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Velocity Distributions of Mhd Poiseuille Fluid Flow in a Rotating Channel

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Abstract— This paper is mainly aimed to study a particular class of MHD problems to cover the steady case and to find the effect of change in the magnetic field, loading parameter, rotational parameter, Hall parameter and Hartmann number in the finite parallel plate and infinite parallel plate geometries of Hartmann flow. As it is important from the theoretical as well as from the practical point of view we have considered the MHD Poiseuille flow between two parallel plates (i.e., the case when one plate is at y=0, and other is y=1, and the case when one plate is at y=0 and other plate is at y=∞). From the data obtained and the graphs drawn for both cases, we discussed the effects of pressure gradient ( ), loading parameter ( ), rotation parameter( ), Hartmann number( ),and Hall parameter( ), taking one at a time and the other parameters are kept fixed for the MHD fluid model.

Keywords— *Pressure gradient ( ), Loading parameter ( ), Rotation parameter ( ), Hartmann number ( ), Hall parameter ( ).*

Introduction

A steady MHD plane Poiseuille flow between two parallel plates when a transverse magnetic field is applied perpendicular to the plates, is investigated. The fluid between the parallel plates is set in motion not by the upper plate but by a pressure gradient P in the X-direction. It is intended, in this paper to study the effects of the Hall currents on velocity and temperature fields. The Hall effect is caused by electromagnetic field and the hall electric field is represented by cross product $\bar{J}\ X\ \bar{B}$,which appears in ohm's law because the hall current is perpendicular to both $\bar{B}$ and$(\bar{E} + \bar{\nabla} X\ \bar{B})$, it is that component of the electric current which is parallel but opposite in direction to the flow velocity, where as the conduction current is in the direction of $\bar{\nabla} X\ \bar{B}$. The magnitude of the hall effect is determined by the product $\omega_c\tau$. Where ω_c is electronic frequency and τ is the mean free time. The hall effects become important in engineering applications, when the value of the product $\omega_c\tau$ approaches or exceeds unity. The rotating channel that we considered in paper has the following physical meaning. If we introduce channel into a rotating flow which type of situation occurs in geophysical problems, then the channel also rotates. This causes a primary and secondary velocity for the flow. This branch of fluid mechanics has developed rapidly in recent years as an obvious consequence of interest in geophysical flow problems. Evaluation of the parameters show that the motions, particularly on the large scale, of the Earth's atmosphere, oceans, and core of the earth and of stars and galaxies will all exhibit the effects discussed in paper. The rotation gives rise to a range of new phenomena, here we consider a small selection of these.

The whole subject could be formulated as seen by an observer external to the rotation. Since however, all the boundary conditions will be specified in terms of the rotating frame of reference, it is easier to modify the equation of motion so that they apply in such a frame, if one takes a body of fluid and rotates its boundaries at a constant angular velocity Ω, Then at any time sufficiently long after starting the rotation, the whole body is rotates with angular velocity, moving as if it were a rigid body. There are no viscous stresses acting with in the fluid. Any disturbance i.e., anything that would produce a motion in a non-rotating system-will produce motion can be considered as the flow pattern, it is the pattern that will e observed by an observer to fixed the rotating boundaries. The effect of using a rotating frame of reference is well known from the mechanics of solid systems, there are accelerations associated with the use of a non-inertial frame that can be taken into account by introducing centrifugal and coriolis forces. The difference between the dynamics of nonrotating and rotating fluids is the coriolis force. This coriolis force occurs as one of the terms in the Navier-Stoke's equations. Because of the importance of the coriolis force an attempt is made to incorporate rotation in the flow model taken. The problem studied in this paper is to find an exact solution for the magneto dynamic Poiseuille flow and heat transfer of an electrically conducting incompressible fluid taking rotation parameter, loading parameter, pressure gradient, Hall parameter and Hartmann number into consideration for in depth study of this flow model. Sherman and Sutton (1961) studied the Hartmann flow problem, without taking rotation and pressure gradient in the equation. The present paper is an extension of this problem taking the clue from Huges and Young (1966). The effects of non-dimensional pressure gradient for a generalized couette flow i.e., the superposition of a plane Poiseuille flow over a plane couette flow are studied.

