensity factors.

This study focuses on numerical solution for a slanted crack issues in TEBMs under diverse mechanical loading conditions such as shear ($\sigma_x^\infty = p$) and normal ($\sigma_y^\infty = p$) stresses as shown in Fig. 1. This study is the extended from Mohd Nordin et al. [19] that only focus on the formulation.

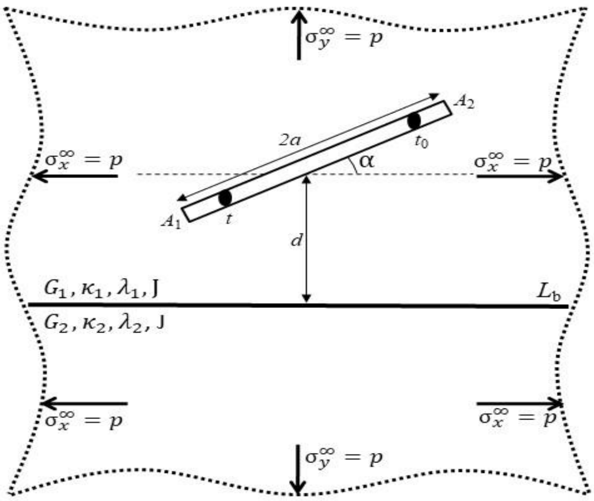


Fig. 1 A slanted crack in thermoelectric bonded materials

2. Mathematical Formulation

According to Mohd Nordin et al. [19] the determination of normal and tangential components $N+iT$ along the alternative segment of a crack is influenced by the traction, revealing the HSIEs for a slanted crack positioned in the upper part of bonded materials as follows

$$\left[N(t_0)+iT(t_0)\right]_1 = \frac{1}{\pi}\int_L \frac{g(t)dt}{\left(\bar{t}-\bar{t}_0\right)^2} + \frac{1}{2\pi}\int_L M_1(t,t_0)g(t)dt + \frac{1}{2\pi}\int_L M_2(t,t_0)\overline{g(t)}d\bar{t} + M_3(a,t_0) \tag{1}$$

where

$$M_1(t,t_0) = -\frac{1}{(t-t_0)^2} + \frac{1}{(\bar{t}-\bar{t}_0)^2}\frac{d\bar{t}}{dt}\frac{d\bar{t}_0}{dt_0}$$
$$+\Gamma_1\left[\frac{1}{(t-\bar{t}_0)^2} + \frac{2(\bar{t}_0-t)}{(t-\bar{t}_0)^3} + \left(\frac{1}{(\bar{t}-t_0)^2} + \frac{1}{(t-\bar{t}_0)^2}\right)\frac{d\bar{t}}{dt}\right.$$
$$+\left(\frac{2(2t_0-3\bar{t}_0+\bar{t})}{(t-\bar{t}_0)^3} - \frac{6(\bar{t}_0-\bar{t})(\bar{t}_0-t)}{(t-\bar{t}_0)^4}\right.$$
$$\left.\left.-\left(\frac{1}{(t-\bar{t}_0)} + \frac{2(\bar{t}_0-t_0)}{(t-\bar{t}_0)^3}\right)\frac{d\bar{t}}{dt} - \frac{1}{(t-\bar{t}_0)^2}\right)\frac{d\bar{t}_0}{dt_0}\right]$$

$$M_2(t,t_0) = \frac{1}{(\bar{t}-\bar{t}_0)^2}\frac{d\bar{t}}{dt} + \left(\frac{1}{(\bar{t}-\bar{t}_0)^2} + \frac{2(t_0-t)}{(\bar{t}-\bar{t}_0)^3}\frac{d\bar{t}}{dt}\right)\frac{d\bar{t}_0}{dt_0}$$
$$+\Gamma_1\left[\frac{1}{(\bar{t}-t_0)^2} + \frac{1}{(t-\bar{t}_0)^2} + \left(\frac{1}{(\bar{t}-t_0)^2} + \frac{2(t_0-t)}{(\bar{t}-t_0)^3}\right)\frac{d\bar{t}}{dt}\right.$$
$$\left.+\left(\frac{2(t_0-\bar{t}_0)}{(t-\bar{t}_0)^3} - \frac{1}{(t-\bar{t}_0)^2}\right)\frac{d\bar{t}_0}{dt_0}\right]$$

