

Thermal Slip Effects on Couple Stress Fluid over a Deformable Surface with Aluminium Alloys Nanoparticles

Iskandar Waini¹, Khairum bin Hamzah², Farah Nadzirah Jamrus³, Anuar Ishak⁴, Ioan Pop⁵

¹ *Fakulti Teknologi dan Kejuruteraan Industri dan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, Durian Tunggal, Melaka, Malaysia,*

² *Fakulti Teknologi dan Kejuruteraan Industri dan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, Durian Tunggal, Melaka, Malaysia*

³ *Kolej Pengajian Pengkomputeran, Informatik dan Matematik, Universiti Teknologi MARA (UiTM) Cawangan Melaka Kampus Jasin, Merlimau, Melaka, Malaysia*

⁴ *Department of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, UKM, Bangi, Selangor, Malaysia*

⁵ *Department of Mathematics, Babes-Bolyai University, Romania*

Abstract:- In the current research, a mathematical analysis of couple stress fluid flow and heat characteristics through a deformable surface with hybrid nanoparticles and thermal slip effects is conducted. The solid nanoparticles of the aluminium alloys (AA7072 and AA7075) are suspended in methanol to create the hybrid nanofluid. The similarity approach is used to reduce the governing equations into the similarity equations. Then, MATLAB's bvp4c function is employed to solve the resulting equations. The solutions for the flow and temperature fields, as well as the skin friction coefficient and Nusselt number are presented in table and graphical forms. The results demonstrate that hybrid nanofluids excel as thermal conductors, significantly augmenting the heat transfer rate. The heat transfer rates was occurred almost at the same value for regular fluid and hybrid nanofluid when no thermal slip is considered. However, their values are significantly changed by the imposition of the thermal slip with 5.19% increment when 2% of hybrid nanoparticle is added to the base fluid. The thermal slip contributes to the reduction of the fluid temperature where the thermal gradient on the surface is declined. This led to the reduction of the heat transfer rate of the hybrid nanofluid.

Keywords: *Couple Stress Fluid, Hybrid Nanofluid, Shrinking Sheet, Thermal Slip*

1. Introduction

In recent years, there has been a notable surge in the interest of researchers and scientists towards the advancement of heat transfer fluids. While conventional fluids widespread use in engineering and industrial applications, their heat transfer capabilities are limited due to their low thermal conductivity. To overcome this limitation, researchers introduced a solution known as 'nanofluid' in 1995, pioneered by Choi and Eastman [1]. This innovative approach involves incorporating nanosized particles into the aforementioned fluids. Then, Khanafer et al. [2], and Oztop and Abu-Nada [3] studied the nanofluid flow in a rectangular enclosure. However, in order to further enhance the thermal properties of nanofluids, researchers have developed a novel type of fluid known as 'hybrid nanofluid'. Early studies focusing on the utilization of hybrid nano-composite particles can be attributed to Jana et al. [4]. Unlike conventional nanofluids, hybrid nanofluids consist of multiple types of nanoparticles, which synergistically enhance the heat transfer rate [5,6]. By combining or hybridizing appropriate nanoparticles, the desired level of heat transfer can be achieved, see Suresh et al. [7]. Takabi and Salehi [8] conducted a study on enhancing the heat transfer efficiency of a sinusoidal corrugated enclosure using a hybrid nanofluid. Numerous papers on hybrid nanofluids have been published and are accessible in the literature, see Refs [9–13].

The theory of couple stress fluids serve as an extension of the classical Newtonian fluids, allowing for the inclusion of polar effects such as body couples and couple stresses, see Stokes [14]. Couple stress fluids find practical applications in various fields, including biomechanics, chemical and petroleum industries, geothermal energy extraction, geohydrology, and medicine. Several researchers have conducted simulations and investigations pertaining to couple stress flows in different contexts. For instance, Turkyilmazoglu [15] investigated the flow and heat characteristics of couple stress fluid over surfaces that can stretch or shrink. Khan et al. [16] computationally studied flow of couple stress with several surface temperature conditions. Additionally, Das et al. [17] reported the effects of the Darcy-Forchheimer on couple stress fluid past an inclined sheet.