Formulation Of The Flow Model And Its Solution

Consider the steady fluid flow of electrically conducting fluid between two infinite parallel plates, when the fluid and plates rotate with angular velocity Ω about an axis normal to the plates. Let a uniform magnetic field B_0acts normal to the plates parallel to the z-axis. The XY- plane coincides with the stationary plate and the plate Z=D moves with a uniform velocity U_0 in the X- direction. For this the Navier-Stoke's equation for the motion of the viscous fluid with Lorentz force for the fluid in the geometry chosen, when the flow quantities are independent of X and Y, are given by

$$-2\Omega V = \frac{\mu}{\rho} \frac{d^2U}{dZ^2} + \frac{J_Y B_0}{\rho} \quad (1)$$

$$-2\Omega U = \frac{\mu}{\rho} \frac{d^2V}{dZ^2} - \frac{J_X B_0}{\rho} \quad (2)$$

Where U, V are the components of velocity of the fluid in XY- direction, J_X, J_Y are the components of current density respectively, μ the viscosity, ρ the fluid density.

In deriving the above equations, the magnetic Reynolds number is assumed to be small so as to ensure that distortions in the magnetic field will not effect the flow field. Therefore B_0 can be substituted for the magnetic field in the Lorentz force which is$\frac{1}{\rho}(\bar{J}\ X\ \bar{B})$.The current density components follow from the modified ohm's law, ignoring the electron pressure and ion slip.

$$\bar{J} = \sigma\left(\bar{E} + \overline{Q}\, X\, \overline{B} - \frac{1}{n_e}(\bar{J}\, X\, \overline{B})\right) \quad (3)$$

Where 'n' is the number density of electrons,-e the electron charge, $\bar{B}(B_X, B_Y, B_0)$ the magnetic induction vector and $\bar{E}(\,E_X = C_1, E_Y = C_2, E_Z\,(Z))$ the electric field relative to the rotating frame.It follows from the equation (4)

$$J_X = \frac{\sigma}{1+h_p^2}\left[\,(\,E_X + V\,B_0 - \,h_p^2\,(\,E_X - U\,B_0)\,)\right] \quad (4)$$

$$J_Y = \frac{\sigma}{1+h_p^2}\left[(\,E_Y - U\,B_0 + \,h_p^2\,(\,E_X + V\,B_0\,)\,)\right] \quad (5)$$

Where $h_p\ (Hall\, parameter) = \frac{\sigma\, B_0}{n_e}$ (6)

Equations (1) and (2) with the value of J_X and J_Y substituted from the equations (4) and (5) assuming the following forms:

$$-R_p\, v = \frac{d^2u}{dz^2} + \frac{H_n^2}{1+h_p^2}\left(E_Y - u + h_p\,(E_X + v)\right) \qquad (7)$$

$$+R_p\, u = \frac{d^2v}{dz^2} + \frac{H_n^2}{1+h_p^2}\left(-E_X - v + h_p\,(E_Y - u)\right) \qquad (8)$$

In terms of the non-dimensional variables defined by

$$z = \frac{Z}{D},$$

$$(u, v, 0) = \frac{(U, V, 0)}{U_0}, [E_x, E_y, E_z\,(z)] = \frac{(E_X, E_Y, E_Z)}{U_0 B_0}$$

$$H_n^2(Hartmann\ number) = B_0^2 D^2 \left(\frac{\sigma}{\rho\gamma}\right),$$

$$R_p(rotation\ parameter) = \frac{2\Omega D^2}{\gamma}, (\,b_x,\ b_y,\ 1) = \frac{(\,B_X,\ B_Y,\ B_0\,)}{B_0},$$

$$(\,j_x,\ j_y,\ 0) = \frac{(\,J_X,\ J_Y,\ 0\,)}{\sigma\ U_0\ B_0} \qquad (9)$$

when the electrodes are infinitely far apart, so that there are no variations in the x and y directions, it can be taken as E_X as equal to 0 and E_Y a constant called the loading parameterL_P. Equations (7) and (8) can be combined into a single equation using the velocity u + iv = V. Since our interest is to find out the effect of non-dimensional pressure gradient P as in Huges and Young (1966), the single equation can be written incorporating the pressure gradient term P as

$$\frac{d^2\bar{V}}{dy^2} - M_1 V = -P - \propto_0\ L_p\ (\,1 + ih_p\,) \qquad (10)$$

