

Mathematical Modelling of a Multi-Stage Feedback Queue with Discretionary Server Downtime

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Abstract

In this section, we take into account an M/G/1x queue in the presence of multiple service tiers, customers who provide feedback at random, and a server who is on vacation. The customer is given the option to progress through a series of k progressively more complex steps in the system. A customer is free to abandon the system after receiving service in the first of k stages ($k > 1$). There is a specific order to the stages, starting with the first stage service and continuing on to the second stage service and so on. The customer has multiple opportunities for subsequent service after each phase. However, the re service can only be used from the very beginning, so it doesn't matter which stage he left. Therefore, the customer who requires a different service should go to the very end of the line and wait there. When a server finishes a service for a customer, they may take a short break or move on to the next customer immediately, depending on their schedule. In this article, we extend this model to accommodate a batch arrival queue that allows for a server to take a break.

Keywords: *Multistage feedback queue, Median vacation, Median queue size and the Median size of the system.*

1. Introduction

This paper examines an M/G/1x queue in the presence of service levels, customer feedback variability, and server absence. The customer is given the option to progress through a series of k progressively more complex steps in the system. The customer is free to leave the system once they have received service in k stages, where $k > 1$. There is a specific order to the stages, starting with the first stage service and continuing on to the second stage service and so on. The customer has multiple opportunities for subsequent service after each phase. However, the re service can only be used from the very beginning, so it doesn't matter which stage he left. Therefore, the customer who requires a different service should go to the very end of the line and wait there. When a server finishes a service for a customer, they may take a short break or move on to the next customer immediately, depending on their schedule. The service of customers may be repeated in many contexts, such as production systems, banking services, and computer and communication networks. Broad information about CRNs, including their architectures and common security problems, was gathered in a study conducted by **S. Parvin et al., 2012**. When it comes to the issue of server failures and repairs, **E. Ever et al. (2013)** presented a new method for analytical modeling of open networks that offers improvements. Production quality was proposed by **M. Colledani et al. in 2014** as a new paradigm that would expand upon the methods used in the six sigma movement. Feedback Queueing system with multistage service of units and random server breakdown was discussed by **V. Ganesan & S. Rita in 2015**. Challenges and opportunities for developing next-generation de- and remanufacturing systems were discussed by **T. Tolio (2017)**. **A. P. G. Scheidegger (2018)** was counted on to back up the selection of the most appropriate simulation method for the system under investigation. During the planning, development, and rollout of the synERGY system, **F. Skopik (2020)** reported on the overarching architecture and the specific challenges that arose and had to be addressed. **T. Sakai (2020)** assessed the effects of several potential logistics

strategies. Individual opinions shared between interacting agents lead to inflated collective expectations for a new idea or innovation, which quickly collapse into dramatic collective disillusionment, as described by the microscopic model proposed by **F. Hashemi (2021)**. Researchers now have a firm grasp of the primary methods and algorithms used to enhance manufacturing processes over the past two decades, thanks to the work of **A. Dogan and D. Birant (2021)**.

2. Model Description-

In a compound Poisson process, customer arrivals occur in varying-sized batches. The server offers k stages of heterogeneous service, at the end of each of which the customer can either proceed to the next stage, return to the end of the queue for re service, or leave the system. Let $p_r (1 \leq i \leq k)$ represent the likelihood of the customer returning to the queue after the i^{th} stage. Let $p_g (1 \leq i \leq k)$ represent the possibility that the customer will advance from the i^{th} to the $(i + 1)^{th}$ stage. Thus, $p_r + p_g + p_l = 1$ indicates that the customer abandons the system after the i^{th} stage with probability p_l . Customers typically leave the system with a probability of $1 - p_{rk}$ after receiving service in the k^{th} stage. Assume that the k -stage service times are independent random variables S_i , with LaplaceStieltjes Transform (LST) $S_i^*(s)$ and finite moments $E(S_i^m)$, $m \geq 1$. To calculate the new service time, or the time it takes for a customer to finish the service cycle, we use the formula:

$$S = S_1 + \sum_{i=1}^k p_{g1} p_{g2} p_{g3} \dots p_{g(i-1)} S_i \quad (1)$$

$$S^*(s) = S_1^*(s) \prod_{i=2}^k p_{g1} p_{g2} p_{g3} \dots p_{g(i-1)} S_i^*(s) \quad (2)$$

$$E(S) = S_1 + \sum_{i=2}^k p_{g1} p_{g2} p_{g3} \dots p_{g(i-1)} E(S_i) \quad (3)$$