The slip boundary conditions has been examined in existing literature and they has greater significance for practical applications such as in microelectronic cooling, gas flows in vacuum systems, microfluidics, and microscale viscous pumps, see Zhao [18]. According to Mansur et al. [19], the thermal slip effects has a tendency to decrease the local Nusselt number, which quantifies the heat transfer rate. Similar observation was reported by Khan and Rasheed [20]. Meanwhile, He and Abd Elazem [21] observed that the slip parameter has the potential to amplify the temperature field and augment the thickness of the thermal boundary layer. Among others, Asako et al. [22] and Gholamalizadeh et al. [23] considered the thermal slip effects in a microchannel, while Ahmed et al. (2020) examined the flow over curved surface. Ariffin et al. [24] reported the thermal slip effect shows a reduction in the heat transfer rate of the non-Newtonian fluid flow over a stretching surface.

In recent times, numerous types of nanomaterial have been reported in the literature. Notably, aluminum alloy nanoparticles AA7075 and AA7072 stand out as exceptional nanomaterials due to their remarkable thermal, chemical, and physical properties, see Tlili [25]. The special features and superior properties make them highly desirable for various industrial applications, including aerospace. Also, these alloys find extensive applications in the manufacturing of transportation equipment such as glider aircraft and rocket frames [25]. Different from the previous study of Turkyilmazoglu [15], the objective of this endeavor is to investigate the effects of hybrid nanoparticles over a stretched surface in a couple stress fluid. The hybrid nanoparticles considered in this study are aluminum alloy nanoparticles AA7075 and AA7072, while the base fluid is methanol. The obtained results are presented in graphical and tabular forms for several physical parameters. It should be mentioned that, this problem has not been explored. Hence, this study holds immense importance as a future point of reference for practitioners, scientists, engineers, and fluid mechanists.

2. Mathematical Formulation

In this study, we examine the behavior of an electrically conducting quiescent couple stress fluid interacting with a deformable surface. We consider the influence of hybrid nanoparticles and a uniform external magnetic field, with the coordinates x and y denoting the surface and normal directions, respectively. Here, the velocity and temperature at the wall are taken as $u_w = ax$ and $T_w(x) = T_\infty + T_0x^2$ respectively.

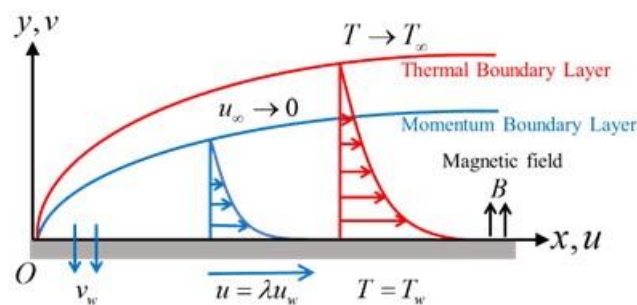


Fig. 1. Schematic Model

The governing equations are then given by, see Turkyilmazoglu [15]

$$u_x + v_y = 0 \quad (1)$$

$$uu_x + vu_y = \frac{\mu_{hnf}}{\rho_{hnf}} u_{yy} - \frac{\eta_0}{\rho_{hnf}} u_{yyy} - \frac{\sigma_{hnf}}{\rho_{hnf}} B_0^2 u \quad (2)$$

$$uT_x + vT_y = \frac{k_{hnf}}{(\rho C_p)_{hnf}} T_{yy} \quad (3)$$

subject to:

$$v = v_w, u = \lambda u_w = \lambda ax, T = T_w + LT_y \text{ at } y = 0$$

$$u \rightarrow 0, u_y \rightarrow 0, u_{yy} \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \quad (4)$$

where L is the slip constant, η_0 is the material constant for the couple stress fluid, ρ is the density, and μ is the dynamic viscosity. The properties of hybrid nanofluid are given in Tables I and II. The following similarity transformations are considered, see Turkyilmazoglu [15].

$$\psi = x\sqrt{av_f}f(\eta), u = axf'(\eta), v = -\sqrt{av_f}f(\eta),$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \eta = y\sqrt{\frac{a}{v_f}} \quad (5)$$