Where

$$M_1 = \frac{H_n}{1 + h_p^2} - i\left[R_p + \frac{H_n\ h_p}{1 + h_p^2}\right]$$

$$\propto_0 = \frac{H_n}{1 + h_p^2}, \quad \text{and } P = -\ \frac{D^2}{\mu\ U_0}\ \frac{dp}{dx},$$

$$L_p = \frac{E}{U\ B}$$

Equation (10) is solved using two artificial boundary conditions which are assumed as

Case (1) V=0 at Y=0, V=1 at Y=1 ,

case(2) V=0 at Y=, $V \to -\frac{R_1}{M_1}$ as$Y \to \infty$,

Where $R_1 = A + iB$,

$$A = (-P + \propto_0 L_p), \qquad B = -\propto_0 L_p h_p$$

The solutions obtained for each case are

Case(1)

$$V(y) = \frac{\sinh\ y\sqrt{M_1}}{\sinh\ \sqrt{M_1}} + \frac{R_1}{M_1}\left[\left(\cosh\ y\sqrt{M_1} - 1\right) + \frac{\sinh\ y\sqrt{M_1}}{\sinh\sqrt{M_1}}\left(1 - \cosh\ \sqrt{M_1}\right)\right],$$

Case(2)

$$V(y) = \frac{R_1}{M_1}\left[\cosh\ y\sqrt{M_1} - \sinh\ y\sqrt{M_1} - 1\right],$$

Since V = u + i v = primary velocity + i Secondary velocity.

Real and imaginary parts are separated out and written as

Case (1)

$$u(y) = \left[\frac{1}{(\propto_0^2+\beta_0^2)(a_2^2+b_2^2)}\right] * \left[\begin{array}{c} (A\propto_0 + B\,\beta_0)\left[\begin{array}{c}(1-a_4)\ \ (a_1a_2+b_1b_2)\\ +(a_3-1)\ \ (a_2^2+b_2^2)\\ +b_4(a_2b_1-a_1b_2)\end{array}\right] \\ -(A\beta_0 - B\propto_0)\left[\begin{array}{c}(1-a_4)\,(a_1b_2-a_2b_1)\\ -b_3\,(a_2^2+b_2^2)\\ +b_4\,(a_1a_2+b_1b_2)\end{array}\right] + \\ +(\propto_0^2+\beta_0^2)\,(a_1a_2+b_1b_2)\end{array}\right]$$

$$\mathrm{v(y)} = \left[\frac{1}{(\propto_0^2+\beta_0^2)(a_2^2+b_2^2)}\right] * \left[\begin{array}{c} (A\propto_0 + B\,\beta_0)\left[\begin{array}{c}(1-a_4)\,(a_2b_1-a_1b_2)\\ +b_3\,(a_2^2+b_2^2)\\ -b_4\,(a_1a_2+b_1b_2)\end{array}\right] \\ -(A\beta_0 - B\propto_0)\left[\begin{array}{c}(a_3-1)\,(a_2^2+b_2^2)\\ +(1-a_4)\,(a_1a_2+b_1b_2)\\ +b_4\,(a_2b_1-a_1b_2)\end{array}\right] + \\ +(\propto_0^2+\beta_0^2)\,(a_2b_1-a_1b_2)\end{array}\right]$$

