An Extensive Analysis of the Contribution of Laplace Transform in Science and Technology

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Abstract: - The Paramount aim of the research paper is to emphasize the significance and impact of the Laplace Transform in the realm of science and technology. Also, aim to underscore its role and valuable contributions. The goal is to emphasize the importance of the mathematical model in representing and studying intricate systems, and how it has contributed to the progress of science and technology. The paper explores the fundamental principles of the Laplace transform and its mathematical properties. It discusses how the Laplace transform is employed in physics, engineering, and other domains. Overall, this analysis emphasizes the importance of Laplace transform as a fundamental tool for understanding and utilising mathematical models in science and technology.

***Keywords**: Laplace Transform, Mathematical Model, Science, Technology, Analysis, Applications, Physics, Engineering.*

1. Introduction

The employment of the Laplace transform has been instrumental in driving numerous scientific and technological advancements. It provides a powerful tool for transforming time-domain problems into frequency-domain problems, enabling researchers and engineers to tackle a wide range of challenges effectively. By utilizing Laplace transform, it becomes possible to solve differential equations, characterize system behaviour, and predict system responses with remarkable precision. Its applications span diverse fields including engineering technology, fundamental sciences, mathematics, and economics

Mathematical models and their practical applications are frequently employed in our everyday lives. The significant importance lies in the application of the Laplace transform within the mathematical framework used to predict population growth in Albania. (Refer to A. Daci [5] and Alfred Daci, SaimirTola [7]).

About mechanical systems and nuclear physics, Sawant [8] explores the fundamental application of the Laplace transform in engineering fields, focusing on its utilization in analysing transfer functions. Patil [2] explains the application of Laplace transforms in finance, specifically in representing discounted values, and also discusses the utilization of the time derivative property through Laplace transforms. According to Das [1], it is highlighted that the Laplace transform theory goes against a fundamental necessity present in all control systems within the field of engineering.

The theory of the Laplace transform is interconnected with other transforms, as discussed in the works of Rani and Devi [4] and Anumaka [3]. Several researchers, including Ananda and Ganga Dharaiah [6], Stankovi [9], Duz [10], Bhullar [11], Subramanian [12], and Wadkar et al [13], have conducted studies concerning the utilization of the Laplace transform in various domains.

The authors aim to comprehensively analyse the role of Laplace's transformation in science and technology. It explores its applications in various fields, including electrical engineering, and physics. The Laplace transform plays a vital role in electrical engineering as it simplifies differential equations and allows for easy determination of system responses, making the analysis of circuits and systems much more convenient. Control systems benefit from Laplace transform as it allows for the analysis of stability, transient response, and frequency response of feedback systems. Signal processing heavily relies on Laplace transform to analyse and manipulate continuous-time signals, leading to advancements in areas such as audio processing, image processing, and data compression. Additionally, within the realm of physics, the utilization of the Laplace transform holds immense significance in the resolution of partial differential equations, elucidation of physical occurrences, and construction of representations for dynamic systems.

By comprehensively examining the role of the Laplace transform in science and technology, this analysis aims to enhance our understanding of its relevance and impact. It will provide researchers, engineers, and scientists with valuable insights into the applications and potential future advancements of this transformative mathematical tool.

2. Definitions and Properties

2.1 Definition of Laplace Transform:

The term "L T" refers to a mathematical technique that converts a time-dependent function, represented as f(t), into another function F(s).
In this context, s represents a complex variable, and F(s) denotes the Laplace transform of the original function f(t).
In terms of mathematical representation, it can be stated as

$$F(s) = L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\, dt.$$

In this equation, s is a complex variable, and e^{-st} refers to the factor that causes exponential decay. The integral is taken over the entire positive time axis[14].