Here, ψ signifies the stream function with $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ which satisfied (1). Then, using (5), the governing equations (2) and (3) are reduced to the similarity form

$$-Cf'''' + \frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f'''' + ff'' - f'^2 - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} Mf' = 0 \quad (6)$$

$$\frac{1}{Pr} \frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f} \theta'' + f\theta' - 2f'\theta \quad (7)$$

subject to

$$f(0) = S, f'(0) = \lambda, \theta(0) = 1 + \beta\theta'(0) \quad (8)$$

$$f'(\eta) \rightarrow 0, f''(\eta) \rightarrow 0, f'''(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

with $\lambda = 1$ and $\lambda = -1$ represent the stretching and shrinking parameters, respectively. Further, Prandtl number Pr , the mass flux parameter S , the couple stress parameter C , the magnetic parameter M , and the thermal slip factor β are defined as

$$Pr = \frac{(\mu C_p)_f}{k_f}, S = -\frac{v_w}{\sqrt{av_f}}, C = \frac{\eta_0 a}{\rho_{hnf} v_f^2}, \quad (9)$$

$$M = \frac{\sigma_f B_0^2}{a\rho_f}, \beta = L\sqrt{\frac{a}{v_f}}$$

The local Nusselt number Nu_x and the skin friction coefficient C_f are given as

$$Nu_x = \frac{xq_w}{k_f(T_w-T_\infty)}, C_f = \frac{\tau_w}{\rho_f u_w^2} \quad (10)$$

where the surface heat flux and the surface shear stress are defined by

$$q_w = -k_{hnf} \left(\frac{\partial T}{\partial y} \right)_{y=0}, \tau_w = \mu_{hnf} \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (11)$$

Thus, we have:

$$Re_x^{-1/2} Nu_x = -\frac{k_{hnf}}{k_f} \theta'(0), Re_x^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} f''(0) \quad (12)$$

where $Re_x = u_w x / v_f$ denote the local Reynolds number.

3. Results And Discussion

This section presents a comprehensive analysis of the numerical results obtained from solving Eqs. (6)-(8) using the MATLAB software's bvp4c package [26]. Detailed discussions are provided regarding the impact of various physical parameters associated with the proposed model. The values of the parameters are referred from the

previous study, see Turkyilmazoglu [15]. For the limiting cases, the models of Turkyilmazoglu [15] are equivalent to the present model. The validation processes yielded excellent agreement across all aspects, demonstrating a high level of consistency, see Table III.

Table I The properties of AA7075, AA7072, and methanol [25]

Properties	Nanoparticles		Base fluid
	AA7075	AA7072	Methanol
ρ (kg/m ³)	2810	2720	792
C_p (J/kgK)	960	893	2545
k (W/mK)	173	222	0.2035
σ (S/m)	26.77×10 ⁶	34.83×10 ⁶	0.5×10 ⁻⁶
Prandtl number, Pr			7.38

Table II The thermophysical properties of hybrid nanofluid [10]

Thermophysical properties	Hybrid Nanofluid
Dynamic viscosity	$\mu_{hnf} = \frac{\mu_f}{(1 - \phi_{hnf})^{2.5}}$
Heat capacity	$(\rho C_p)_{hnf} = (1 - \phi_{hnf})(\rho C_p)_f + \phi_1(\rho C_p)_{n1} + \phi_2(\rho C_p)_{n2}$
Density	$\rho_{hnf} = (1 - \phi_{hnf})\rho_f + \phi_1\rho_{n1} + \phi_2\rho_{n2}$
Thermal conductivity	$\frac{k_{hnf}}{k_f} = \frac{\frac{\phi_1 k_{n1} + \phi_2 k_{n2}}{\phi_{hnf}} + 2k_f + 2(\phi_1 k_{n1} + \phi_2 k_{n2}) - 2\phi_{hnf}k_f}{\frac{\phi_1 k_{n1} + \phi_2 k_{n2}}{\phi_{hnf}} + 2k_f - (\phi_1 k_{n1} + \phi_2 k_{n2}) + \phi_{hnf}k_f}$
Electric conductivity	$\frac{\sigma_{hnf}}{\sigma_f} = \frac{\frac{\phi_1 \sigma_{n1} + \phi_2 \sigma_{n2}}{\phi_{hnf}} + 2\sigma_f + 2(\phi_1 \sigma_{n1} + \phi_2 \sigma_{n2}) - 2\phi_{hnf}\sigma_f}{\frac{\phi_1 \sigma_{n1} + \phi_2 \sigma_{n2}}{\phi_{hnf}} + 2\sigma_f - (\phi_1 \sigma_{n1} + \phi_2 \sigma_{n2}) + \phi_{hnf}\sigma_f}$