Given an elapsed time x , we can calculate the conditional probability density $\mu_i(x)dx$ of the i^{th} service being completed within the interval $(x, x + dx)$ as follows:

$$\mu_i(x) = \frac{s_i(x)}{1 - S_i(x)}, i = 1, 2, \dots, k \quad (4)$$

$$\text{where } s_i(x) = \mu_i(x) e^{-\int_0^x \mu_i(x) dx}, i = 1, 2, \dots, k \quad (5)$$

After the current service (including all stages the customer adopted) is complete, the server may take a vacation with probability r , attend the next customer if any, or sit idle in the system if the system is free with probability $1 - r$.

Probability Generating Functions

$$P_i(x, z) = \sum_{n=0}^{\infty} z^n P_{i,n}(x) P_i(z) = \sum_{n=0}^{\infty} z^n P_n \quad i = 1, 2, \dots, k; \quad x \geq 0; \quad |z| < 1$$

$$P_i(x, z) = \sum_{n=0}^{\infty} z^n P_{i,n}(x) P_i(z) = \sum_{n=0}^{\infty} z^n P_n \quad i = 1, 2, \dots, k; \quad x \geq 0; \quad |z| < 1$$

$$Q(x, z) = \sum_{n=0}^{\infty} z^n V_n(x) \quad Q(z) = \sum_{n=0}^{\infty} z^n V_n \quad x > 0; \quad |z| < 1$$

$$R(z) = \sum_{n=0}^{\infty} z^n R_n, \quad |z| < 1$$

$$X(z) = \sum_{k=0}^{\infty} z^k a_k$$

$$\frac{d}{dx} P_{i,n}(x) + [\lambda + \mu_i(x)] P_{i,n}(x) = \lambda \sum_{j=1}^n a_j P_{i,n-j}(x) \quad x \geq 0; n \geq 1; i = 1, 2, \dots, k \quad (6)$$

$$\frac{d}{dx} P_{i,0}(x) + [\lambda + \mu_i(x)] P_{i,0}(x) = 0 \quad (7)$$

$$\frac{d}{dx} Q_n(x) + [\lambda + v(x)] Q_n(x) = \lambda \sum_{j=1}^n a_j Q_{n-j}(x) \quad x \geq 0; n \geq 1 \quad (8)$$

$$\frac{d}{dx} Q_0(x) + [\lambda + \beta(x)] Q_0(x) = 0 \quad (9)$$

$$\lambda R_0 = \int_0^\infty v(x) Q_0(x) dx + (1-r) \sum_{i=1}^{k-1} \mu_i(x) P_{i,0}(x) dx + (1-r)(1-p_k) \int_0^\infty \mu_k(x) P_{k,0}(x) dx \quad (10)$$

The above equations are to be solved subject to the following boundary Conditions:

$$P_{i,n}(0) = \lambda a_n R_0 + (1-r) \sum_{i=1}^k p_i \int_0^\infty \mu_i(x) P_{i,n}(x) dx + (1-r) \sum_{i=1}^{k-1} q_i \int_0^\infty \mu_i(x) P_{i,n+1}(x) dx + (1-r)(1-p_k) \int_0^\infty \mu_k(x) P_{k,n+1}(x) dx + \int_0^\infty v(x) Q_{n+1}(x) dx \quad (11)$$

$$P_{i,0}(0) = \lambda R_0 + (1-r) \sum_{i=1}^k p_i \int_0^\infty \mu_i(x) P_{i,0}(x) dx + (1-r) \sum_{i=1}^{k-1} q_i \int_0^\infty \mu_i(x) P_{i,1}(x) dx + (1-r)(1-p_k) \int_0^\infty \mu_k(x) P_{k,1}(x) dx + \int_0^\infty v(x) Q_1(x) dx$$

$$P_{i,n}(0) = \theta_{i-1} \int_0^\infty \mu_{i-1}(x) P_{i-1,n}(x) dx, \quad n \geq 0; i = 2, 3, \dots, k \quad (13)$$

$$Q_n(0) = r \sum_{i=1}^{k-1} q_i \int_0^\infty \mu_i(x) P_{i,n}(x) dx + r \sum_{i=1}^k p_i \int_0^\infty \mu_i(x) P_{i,n-1}(x) dx + r(1-p_k) \int_0^\infty \mu_k(x) P_{k,n}(x) dx \quad (14)$$