Case (2)

u(y)= $\frac{1}{(\propto_0^2+\beta_0^2)}\left[(A\propto_0 + B\,\beta_0)(a_3-a_1-1) - (B\propto_0 - A\beta_0)(b_3-b_1)\right]$ and

v(y) = $\frac{1}{(\propto_0^2+\beta_0^2)}\left[(B\propto_0 - A\beta_0)(a_3-a_1-1) + (A\propto_0 + B\,\beta_0)(b_3-b_1)\right]$

where $a_1 = \sinh yl_1 \cosh yl_2$

$a_2 = \sinh l_1 \cos l_2$

$a_3 = \cosh yl_1 \cos yl_2$

$a_4 = \cosh l_1 \cos l_2$

$b_1 = \cosh\ yl_1 \sin \mathrm{y}\, l_2$

$b_2 = \cosh l_1 \sin\ l_2$

$b_3 = \sinh \mathrm{y}\, l_1 \sin \mathrm{y}\, l_2$

$b_4 = \sinh\ l_1 \sin\ l_2$

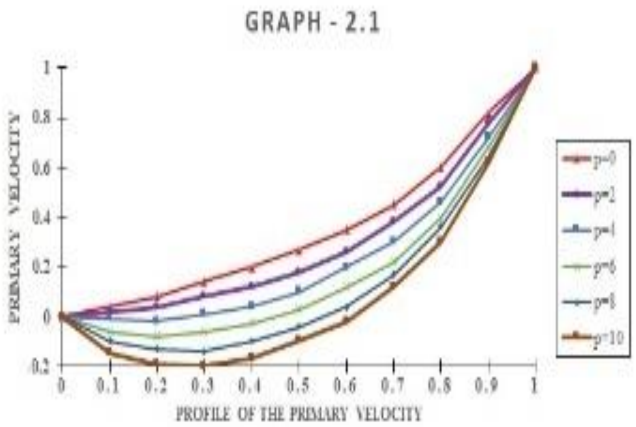
$\beta_0 = \left(R_p + \alpha_0 h_p\right)$

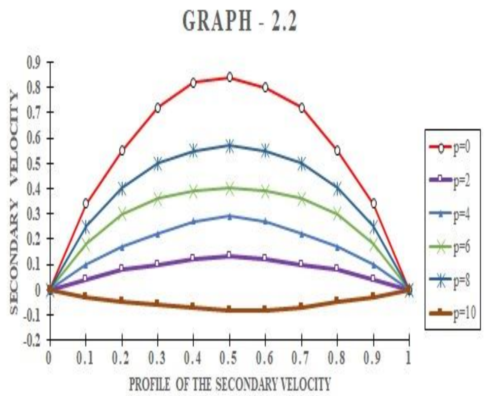
$$l_1 = \pm\sqrt{\frac{\sqrt{{\alpha_0}^2 + {\beta_0}^2} + \alpha_0}{2}}$$

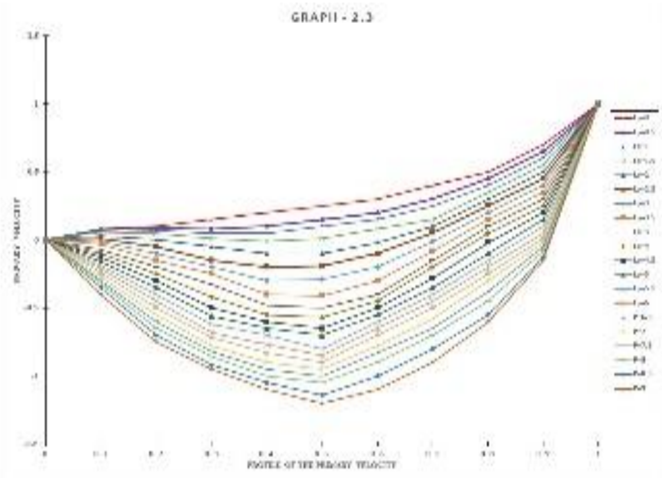
$$l_2 = \pm\sqrt{\frac{\sqrt{{\alpha_0}^2 + {\beta_0}^2} - \alpha_0}{2}}$$

Discussions And Results

Since the paper is aimed to bring out the effects of pressure gradient(P),loading parameter (L_p), rotation parameter(R_p), Hartmann number(H_n),and Hall parameter(h_p), taking one at a time for the MHD fluid model, extensive calculations have been made with the help of computers. From the data obtained the graphs are drawn for the primary, secondary velocities.