Table III Values of $-f''(0)$ and $-\theta'(0)$ for various values of M when $C = 0.01$, $\delta = \phi_1 = \phi_2 = 0$, $S = 2$, $Pr = 1$, and $\lambda = -1$

M	Turkyilmazoglu [15]	Present results		
	(Exact Solution)	(bvp4c)		
	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
0	8.87298335	1.71880075	8.87298335	1.71880075
1	8.78885066	1.71651765	8.78885066	1.71651766
3	8.60576725	1.71141583	8.60576725	1.71141583
6	8.27949145	1.70184151	8.27949146	1.70184151

Table IV Values of $Re_x^{-1/2}Nu_x$ for various values of β and ϕ_{hnf} when $C = 0.01$, $M = 0$, $S = 1$, $Pr = 7.38$ and $\lambda = -1$

β	$\phi_{hnf} = 0\%$	$\phi_{hnf} = 2\%$	$\phi_{hnf} = 4\%$
	($\phi_1 = \phi_2 = 0$)	($\phi_1 = \phi_2 = 0.01$)	($\phi_1 = \phi_2 = 0.02$)
0	5.97289575	5.97511385	5.98008741
0.5	1.49830023	1.56592486	1.63445483
1	0.85658756	0.90103105	0.94658608

The variations of $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ against ϕ_{hnf} and M are presented in Figs. 2 and 3. The increasing values of M tend to decline the values of $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$. However, the values of these physical quantities are enhanced with the rising of ϕ_{hnf} and their values are intensified when ϕ_{hnf} become larger. This finding supports the notion that hybrid nanofluids exhibit superior thermal characteristics owing to the synergistic effects observed.

Figure 4 shows the variations of $Re_x^{-1/2}Nu_x$ with ϕ_{hnf} and β . Note that the heat transfer rate is decreased with the increasing of β . The heat transfer rates was occurred almost at the same value for regular fluid and hybrid nanofluid when no thermal slip is considered. However, their values are significantly changed by the imposition of the thermal slip. Additionally, the numerical values of $Re_x^{-1/2}Nu_x$ which corresponded to Fig. 4 is given in Table IV. For fixed value of $\beta = 1$, the heat transfer rate is increased by 5.19% when 2% of hybrid nanoparticle is added to the base fluid. The increment is pronounced when 4% of hybrid nanoparticle is considered with 10.51% increment if compared to regular fluid.

Figure 5 illustrates the impact of the thermal slip parameter, β on the temperature field. The increasing value of β leads to decrease the temperature profiles. Physically, the temperature jump is caused by the transfer of heat carriers, known as phonons, across the interface. As the thermal slip parameter increases, the temperature profile decreases, while the thermal