Applying standard manipulations to (6), (7), (8) and (9)

$$P_1(x, z) = P_1(0, z) e^{-[\lambda - \lambda X(z)]x} (1 - S_1(x)) \quad (15)$$

$$P_i(x, z) = Q_{i-1} P_{i-1}(0, z) e^{-[\lambda - \lambda X(z)]x} (1 - S_i(x)); \quad i = 2, 3, \dots, k \quad (16)$$

$$Q(x, z) = Q(0, z) e^{-[\lambda - \lambda X(z)]x} (1 - v(x)) \quad (17)$$

By applying the appropriate powers of z to the products in (11) and (13) and summing up the values of n and then adding (12), we get:

$$z P_i(0, z) = \lambda R_0 z [X(z) - 1] + (1-r) \sum_{i=1}^k p_i z P_i(0, z) S_i^* + (1-r) \sum_{i=1}^{k-1} q_i P_i(0, z) S_i^* + (1-r)(1-p_k) P_k(0, z) S_k^* + Q(0, z) V^* \quad (18)$$

where $S_i^* = S_i^*[\lambda - \lambda X(z)]$ & $V^* = V^*[\lambda - \lambda X(z)]$ are the Laplace-Stieltjes transforms of the i^{th} stage service time and vacation time respectively.

Consider

$$P_i(0, z) = \theta_{i-1} P_{i-1}(0, z) S_{i-1}^* = \theta_1 \theta_2 \dots \theta_{i-1} S_1 S_2 \dots S_{i-1}^* P_1(0, z) \quad (19)$$

From eq (14)

$$Q(0, z) = r \sum_{i=1}^k q_i P_i(0, z) S_i^* + r z \sum_{i=1}^k p_i z P_i(0, z) S_i^* + r(1-p_k) P_k(0, z)$$

$$z P_1(0, z) = \lambda R_0 [X(z) - 1] + (1-r) \sum_{i=2}^k p_i z \Phi_{i-1} B_i^* P_1(0, z) + (1-r) \sum_{i=2}^k q_i z \Phi_{i-1} B_i^* P_1(0, z) + (1-r)(1-p_k) \Phi_{k-1} B_k^* P_1(0, z) + (1-r)(p_1 z + q_1) S_1^* + r \sum_{i=2}^k q_i \Phi_{i-1} B_i^* P_1(0, z) V^* + r z \sum_{i=2}^k p_i z \Phi_{i-1} B_i^* P_1(0, z) + r(1-p_k) \Phi_{k-1} B_k^* P_1(0, z) V^* \quad (20)$$

Also

$$P_1(0, z) = \frac{\lambda R_0 [X(z) - 1]}{z - [(p_1 z + q_1) S_1^* + \sum_{i=2}^{k-1} (p_i z + q_i) \Phi_{i-1} B_i^* + (1 - p_k + p_k z) \Phi_{k-1} B_k^*]} \quad (21)$$

$$\begin{aligned} P_1(z) &= \int_0^\infty P_1(x, z) dx = P_1(0, z) \frac{1 - S_1^*(\lambda - \lambda X(z))}{\lambda(1 - X(z))} \\ P_i(z) &= P_i(0, z) \frac{(1 - S_i^*)(\lambda - \lambda X(z))}{\lambda(1 - X(z))} = P_1(0, z) \frac{\Phi_{i-1} B_i^* (1 - S_i^*)(\lambda - \lambda X(z))}{\lambda(1 - X(z))} \\ Q(z) &= Q(0, z) \frac{1 - V^*}{\lambda - \lambda X(z)} \\ &= \frac{P_1(0, z)}{\lambda - \lambda X(z)} [r(p_1 z + q_1) S_1^* + \sum_{i=2}^{k-1} (p_i z + q_i) \Phi_{i-1} B_i^* + r(1 - p_k + p_k z) \Phi_{k-1} B_k^* (1 - V^*)] \end{aligned}$$