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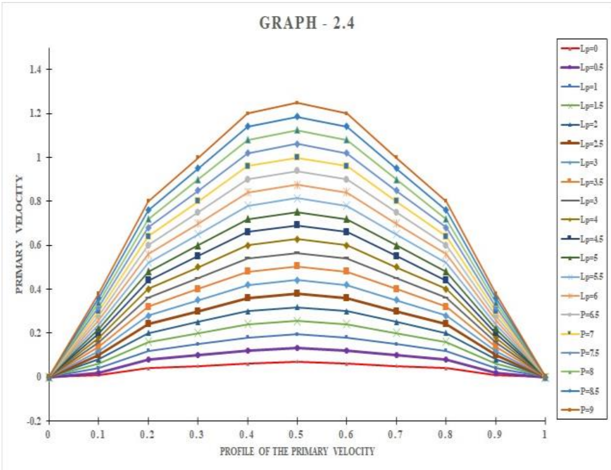
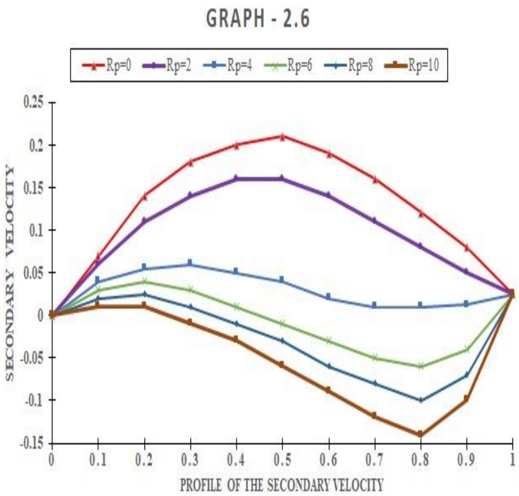
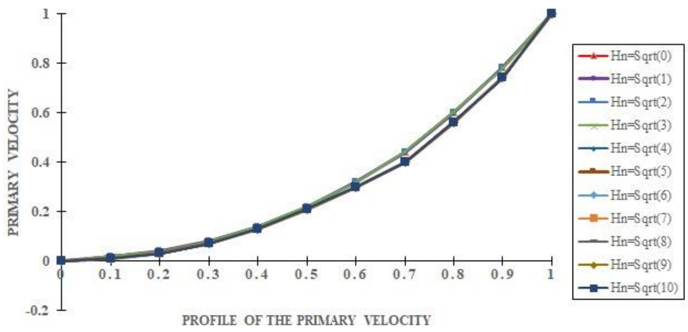
PROFILE OF THE PRIMARY VELOCITY

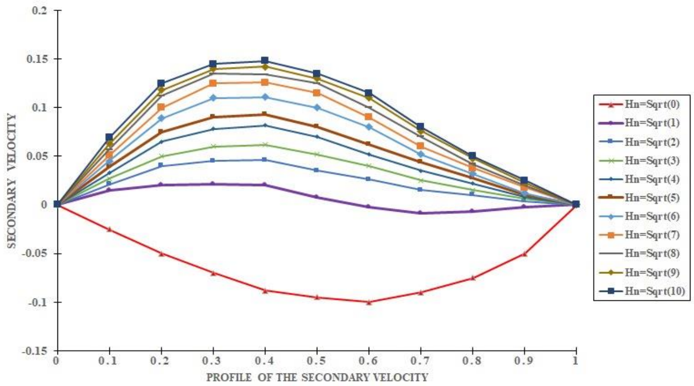

GRAPH - 2.4
PRIMARY VELOCITY
PROFILE OF THE PRIMARY VELOCITY
Lp=0
Lp=0.5
Lp=1
Lp=1.5
Lp=2
Lp=2.5
Lp=3
Lp=3.5
Lp=3
Lp=4
Lp=4.5
Lp=5
Lp=5.5
Lp=6
P=6.5
P=7
P=7.5
P=8
P=8.5
P=9