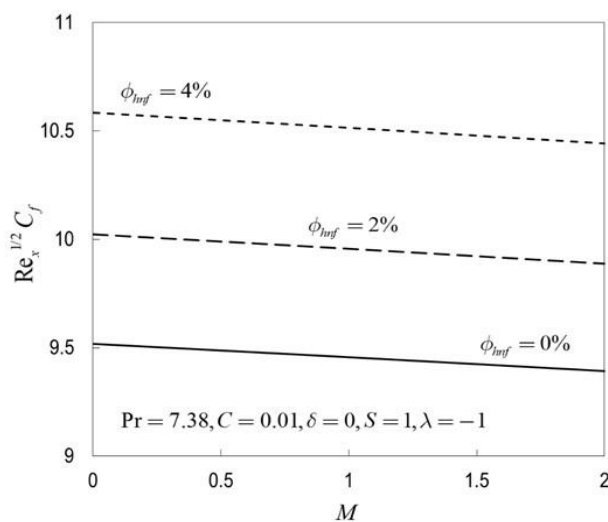


Fig. 2. Variations of $Re_x^{1/2}C_f$ with ϕ_{hnf} and M

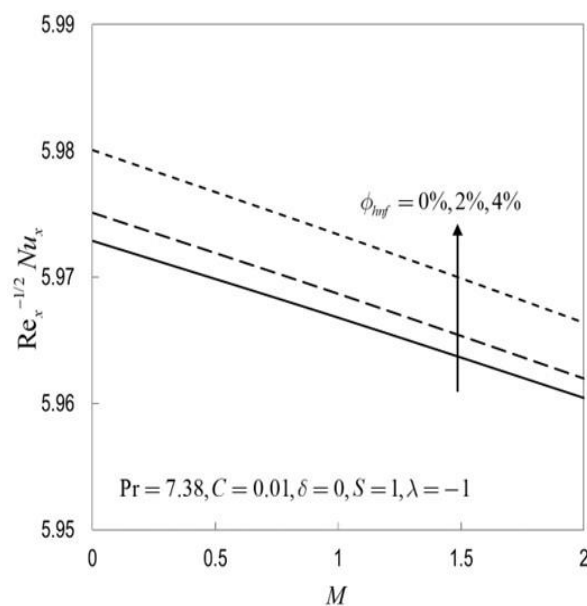


Fig. 3. Variations of $Re_x^{-1/2}Nu_x$ with ϕ_{hnf} and M

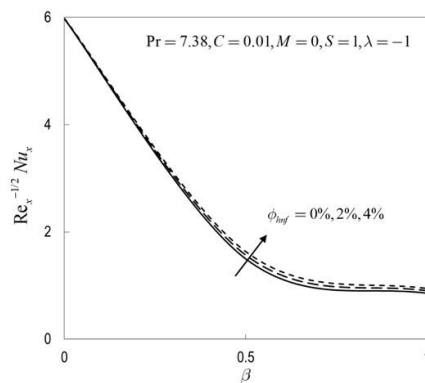


Fig. 4. Variations of $Re_x^{-1/2} Nu_x$ with ϕ_{hnf} and β

boundary layer thickness undergoes minimal change. Note that the temperature gradient on the surface is lower, which leads to a reduction in the heat transfer rate. The temperature profiles become nearer to each other, and the sensitivity to β clearly diminishes. The temperature remains highest in the absence of a temperature jump, highlighting that the temperature jump condition weakens thermal transport.

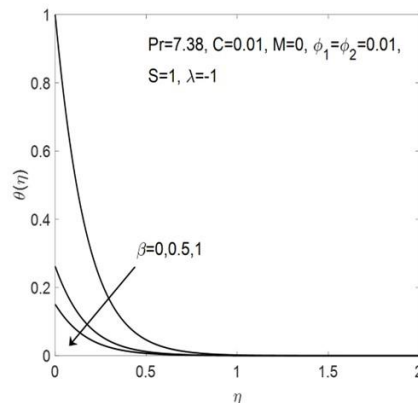


Fig. 5. Temperature profiles $\theta(\eta)$ for different values of β

4. Conclusion

This study focuses on investigating the steady flow of a couple stress fluid over a deformable surface in the presence of hybrid nanoparticles and thermal slip. The governing problem is transformed using similarity transformation, and the numerical solution is obtained utilizing the bvp4c solver. The findings suggest that hybrid nanofluids exhibit exceptional thermal conductivity and play a vital role in augmenting the rate of heat transfer. It is revealed that the heat transfer rate is increased by 5.19% when 2% of hybrid nanoparticle is added to the base fluid, while 10.51% increment is observed for 4% of hybrid nanoparticle if compared to regular fluid. It is worth noting that the surface temperature gradient is lower, resulting in a decrease in the rate of heat transfer by the imposition of the thermal slip effects.