Assuming that the queue size distribution is a normal distribution over time regardless of the server's health, we can define it as $P(z)$ to be

$$\begin{aligned} P(z) &= P_1(z) + \sum_{i=2}^k P_i(z) + Q(z) + R_0 \\ &= \frac{R_0 [z \{S_1^* + \sum_{i=2}^k (1 - S_i^*) \Phi_{i-1} B_{i-1}^*\} + \sum_{i=2}^{k-1} (p_i z + q_i) \Phi_{i-1} B_{i-1}^* + (1 - p_k + p_k z) \Phi_{k-1} B_{k-1}^*] (1 - 2r(1 - V^*))}{[(p_1 z + q_1) S_1^* + \sum_{i=2}^{k-1} (p_i z + q_i) \Phi_{i-1} B_i^* + (1 - p_k + p_k z) \Phi_{k-1} B_k^*] (1 - r + rV^*) - z} \quad (22) \end{aligned}$$

R_0 can be calculated by setting $P(1) = 1$, the normalization condition.

After simplification let

$$\begin{aligned} P_1(1) + \sum_{i=2}^k P_i(1) + Q(1) + R_0 &= 1 \\ R_0 &= \frac{1 - [p_1 + \sum_{i=2}^{k-1} p_i \Phi_{i-1}]}{1 - [p_1 + \sum_{i=2}^{k-1} p_i \Phi_{i-1}] + \lambda E(X) [E(S_1) + \sum_{i=2}^k E(S_i) + r E(V)]} \quad (23) \end{aligned}$$

$P(z)$ can be found in closed form while substituting the value of R_0

The Mean Queue Size

Let's say the average number of people waiting in line during the steady state is denoted by the symbol L_q .

$$L_q = \frac{d}{dz} P(z) \text{ at } z = 1$$

$$\text{then } P(z) = Q \frac{N(z)}{D(z)}$$

where $N(z)$ and $D(z)$ are the numerator & denominator of R. H. S. of $P(z)$ excluding Q . After applying L' Hospital's rule, we have:

$$L_q = Q \frac{D'(1)N(1) - N'(1)D''(1)}{[D''(1)]^2}$$

here

$$\begin{aligned} N'(1) &= (1 - S_1^*(\alpha)) S_{i+1}^*(\alpha) \beta (-\lambda a_1) + \{\alpha \beta \gamma S_i^*(\alpha) (-\lambda a_1)\} - (1 - S_i^*(\alpha)) \\ N''(1) &= 2\lambda^2 a_1^2 \beta \{S_{i+1}^{*'}(\alpha) (1 - S_i^*(\alpha)) - S_{i+1}^*(\alpha) S_i^{*'}(\alpha)\} \\ &\quad + (2\lambda^2 a_1^2 - \lambda a_2 \beta) S_{i+1}^*(\alpha) (1 - S_i^*(\alpha)) \\ &\quad + [\alpha \beta \{\gamma (\lambda^2 a_1^2 n - \lambda a_2) S_i^*(\alpha) + (\gamma + 1) \lambda^2 a_1^2 S_i^{*'}(\alpha)\} + 2\gamma \lambda^2 a_1^2 (\alpha + \beta) S_i^*(\alpha) - 2\alpha \beta \gamma p \lambda a_1 S_i^*(\alpha)] \\ &\quad + \alpha [-2\lambda a_1 (1 - S_i^*(\alpha)) - \lambda a_2 (1 - S_i^*(\alpha))] \end{aligned}$$

$$D'(1) = \alpha\beta \left[1 - \gamma p + p^2 - \gamma\lambda a_1 (S_i^{*'}(\alpha) + S_i^*(\alpha)) + \gamma p S_i^*(\alpha) \right] \\ - \lambda a_1 (\alpha + \beta) \{ \gamma p (\gamma - 2) + \gamma + p + \gamma S_i^*(\alpha) \} \\ - [\alpha\beta \{ (-\lambda a_1) \{ (1 - S_i^*(\alpha)) S_{i+1}^{*'}(\alpha) + S_{i+1}^*(\alpha) S_i^{*'}(\alpha) \} + (1 - S_i^{*'}(\alpha)) S_{i+1}^*(\alpha) \} + (1 - S_i^*(\alpha)) S_{i+1}^*(\alpha) \}]$$