$$M_3(a,t_0) = G_2\frac{J^2}{4l_1^2}\left(t_0^2 - a^2 + \frac{t_0}{2\sqrt{t_0^2-a^2}}\left(t_0\sqrt{t_0^2-a^2} - a^2\ln\left(t_0+\sqrt{t_0^2-a^2}\right)\right)\right)$$
$$+\left(G_2 + (G_6 - G_2)\frac{d\bar{t}_0}{dt_0}\right)\frac{J^2}{4l_1^2}$$
$$\cdot\left(\bar{t}_0^2 - a^2 + \frac{t_0}{2\sqrt{t_0^2-a^2}}\left(\bar{t}_0\sqrt{t_0^2-a^2} - a^2\ln\left(\bar{t}_0+\sqrt{\bar{t}_0^2-a^2}\right)\right)\right)$$
$$+G_2\frac{J^2}{4l_1^2}\left(t_0 - \bar{t}_0\right)$$
$$\cdot\left(3\bar{t}_0 + \left(t_0\sqrt{\bar{t}_0^2-a^2} - a^2\ln\left(\bar{t}_0+\sqrt{\bar{t}_0^2-a^2}\right)\right)\right.$$
$$\left.\cdot\frac{\left(\bar{t}_0^2 - \bar{t}_0 - a^2\right)}{2\left(\sqrt{\bar{t}_0^2-a^2}\right)^3}\right)\frac{d\bar{t}_0}{dt_0} + G_3\frac{J^2}{4l_1^2}$$
$$\cdot\left(2a^2 - t_0^2 - \bar{t}_0^2 + \left(3\bar{t}_0^2 - 2t_0\bar{t}_0 - a^2\right)\frac{d\bar{t}_0}{dt_0}\right)$$
$$+G_4\frac{iU}{2}\left(\sqrt{t_0^2-a^2} - \sqrt{\bar{t}_0^2-a^2} + \frac{2\bar{t}_0^2 - \bar{t}_0 t_0 - a^2}{\sqrt{\bar{t}_0^2-a^2}}\frac{d\bar{t}_0}{dt_0}\right)$$
$$+\left[\left(\frac{G_7}{k_1} - \frac{G_8}{l_1}\right)\frac{J^2}{4l_1}\left(\bar{t}_0^2 - a^2\right) + \left(\frac{G_7}{k_1} - \frac{G_9}{2}\right)\right.$$
$$\left.\cdot iU\sqrt{\bar{t}_0^2-a^2}\right]\frac{d\bar{t}_0}{dt_0} + (G_{17}+G_{18})\frac{J^2}{4l_1^2}$$
$$\cdot\left[\sqrt{t_0^2-a^2}\sqrt{\bar{t}_0^2-a^2} + \left(t_0\sqrt{t_0^2-a^2} - a^2\ln\left(t_0+\sqrt{t_0^2-a^2}\right)\right)\right.$$
$$\left.\frac{\bar{t}_0}{2\sqrt{\bar{t}_0^2-a^2}}\frac{d\bar{t}_0}{dt_0}\right]$$

Note that, $g(t)$ is COD, G_j is shear modulus, Γ_j is bi-elastic constant ratio, J is electric current density vector, and U is energy flux load.

Examine the geometric considerations relevant to slanted crack phenomena in TEBMs under diverse mechanical loading conditions, encompassing both shear and normal stresses, as depicted in Fig. 1. Keep in mind that d represents the distance between the crack center and the boundary L_b, while $2a$ denotes the length of the crack. To calculate DSIFs, it is crucial to incorporate the distinct types of mechanical loading associated with the crack issues. In the case of shear stress, define the conditions for the normal and tangential components at an angle of the crack α as follows [20]

$$\frac{1}{E_1}\sigma_{x_1} = \frac{1}{E_2}\sigma_{x_2}, \quad N + iT = -p\sin^2\alpha - ip\sin\alpha\cos\alpha \tag{2}$$