2.2 Properties of Laplace Transforms:

The Laplace transform possesses numerous characteristics that render it valuable in the resolution of DE and analysis of dynamic systems [14]:

Table 1:Properties of Laplace Transforms

Property	Mathematical form
Linearity	$L\{af(t) + bg(t)\} = aF(s) + bG(s)$, where a and b are constants.
Time Shifting	$L\{f(t-a)\} = e^{-as}F(s)$
Scaling	$L\{f(at)\} = (1/a)F(s/a)$
Derivative Property	$L\{f'(t)\} = sF(s) - f(0)$
Integral Property	$L\{\int_0^t f(u)\, du\} = (1/s)F(s)$
Multiplication by t^n	$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)]$, wheren=1,2,3…….
Division by t	$L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)\, ds$
Convolution Property	$L\{f(t) * g(t)\} = F(s) * G(s)$, where * denotes convolution.

By applying the LT, it is possible to convert differential equations into algebraic equations, which are typically more straightforward to solve. The modified equation can be manipulated through algebraic operations, and subsequently, the reverse Laplace transform can be utilized to acquire the solution in the time realm. The Laplace transform also offers a means to examine the behaviour of dynamic systems. Utilizing the LT on the DEs that describe a system allows us to examine the altered equations within the realm of complex frequency analysis. This facilitates the assessment of system stability, transient response, frequency response, and various other characteristics of the system.

2.3 Laplace Transform Pairs and Table:

The Laplace transform pairs represent the transformations of commonly encountered functions. Here are some important Laplace transform pairs [14]:

Table 2: Function and its Laplace Transform

Function	Laplace Transform
$f(t) = 1$	$F(s) = 1/s$
$f(t) = e^{at}$	$F(s) = 1/(s - a)$
$f(t) = t^n$	$F(s) = n!/s^{(n+1)}$, where n is a non-negative integer.
$f(t) = sin(at)$	$F(s) = a/(s^2 + a^2)$
$f(t) = cos(at)$	$F(s) = s/(s^2 + a^2)$
$f(t) = sinh(at)$	$F(s) = a/(s^2 - a^2)$
$f(t) = cosh(at)$	$F(s) = s/(s^2 - a^2)$

2.4 Definition of Exponential Order:

Exponential order refers to a concept in mathematics often used to analyse functions or sequences. It provides a measure of how fast a function or sequence grows as its input or index increases. An exponential order is attributed to a function if it can be confined or limited by an exponential function.

In a more precise manner, we can define a function f(x) as having an exponential order if there are non-negative constant values M and c (where c is greater than zero) such that for all x greater than or equal to a certain real number x_0, the absolute value of f(x) is less than or equal to $Me^{(cx)}$. This implies that the function's magnitude is restricted by the product of an exponential function and a constant factor.

2.5 Initial Value Theorem(IVT):

The Initial Value Theorem establishes a connection between the starting conditions of a signal or system and its Laplace transform. In the realm of Laplace transformations, the IVT states that the initial value of a function f(t) at t = 0 can be determined by evaluating the LT, F(s) at s = ∞:

$$lim\_(s \to \infty)\ [sF(s)] = f(0+)$$

2.6 Final Value Theorem (FVT):

The Final Value Theorem establishes a connection between the long-term behaviour of a signal or system and its Laplace Transform.
Mathematically, the FVT states:
If the LT of a function f(t) is F(s), and all poles of F(s) have negative real parts, then the steady-state value of f(t) as t approaches infinity is given by:

F(s) at s = 0:

$$lim(s \to 0)\ [sF(s)] = lim(t \to \infty)\ f(t)$$

2.7 Inverse Laplace Transform:

The inverse LT refers to the procedure of deriving the original function f(t) from its LT is F(s).
It is denoted by $L^{-1}\{F(s)\} = f(t)$ or sometimes written as $L^{-1}\{F\} = f$.

3. Mathematical model

3.1 Mathematical Model of Linear Differential Equation:

Consider a simple first-order linear ODE;

$$dy/dt + ay = f(t),$$

Where: y (t) represents the unknown function, we want to solve it for t and t is the independent variable (usually time).