GRAPH - 2.6
Rp=0
Rp=2
Rp=4
Rp=6
Rp=8
Rp=10
SECONDARY VELOCITY
PROFILE OF THE SECONDARY VELOCITY

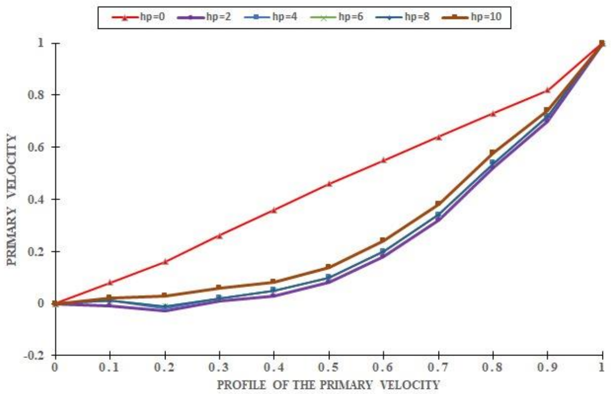
GRAPH - 2.7



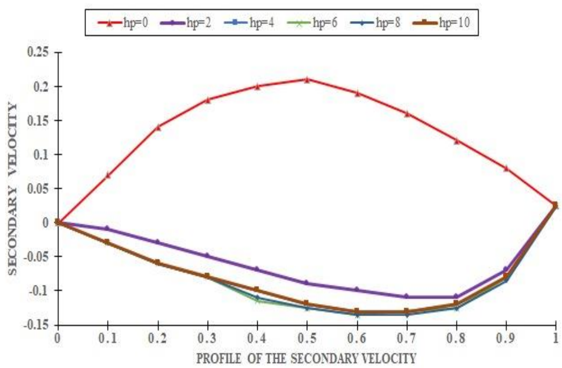
GRAPH - 2.8



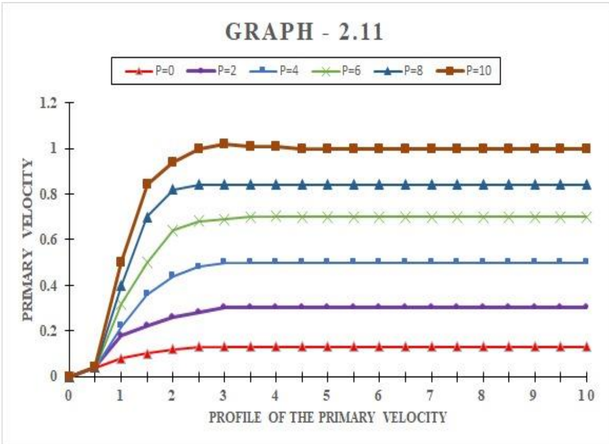
GRAPH - 2.9

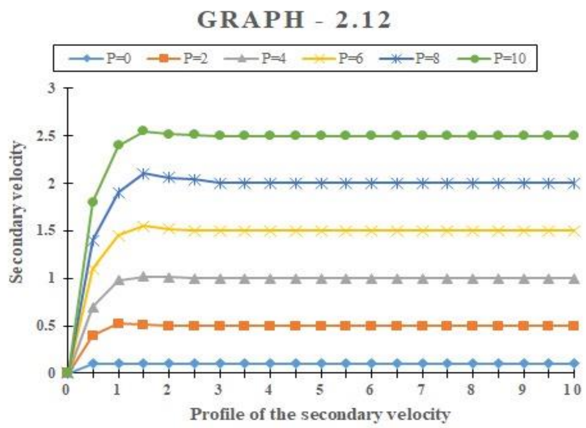


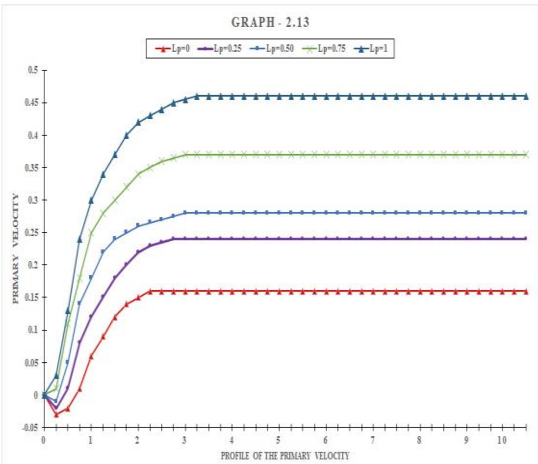
GRAPH - 2.10

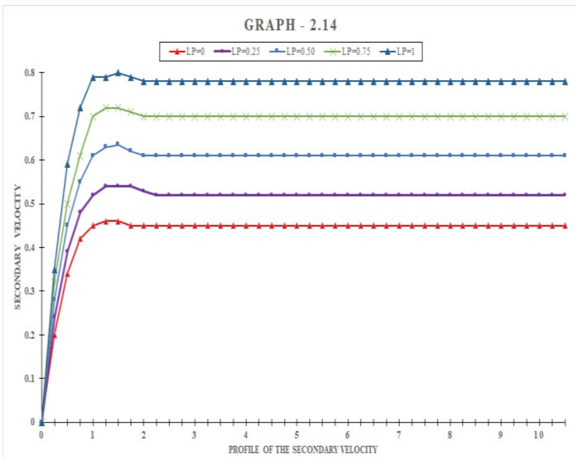


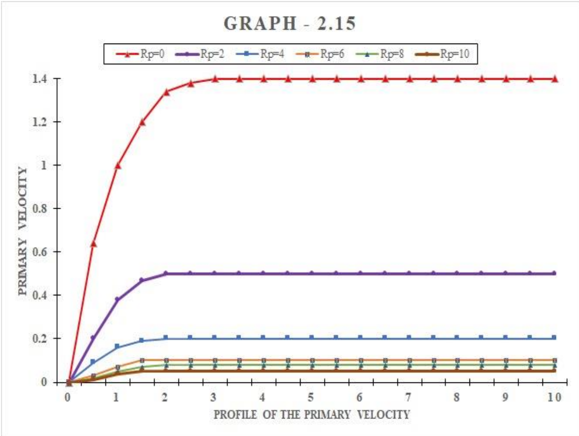
Graphs (2.1 - 2.10) drawn for the case when one plate is at Y=0 and another is at Y=1, it is observed that primary velocity is decreasing at any point of the fluid as pressure gradient increases keeping loading parameter(L_p), rotation parameter (R_p), Hartmann number(H_n), Hall parameter(h_p) are kept fixed. Whereas the secondary velocity is found increasing at any point of the fluid for increasing values of pressure gradient. It is seen that the primary velocity is decreasing, and the secondary velocity is increasing at any point of the fluid as loading parameter increases. It is notice that both the primary and secondary velocities are found decreasing at any point of fluid as rotation parameter increases. Further, it is found that the primary velocity decreases and secondary velocity increases at any point of the fluid for increasing values of the Hartmann number. However, the primary velocity is decreasing, and secondary velocity is increasing at any point of the fluid, in general, as hall parameter increases.