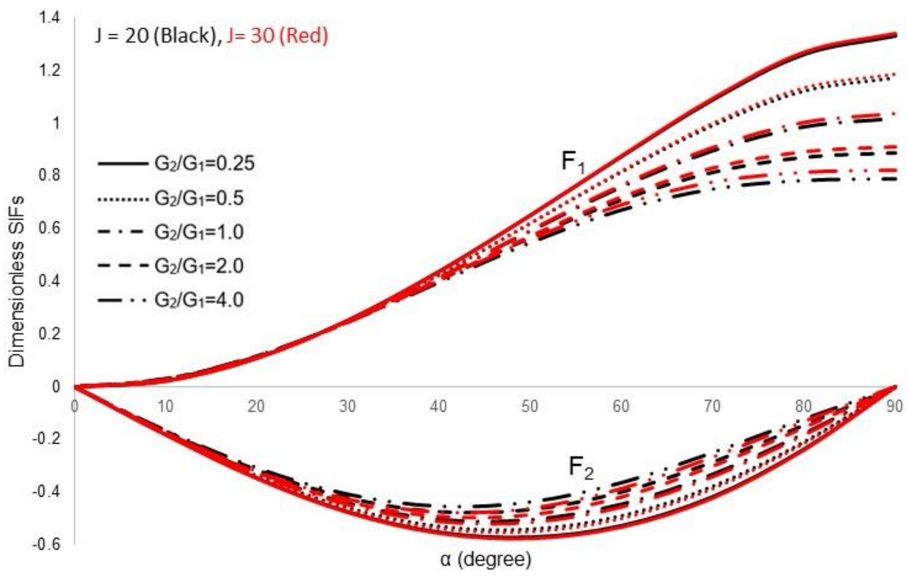
whereas normal stress is defined as follows

$$\frac{1}{E_1}\sigma_{y_1} = \frac{1}{E_2}\sigma_{y_2}, \quad N + iT = -p\cos^2\alpha + ip\sin\alpha\cos\alpha \tag{3}$$

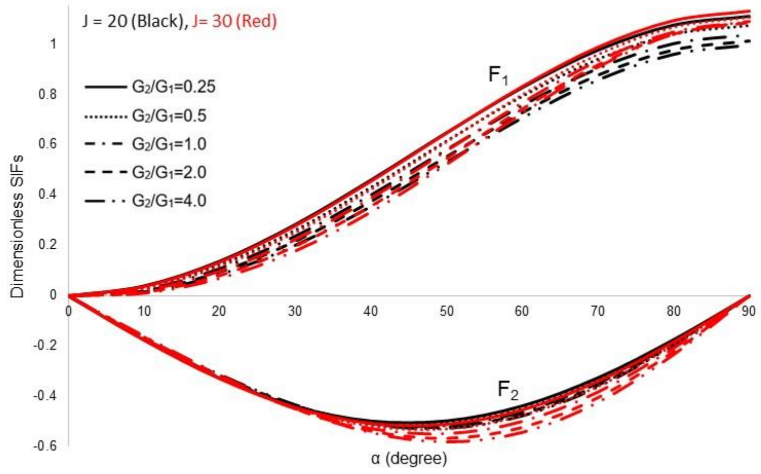
Note that, $E_j = 2G_j\left(1+\tau_j\right)$ and $j = 1,2$ are Young's modulus of elasticity for top and bottom parts of TEBMs.

3. Results And Discussion

Fig. 2 displays the DSIFs ($F_1 + iF_2$) at all crack tips for a slanted crack issues in TEBMs under shear ($\sigma_x^\infty = p$) stress (Fig. 1) for J = 20 (Black), J = 30 (Red), U = 0, $d = a/0.9$, and α is varies. It is found that as α increases the F_1 increases at all crack tips, however F_2 increases for $\alpha > 45°$.