"a" represents a coefficient that remains constant, while "f(t)" denotes a forcing function with a predetermined or well-known value. [16]. By utilizing the LT on the given DE, we obtain the following expression:

$$L[dy/dt] + aL[y] = L[f(t)]$$

By utilizing the characteristics of the Laplace transform, we can streamline the equation.
The LT of the derivative of a function is given by:

$$L[dy/dt] = sY(s) - y(0)$$

Y(s) represents the Laplace transformation of the function y(t), while y(0) denotes the initial value of y(t).
Applying this transformation and rearranging the equation, we get:

$$sY(s) - y(0) + aY(s) = F(s)$$

Now, we can solve for Y(s), which represents the transformed function Y(s):

Y(s) = (y(0) + F(s)) / (s + a)

In the end, we derive the solution y(t) by performing the inverse LT on Y(s).
Note that to fully solve the equation, initial conditions x(0) and dx(0)/dt must be known.

Example: Given the differential equation:

$$y'(t) + 3y(t) = 5, y(0) = 2.$$

By employing the LT on both sides of the eqn., we obtain the following result:

$sY(s) - y(0) + 3Y(s) = 5 / s^2$, Y(s) denotes the LT of y(t).

Rearranging eqn., we get:

$$Y(s) = (5 + 2s) / (s(s + 3)).$$

Express Y(s) as a sum of partial fractions:

$Y(s) = A / s + B / (s + 3).$

Solving for A and B, we find A = 1 and B = 1.

Therefore, $Y(s) = 1 / s + 1 / (s + 3).$

By performing the inverse LT, we derive the solution: $y(t) = 1 + e^{-3t}.$

3.2 Mathematical Model of Second-order Differential Equation:

Let's consider a generic second-order linear differential equation of the form:

$$p\frac{d^2x}{dt^2} + q\frac{dx}{dt} + r\,x(t) = f(t)$$

Constants p, q, and r, are given, and our objective is to determine the unknown function x(t), while considering f(t) as the input function.

Taking LT on both sides, we get

$$L\left\{p\frac{d^2x}{dt^2}\right\} + L\left\{q\frac{dx}{dt}\right\} + L\{r\,x(t)\} = L\{f(t)\}$$

$$p\{s^2X(s) - sx(0) - x'(0)\} + q\{sX(s) - x(0)\} + r\,X(s) = F(s)$$

$$X(s)\{ps^2 + qs + r\} - x(0)\{ps + q\} - px'(0) = F(s)$$

Finally, solve for X(s) to obtain the Laplace transform of x(t):

$$X(s) = \frac{F(s) + x(0)\{ps + q\} + px'(0)}{(ps^2 + qs + r)}$$

After obtaining the LT of X(s), you can apply the inverse LT to determine the solution x(t).

It's important to remember that the final solution in the time domain, x(t), will depend on the initial conditions $x(0)$ and $x'(0)$

Example: Given the differential equation:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y(t) = 0,$$

$y(0) = 1, y'(0) = -2.$

By employing the LT on both sides of the eqn., we obtain the following result:

$s^2Y(s) - sy(0) - y'(0) + 4(sY(s) - y(0)) + 4Y(s) = 0.$

Simplifying and substituting the initial conditions, we get:

$$(s^2 + 4s + 4)Y(s) - (s + 2) = 0.$$

Solving for Y(s), we have:$Y(s) = (s + 2) / (s^2 + 4s + 4).$

Factoring the denominator, we get:

$$Y(s) = (s + 2) / [(s + 2)^2].$$

This mathematical expression provides us with the Laplace transformation of x, and by employing methods for the inverse LT, we can determine the solution x(t) in the time domain: $y(t) = te^{-2t}$.

3.3 Mathematical Model of Simultaneous Differential Equations:

Let's contemplate a setup comprising of a pair of simultaneous differential equations:

$$dx/dt = adx/dt + bdy/dt$$

$$dy/dt = cdx/dt + ddy/dt$$

Utilize the LT on both sides of each equation by applying it. You can find the LT of a derivative by making use of the following property:

$$L\{d^n(f(t))/dt^n\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

In this eqn. F(s) represent the LT of the function f(t), while f(0), f'(0), ..., f^((n-1))(0) represent the initial values or derivatives of ‘f(t)’ at t=0.

To solve the differential equations, we need to apply the Laplace transform to each term. This involves converting the derivatives into algebraic expressions using the Laplace transform property mentioned earlier. Additionally, the Laplace transform needs to convert any initial conditions into suitable terms.