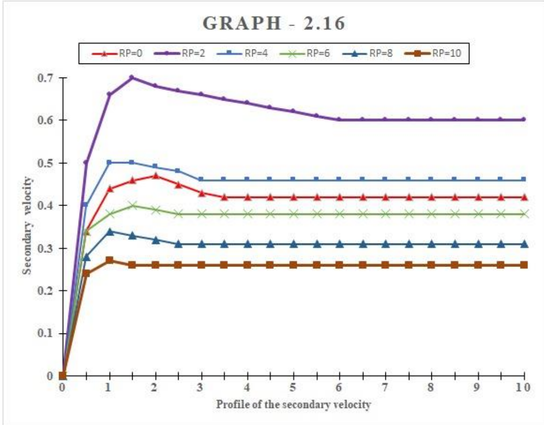


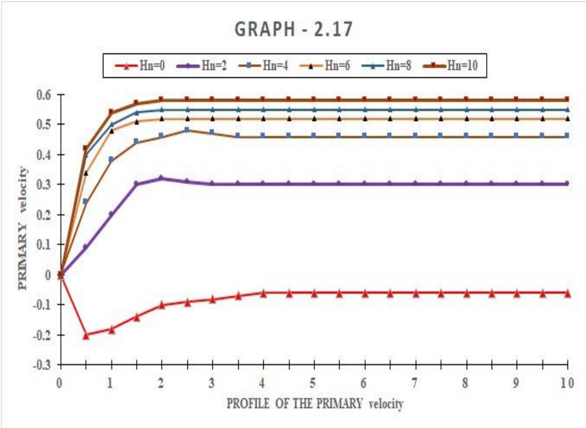


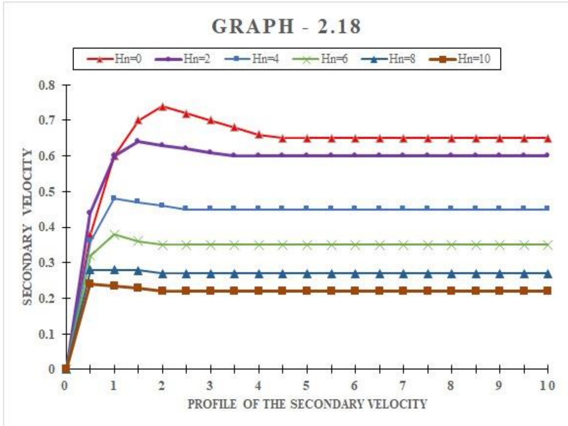


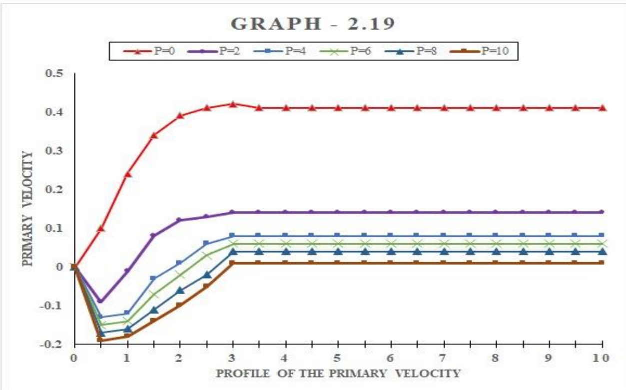


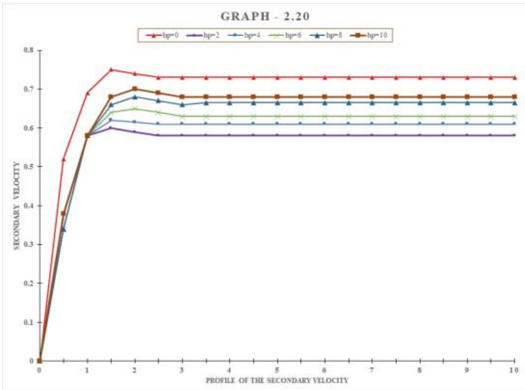












Graphs (2.11 to 2.20) drawn for the case where one plate at Y=0and another plate is at infinity velocity changes are calculated. From these it is observed that both the primary and secondary velocities are increase keeping the other parameters fixed. It is seen that both the primary and secondary velocities are increasing at any point of the fluid as loading parameter increases. It is noticed that both the primary and secondary velocities are found decreasing at any point of the fluid as rotation parameter increases. Further it is found that the primary velocities increases and secondary velocity decreases at any point of the fluid for increasing values of the Hartmann number.

Conclusions

However, the primary velocity is decreasing, and secondary velocity is increasing at any point of the fluid in general as Hall parameter increases. In all these cases only one parameter is changed while the other parameters are kept fixed.

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