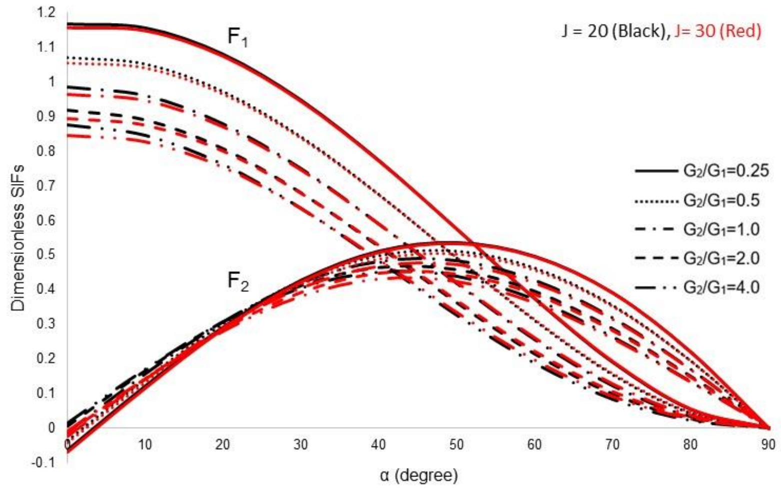


(a) Crack tip A_1

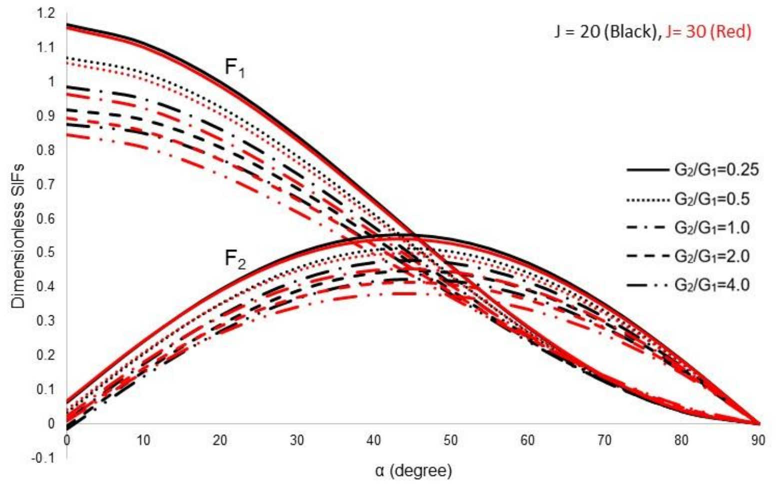


(b) Crack tip A₂

Fig. 2 DSIFs subject to shear stress at all crack tips.



(a) Crack tip A_1



(b) Crack tip A_2

Fig. 3 DSIFs subject to normal stress at all crack tips.

Whereas as G_2/G_1 increases F_1 decreases at all crack tips, however F_2 increases at crack tip A_1 and decreases at crack tip A_2. For J = 30 (Red) is higher than J = 20 (Black) at all crack tips. The numerical outcomes present indications that an increase in α and J results in a reduction in the materials' strength, whereas an increases in G_2/G_1 leads to greater stability in the materials strength.

Fig. 3 displays the DSIFs ($F_1 + iF_2$) at all crack tips for a slanted crack issues in TEBMs under normal ($\sigma_y^\infty = p$) stress (Fig. 1) for J = 20 (Black), J = 30 (Red), U = 0, $d = a/0.9$, and α is varies. It is found that as α increases the F_1 decreases at all crack tips, however F_2 increases for $\alpha < 45°$. Whereas as G_2/G_1 increases F_1 and F_2 decrease at all crack tips. For J = 20 (Black) is higher than J = 30 (Red) at all crack tips. The numerical outcomes present indications that an increase in α, G_2/G_1 and J leads to greater stability in the materials strength.

4. Conclusion

In this specific investigation, our attention has been directed towards addressing issues related to slanted cracks in TEBMs under various mechanical loading conditions, including both shear and normal stresses. The analysis of numerical results leads to the conclusion that the behavior of DSIFs for a slanted crack situated in the upper part of TEBMs is contingent upon multiple factors. These factors encompass the bi-elastic constant ratio, electric current density, crack geometries, and the distance between the crack and the boundary, all of which exert an influence on the strength of the materials. Drawing from this study, we envision several potential extensions, including the exploration of cohesive models, cracks at the interface of bonded materials, cracks induced by inclusions, three-dimensional crack problems in TEBMs, and more. A comprehensive formulation, accompanied by numerical analysis derived from this research, will be detailed in a separate publication. Further research endeavors are underway to broaden the application scope of the developed concept.

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