Suppose we consider X(s) and Y(s) to be the LT of x(t) and y(t) correspondingly. By utilizing the property of the Laplace transform, we can convert the system of differential equations into an algebraic equation:

$$sX(s) - x(0) = a(sX(s) - x(0)) + b(sY(s) - y(0))$$

$$sY(s) - y(0) = c * (sX(s) - x(0)) + d(s * Y(s) - y(0))$$

Find the solutions for X(s) and Y(s) in the obtained algebraic equations. Rearrange the equations to separate X(s) and Y(s) on a single side.

$$(s - a)X(s) - bY(s) = x(0) - ax(0) - by(0)$$

$$-cX(s) + (s - d)Y(s) = -cx(0) + y(0) - dy(0)$$

Find the solutions for X(s) and Y(s) by solving these equations, which depend on the initial values x(0) and y(0), as well as the coefficients a, b, c, and d.

Afterwards, apply the inverse LT to X(s) and Y(s) for getting x(t) and y(t).

You can use LT tables or symbolic manipulation software to perform the inverse LT.

Assuming that x(t) and y(t) represents the inverse LT of X(s) and Y(s) respectively, calculate the inverse LT to express x(t) and y(t) in terms of the variable t.

$$x(t) = L^{-1}\{X(s)\}$$

$$y(t) = L^{-1}\{Y(s)\}$$

Ultimately, validate the acquired solutions by substituting them back into the initial system of differential equations and confirming that they meet the given initial conditions.

3.4 Mathematical Model of Partial Differential Equation:

Let's consider a simple example of a one-dimensional heat equation: $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$, where u(x, t) represents the temperature distribution in a solid medium, α is the thermal diffusivity, and x and t are the spatial and temporal variables, respectively[15].

To employ the LT for solving this equation, we can proceed by adhering to these steps:

Step 1: Utilize the LT on both parts of the equation concerning the time variable, t. We will use L{u(x,t)} to represent the LT of u(x, t).

The transformed equation becomes

$$s\, L\{u\}(x,s) - u(x,0) = \alpha \frac{d^2}{dx^2[L\{u\}(x,s)]},$$

as "s denotes the variable used in the Laplace transform." And u(x, 0) signifies the initial distribution of temperature.

Step 2: After acquiring the LT of u(x, t) as L{u}(x, s), we can employ the inverse LT to determine the solution u(x, t). The inverse LT operation can be carried out using various techniques such as partial fraction decomposition, table lookup, or contour integration.

4. Main Results

In this segment, the authors utilized the Laplace transform in diverse fields such as physics, statistics, and engineering. Additionally, they illustrated the significance and analytical power of the Laplace Transform by highlighting several important applications.

4.1 The Utilization of the Laplace Transform in Electrical Circuits:

The practical uses of the Laplace transform extend to the resolution of both ordinary and partial differential equations, which frequently arise when studying electrical circuits [17]. By converting a differential equation into an algebraic equation, specific techniques for manipulation and solving in the s-domain can be employed.

Suppose we consider a series circuit comprising three components: capacitance (C), resistance (R), and inductance (L). The circuit incorporates an electromotive power source with a voltage denoted as E. Furthermore, there is a switch integrated within the circuit. By Kirchhoff's law, we can derive the following information.

$L\, di/dt + Ri + q/C = E$ 4.1.1,

If the circuit contains no capacitor, then eqn. 4.3.1 reduce to

$L\, di/dt + Ri = E$ 4.1.2,

Example: In a circuit, there is a sequence of components consisting of an electromotive force described by E = 100sin (40t) V, a resistor with an inductor with an inductance of 0.5 henries, and a resistance of 10 ohms. Given that the initial current is zero, determine the current at a specific time t greater than zero.