and

$$D''(1) = \alpha\beta [\gamma \{ \lambda^2 a_1^2 S_i^*(\alpha) - \lambda a_2 S_i^{*'}(\alpha) + (3\lambda^2 a_1^2 - \lambda a_2) S_i^*(\alpha) \} - 2\lambda a_1 \gamma p (S_i^{*'}(\alpha) + S_i^*(\alpha))] \\ - [1 - \gamma p + \gamma^2 p - \gamma\lambda a_1 \{ S_i^{*'}(\alpha) + S_i^*(\alpha) \} + \gamma p (S_i^*(\alpha))] (\lambda a_1) (\alpha + \beta) \\ + \{ \gamma^2 p - 2\gamma p + p + \gamma + \gamma S_i^*(\alpha) \} \{ (2\lambda^2 a_1^2 - \lambda a_2) (\alpha + \beta) \} \\ - (\lambda a_1) (\alpha + \beta) \{ -\lambda a_1 (\alpha + \beta) \{ 1 - \gamma p + \gamma^2 p - \gamma\lambda a_1 \{ S_i^{*'}(\alpha) + S_i^*(\alpha) \} + \gamma p (S_i^*(\alpha)) \} \\ - [\alpha\beta \{ 1 - S_i^*(\alpha) \{ \lambda^2 a_1^2 S_{i+1}^{*'}(\alpha) - \lambda a_2 S_{i+1}^{*'}(\alpha) \} - 2\lambda^2 a_1^2 S_i^{*'}(\alpha) S_{i+1}^{*'}(\alpha) \} \\ - S_{i+1}^{*'}(\alpha) \{ \lambda^2 a_1^2 S_i^*(\alpha) - \lambda a_2 S_i^{*'}(\alpha) \} - 2\lambda a_1 \{ (1 - S_i^*(\alpha)) S_{i+1}^{*'}(\alpha) \} - S_{i+1}^*(\alpha) S_i^{*'}(\alpha) \} \}]$$

Queue With Single Arrival And Without Vacation

Let $r = 0$, thus $E(X) = 1$, $E(X(X-1)) = 0$, and there is no vacation (the probability that the server goes on vacation is zero). The above outcomes imply a singular arrival,

$$P(z) = \frac{R_0 [z \{ S_1^* + \sum_{i=2}^k (1 - S_i^*) \Phi_{i-1} B_{i-1}^* \} + \sum_{i=2}^{k-1} (p_i z + q_i) \Phi_{i-1} B_{i-1}^* + (1 - p_k + p_k z) \Phi_{k-1} B_{k-1}^*]}{[(p_1 z + q_1) S_1^* + \sum_{i=2}^{k-1} (p_i z + q_i) \Phi_{i-1} B_i^* + (1 - p_k + p_k z) \Phi_{k-1} B_k^*]} \\ \text{where } R_0 = \frac{1 - [p_1 + \sum_{i=2}^{k-1} p_i \Phi_{i-1}]}{1 - [p_1 + \sum_{i=2}^{k-1} p_i \Phi_{i-1}] + \lambda [E(S_1) + \sum_{i=2}^k E(S_i)]}$$

$$N^I(1) = 1 - \left[p_1 + \lambda E(S_1) + \sum_{i=2}^{k-1} p_i \Phi_{i-1} \right] + \lambda \sum_{i=2}^k E(S_i) \Phi_{i-1} - \sum_{i=3}^k \Phi_{i-1} (B_{i-1}^*)^I + \sum_{i=2}^{k-1} \Phi_i (B_{i-1}^*)^I$$

$$N^{II}(1) = \lambda E(S_1) (2 - p_1) + \theta_1 \lambda^2 E(S_1^2) - 2\Phi_{k-1} [p_k (B_{k-1}^*)^I + (B_{i-1}^*)^{II}] + 2\lambda \sum_{i=2}^k [E(S_i) (1 + (B_{i-1}^*)^I) \\ + \lambda E(S_i^2)] \Phi_{i-1}$$

$$D^I(1) = 1 - \left[p_1 + \lambda E(S_1) + \sum_{i=2}^k p_i \Phi_{i-1} + \sum_{i=2}^k \Phi_{i-1} (B_i^*)^I + \sum_{i=1}^{k-1} \Phi_i (B_i^*)^I \right]$$

$$D^{II}(1) = -\lambda^2 E(S_1^2) - 2\lambda p_1 E(S_1) - \sum_{i=2}^k 2 p_i \Phi_{i-1} (B_i^*)^I - \sum_{i=2}^k \Phi_{i-1} (B_i^*)^{II} + \sum_{i=1}^{k-1} \Phi_i (B_i^*)^{II}$$