5. Acknowledgement

A grateful acknowledgement goes to Universiti Teknikal Malaysia Melaka, Universiti Teknologi MARA, and Universiti Kebangsaan Malaysia. This research was funded by Universiti Teknikal Malaysia Melaka (JURNAL/2019/FTKMP/Q00042).

References

- [1] S.U.S. Choi, J.A. Eastman, Enhancing thermal conductivity of fluids with nanoparticles, Proceedings of the 1995 ASME International Mechanical Engineering Congress and Exposition, FED 231/MD. 66 (1995) 99–105. <https://doi.org/10.1115/1.1532008>.

- [2] K. Khanafer, K. Vafai, M. Lightstone, Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids, *Int J Heat Mass Transf.* 46 (2003) 3639–3653. [https://doi.org/10.1016/S0017-9310\(03\)00156-X](https://doi.org/10.1016/S0017-9310(03)00156-X).
- [3] H.F. Oztop, E. Abu-Nada, Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids, *Int J Heat Fluid Flow.* 29 (2008) 1326–1336. <https://doi.org/10.1016/j.ijheatfluidflow.2008.04.009>.
- [4] S. Jana, A. Salehi-Khojin, W.H. Zhong, Enhancement of fluid thermal conductivity by the addition of single and hybrid nano-additives, *Thermochim Acta.* 462 (2007) 45–55. <https://doi.org/10.1016/j.tca.2007.06.009>.
- [5] J.A.R. Babu, K.K. Kumar, S.S. Rao, State-of-art review on hybrid nanofluids, *Renewable and Sustainable Energy Reviews.* 77 (2017) 551–565. <https://doi.org/10.1016/j.rser.2017.04.040>.
- [6] N.A.C. Sidik, I.M. Adamu, M.M. Jamil, G.H.R. Kefayati, R. Mamat, G. Najafi, Recent progress on hybrid nanofluids in heat transfer applications: A comprehensive review, *International Communications in Heat and Mass Transfer.* 78 (2016) 68–79. <https://doi.org/10.1016/j.icheatmasstransfer.2016.08.019>.
- [7] S. Suresh, K.P. Venkitaraj, P. Selvakumar, M. Chandrasekar, Synthesis of Al₂O₃-Cu/water hybrid nanofluids using two step method and its thermo physical properties, *Colloids Surf A Physicochem Eng Asp.* 388 (2011) 41–48. <https://doi.org/10.1016/j.colsurfa.2011.08.005>.
- [8] B. Takabi, S. Salehi, Augmentation of the heat transfer performance of a sinusoidal corrugated enclosure by employing hybrid nanofluid, *Advances in Mechanical Engineering.* 6 (2014) 147059. <https://doi.org/10.1155/2014/147059>.
- [9] I. Waini, A. Ishak, I. Pop, Hybrid nanofluid flow on a shrinking cylinder with prescribed surface heat flux, *Int J Numer Methods Heat Fluid Flow.* 31 (2021) 1987–2004. <https://doi.org/10.1108/HFF-07-2020-0470>.
- [10] I. Waini, A. Ishak, I. Pop, Hybrid nanofluid flow over a permeable non-isothermal shrinking surface, *Mathematics.* 9 (2021) 538. <https://doi.org/https://doi.org/10.3390/math9050538>.
- [11] I. Waini, A. Ishak, I. Pop, Symmetrical solutions of hybrid nanofluid stagnation-point flow in a porous medium, *International Communications in Heat and Mass Transfer.* 130 (2022) 105804. <https://doi.org/10.1016/j.icheatmasstransfer.2021.105804>.
- [12] N.S. Khashi'ie, N.M. Arifin, I. Pop, Magnetohydrodynamics (MHD) boundary layer flow of hybrid nanofluid over a moving plate with Joule heating, *Alexandria Engineering Journal.* 61 (2022) 1938–1945. <https://doi.org/10.1016/j.aej.2021.07.032>.
- [13] N.S. Khashi'ie, I. Waini, A. Ishak, I. Pop, Blasius Flow over a Permeable Moving Flat Plate Containing Cu-Al₂O₃ Hybrid Nanoparticles with Viscous Dissipation and Radiative Heat Transfer, *Mathematics* 2022, Vol. 10, Page 1281. 10 (2022) 1281. <https://doi.org/10.3390/MATH10081281>.