By eqn. 4.3.2

$$.5 * di/dt + 10i = 100sin40t \Rightarrow di/dt + 20i = 200sin40t$$

Take the LT of eqn.4.3.2 of both sides, we get

$$L\{di/dt\} + 20L\{i\} = 200L\{sin(40t)\}$$

$$sI(s) - i(0) + 20I(s) = 200 \ (40/(s^2 + 40^2)$$

In this context, I(s) denotes the LT of the function i(t), and i(0) represents its initial condition.
Apply the initial condition, i(0) = 0:

$$sI(s) + 20I(s) = 200 \ (40/(s^2 + 40^2))$$

Solve the equation for I(s): $I(s) \ (s + 20) = 8000/(s^2 + 40^2)$

$I(s) = 8000/(s + 20) \ (s^2 + 40^2)$.

To compute I(s) in the time domain, apply the inverse LT. The expression can be simplified by utilizing partial fraction decomposition.

$$i(t) = L^{-1}\left\{\frac{4}{(s+20)}\right\} + L^{-1}\{80/(s^2 + 40^2)\} - 4L^{-1}\{s/(s^2 + 40^2)\}$$

Finally, the solution to the given differential equation is:

$i(t) = 4e^{-20t} + 2(sin40t - 2cos40t)$ 4.1.3

4.2 The Utilization of Laplace Transform in the Context of Population Growth:

The Laplace transform is used to analyse and solve differential equations that describe economic systems, such as models of economic growth. The logistic equation can be expressed as a separable equation, and through the application of the Laplace transform and its properties, an exponential expression can be established.

Let's consider an example of population growth described by the following differential equation: $dP/dt = kP$, where P represents the population size, t represents time, and k is a constant representing the growth rate[6].

Now, let's apply the LT to both sides of the DE:

$$L\{dP/dt\} = L\{kP\}$$

$$sP(s) - P(0) = kP(s)$$

$$P(s) = P(0) \,/\, (s - k)$$

Taking the inverse LT of P(s), we have:

$$L^{-1}\{P(s)\} = L^{-1}\{P(0) \,/\, (s - k)\}$$

$$P(t) = P(0)e^{kt} \qquad 4.2.1$$

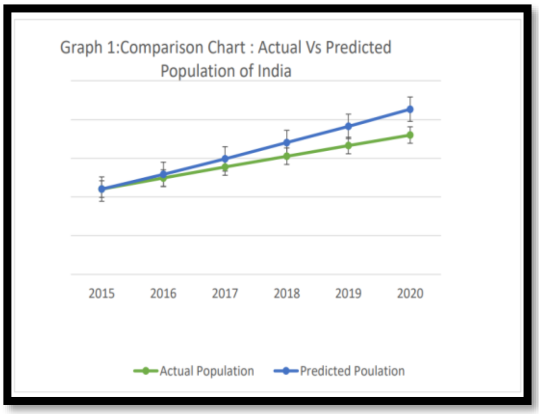
For calculating the value of k, we considered 2011 as the base year and 2001 as the final year. Then we have, P(t) = 1,028,737,436, P (0) = 1,210,854,977, t = -10, which gives a value of k is approximately 0.01629.

(Data link: https://www.worldometers.info/world-population/india-population/).

By employing equation 4.2.1, we can ascertain the value of k and subsequently assess the population for various years (predicated and actual population of India from 2015 to 2020):

Table 3: Population Data of India (2015-2020)

Year	Actual Population	Predicted Population
2015	1,31,01,52,403	1,31,01,52,403
2016	1,32,45,17,249	1,32,92,48,998
2017	1,33,86,76,785	1,34,92,76,289
2018	1,35,26,42,280	1,37,02,74,712
2019	1,36,64,17,754	1,39,12,84,825
2020	1,38,00,04,385	1,41,33,49,485



According to the Growth rate coefficient (0.01629), India's estimated population in 2025 is projected to be approximately 1,494,149,283.

4.3 Utilizing Laplace Transform in the Context of Classical Mechanics:

Typical usage of the Laplace transform in classical mechanics involves studying the behaviour of undamped harmonic motion under the influence of external damping. In this case, a system, such as a mass-spring-damper system, is subjected to an external force and experiences damping due to friction or other dissipative effects [13].

The equation of motion for a damped harmonic oscillator can be written as:

$m\ \frac{d^2x(t)}{dt^{\,2}} + c\ \frac{dx}{dt} + k\,x(t) = F(t)$, 4.3 .1

The variables used in the equation are defined as follows:

b =Damping coefficient,
k =Spring constant,
x = Displacement of the mass from its equilibrium position,
t= corresponds to time, and
F(t) = the applied force over time.