Let's pretend there are three tiers of support in this system. Therefore, $k=3$ Vacation's impact on a multistage queue can be observed by playing around with the model's parameters and observing how changes in r and average vacation length affect server idle and busy times, queue and system sizes, respectively. Assume arbitrarily $\lambda = 3$, $p_1 = p_2 = p_3 = 0.6$, $\theta_1 = \theta_2 = 0.8$,

$E(B_1) = 0.3$, $E(B_2) = 3$, $E(B_3) = 0.3$, $E(B_1^2) = 0.06$, $E(B_2^2) = 0.2$, $E(B_3^2) = 0.056$. According to the values in the table, if we raise the median vacation, the median queue size and the median size of the system will both decrease, as expected.

r	$E(V)$	$E(V^2)$	R	L_q	ρ	L_s
0.1	0.1	0.02	0.4013	1.2053	0.5986	1.8040
0.2			0.3979	1.2113	0.6020	1.8133
0.3			0.3946	1.2193	0.6053	1.8246
0.4			0.3914	1.2293	0.6085	1.8378
0.5			0.3882	1.2413	0.6117	1.8530
0.1	0.2	0.05	0.3979	1.2108	0.6020	1.8128
0.2			0.3914	1.2287	0.6085	1.8373
0.3			0.3851	1.2546	0.6148	1.8695
0.4			0.3789	1.2882	0.6210	1.9092
0.5			0.3730	1.3291	0.6269	1.9561
0.1	0.3	0.09	0.3946	1.2182	0.6053	1.8235
0.2			0.3851	1.2541	0.6148	1.8690
0.3			0.3759	1.3079	0.6240	1.9319
0.4			0.3672	1.3783	0.6327	2.0110
0.5			0.3589	1.4641	0.6410	2.1052
0.1	0.4	0.2	0.3914	1.2258	0.6085	1.8343
0.2			0.3789	1.2876	0.6210	1.9086
0.3			0.3672	1.3835	0.6327	2.0162
0.4			0.3562	1.5103	0.6437	2.1540
0.5			0.3458	1.6654	0.6540	2.31955
0.1	0.5	0.3	0.3882	1.2367	0.6117	1.8485
0.2			0.3730	1.3324	0.6269	1.9594
0.3			0.3589	1.4817	0.6410	2.1227
0.4			0.3459	1.6787	0.6540	2.3328
0.5			0.3338	1.9185	0.6661	2.5847