- [14] V.K. Stokes, Couple Stresses in Fluids, *Theories of Fluids with Microstructure.* (1984) 34–80. https://doi.org/10.1007/978-3-642-82351-0_4.
- [15] M. Turkyilmazoglu, Exact solutions for two-dimensional laminar flow over a continuously stretching or shrinking sheet in an electrically conducting quiescent couple stress fluid, *Int J Heat Mass Transf.* 72 (2014) 1–8. <https://doi.org/10.1016/j.ijheatmasstransfer.2014.01.009>.
- [16] N.A. Khan, F. Riaz, N.A. Khan, Heat Transfer Analysis for Couple Stress Fluid over a Nonlinearly Stretching Sheet, *Nonlinear Engineering.* 2 (2013) 121–127. <https://doi.org/10.1515/NLENG-2013-0014>.
- [17] S. Das, A. Ali, R.N. Jana, Darcy–Forchheimer flow of a magneto-radiated couple stress fluid over an inclined exponentially stretching surface with Ohmic dissipation, *World Journal of Engineering.* 18 (2021) 345–360. <https://doi.org/10.1108/WJE-07-2020-0258/FULL/PDF>.
- [18] J. Zhao, Axisymmetric convection flow of fractional Maxwell fluid past a vertical cylinder with velocity slip and temperature jump, *Chinese Journal of Physics.* 67 (2020). <https://doi.org/10.1016/j.cjph.2020.08.009>.
- [19] S. Mansur, A. Ishak, I. Pop, Flow and heat transfer of nanofluid past stretching/shrinking sheet with partial slip boundary conditions, *Applied Mathematics and Mechanics (English Edition).* 35 (2014). <https://doi.org/10.1007/s10483-014-1878-7>.
- [20] M. Khan, A. Rasheed, Slip velocity and temperature jump effects on molybdenum disulfide MoS₂ and silicon oxide SiO₂ hybrid nanofluid near irregular 3D surface, *Alexandria Engineering Journal.* 60 (2021). <https://doi.org/10.1016/j.aej.2020.11.019>.
- [21] J.H. He, N.Y. Abd Elazem, Insights into partial slips and temperature jumps of a nanofluid flow over a stretched or shrinking surface, *Energies (Basel).* 14 (2021). <https://doi.org/10.3390/en14206691>.
- [22] Y. Asako, S.Y. Heng, C. Hong, On temperature jump condition for turbulent slip flow in a quasi-fully developed region of micro-channel with constant wall temperature, *International Journal of Thermal Sciences.* 136 (2019). <https://doi.org/10.1016/j.ijthermalsci.2018.10.040>.
- [23] E. Gholamalazadeh, F. Pahlevanzadeh, K. Ghani, A. Karimipour, T.K. Nguyen, M.R. Safaei, Simulation of water/FMWCNT nanofluid forced convection in a microchannel filled with porous material under slip velocity and temperature jump boundary conditions, *Int J Numer Methods Heat Fluid Flow.* 30 (2020). <https://doi.org/10.1108/HFF-01-2019-0030>.
- [24] N.A.N. Ariffin, I. Waini, A.R.M. Kasim, M.H.A. Kamal, M.R. Ilias, S.A. Kechil, Flow and Heat Transfer Analysis on Reiner-Philippoff Fluid Flow over a Stretching Sheet in the Presence of First and Second Order Velocity Slip and Temperature Jump Effects, *CFD Letters.* 15 (2023). <https://doi.org/10.37934/cfdl.15.1.88102>.
- [25] I. Tili, H.A. Nabwey, G.P. Ashwinkumar, N. Sandeep, 3-D magnetohydrodynamic AA702-AA705/methanol hybrid nanofluid flow above an uneven thickness surface with slip effect, *Sci Rep.* 10 (2020) 1–13. <https://doi.org/10.1038/s41598-020-61215-8>.
- [26] L.F. Shampine, I. Gladwell, S. Thompson, *Solving ODEs with MATLAB*, Cambridge University Press, Cambridge, 2003. <https://doi.org/https://doi.org/10.1017/cbo9780511615542>.