Table 1: Measures of various vacation parameters

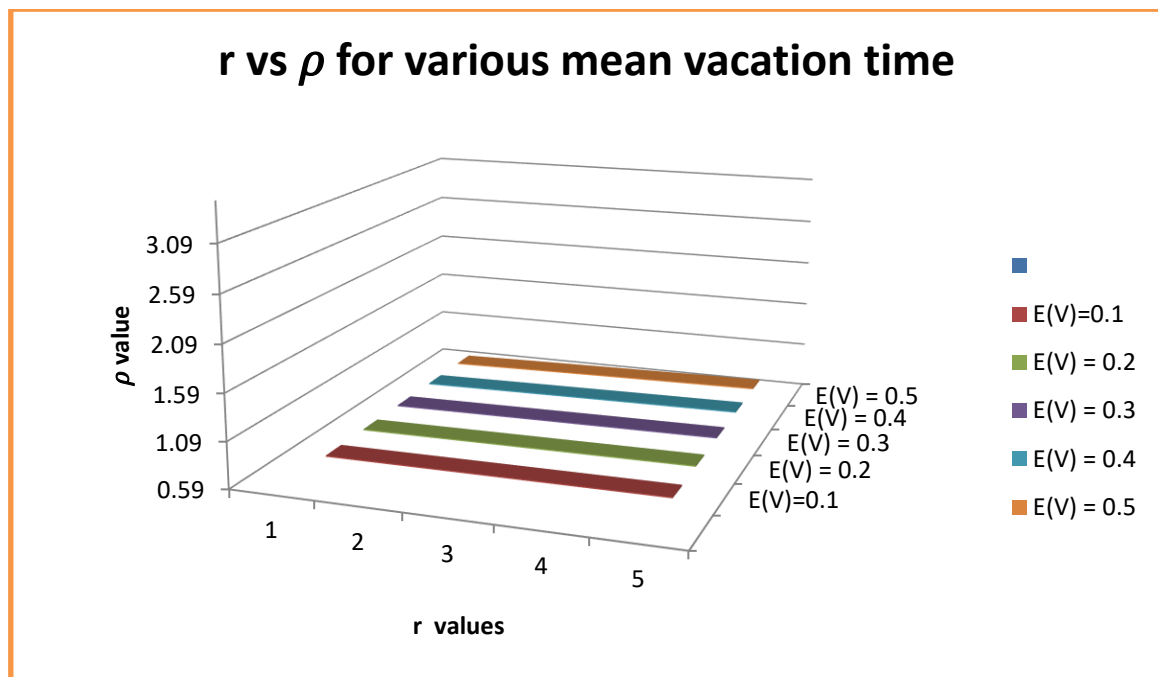


Fig 1: r vs ρ for various mean vacation time

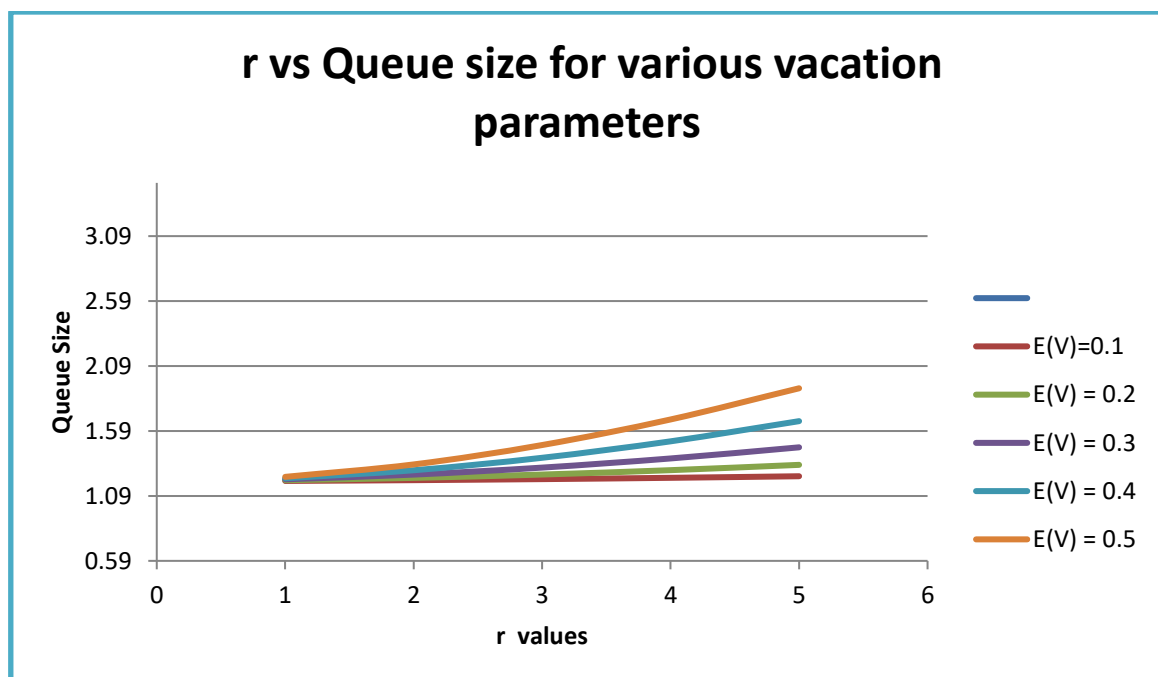


Fig2: r vs Queue size for various vacation parameters

Conclusion

In this paper, single server bulk arrival and batch service queueing system with secondary service, server failure and vacation is analysed. The model so considered is unique in the sense that secondary service and service interruption are introduced for M/G/1x queueing system. For the given model, probability generating function of the queue size at an arbitrary time is obtained by using supplementary variable technique. Various performance measures can be easily understood by the numerical illustrations.

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