The free-damped force differential equation describes the motion of a damped harmonic oscillator without any external forcing. It can be represented by the following equation:

: $m\ \frac{d^2x(t)}{dt^{\,2}} + c\ \frac{dx}{dt} + k\,x(t) = 0,$ 4.3 .2

To solve this equation using the Laplace transform, we apply the transform to both sides of the equation of eqn.4.3.2, we have:

$$L\left\{ m\ \frac{d^2x(t)}{dt^{\,2}} + c\ \frac{dx}{dt} + k\,x(t) \right\} = 0$$

$$L\left\{m \frac{d^2x(t)}{dt^2}\right\} + L\left\{c \frac{dx}{dt}\right\} + L\{k\,x(t)\} = 0$$

Let's assume the initial conditions are $x(0) = \bar{x}(0)$ and $dx(0)/dt = \bar{v}(0)$, where is the initial displacement and $\bar{v}(0)$ is the initial velocity.

Using the properties of the Laplace transform, we have:

$$m\ (s^2 X(s) - s\ x(0) - \bar{v}(0)) + c\ (s\ X(s) - x(0) + k\ X(s) = 0,$$

where X(s) is the LT of x(t) and s is the complex frequency variable.

Rearranging the equation and solving for X(s), we get:

X(s) = (s m $\bar{x}(0)$ + m $\bar{v}(0)$ + c x(0) / (m s^2 + c s + k), 4.3.3,

where X(s) is the LT of x(t).

We need to look up the inverse LT of X(s) to obtain the inverse LT of X(s).

Example of Free Damping Force:

The Laplace Transform equation describes a mathematical model for a system with free damping force. By applying this model, we can tackle the following scenario: A coil spring hangs from the ceiling, and a 32-pound weight is attached to its lower end. Initially, when the weight reaches its balanced position, the spring stretches by 2 feet.

However, after being pulled 6 inches below its resting position, the weight is let go at t=0. The system is not influenced by any external forces, but the resistance from the surrounding medium, measured in pounds, is directly related to eight times the derivative of displacement with respect to time, denoted as dx/dt, where dx/dt represents the instantaneous velocity in feet per second. Our objective is to analyze the subsequent movement of the weight connected to the spring and provide an explanation for the observed behaviour.

Explanation: Using Hook's law, we have$32 = k * 2 \Rightarrow k = 16lb/ft$.

Again $W = mg \Rightarrow 32 = m * 32 \Rightarrow m = 1\ slug.$ Here damping factor, c=8. The initial conditions are: $x_0 = \frac{6}{12} = 0.5$ and $v_0 = 0$.

Put these values in equation 4.3.3, and we get

X(s) = (s *1/2 + 8*1/2) / (s^2 + 8 s +16)=1/2(s + 8) / (s^2 + 8 s +16) 4.3.4,

Taking the inverse Laplace transform of 4.3.4 on both sides we get.

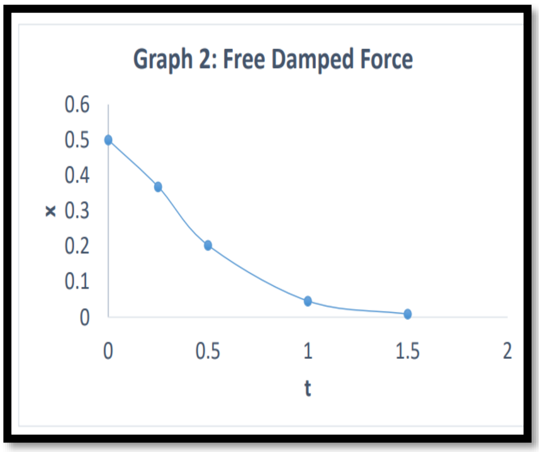
$x(t) = 1/2(1 + 4t)e^{-4t}$ 4.3.5

The movement exhibits a critically damped behaviour, we get,

$$x = 0 \Leftrightarrow t = -1/4.$$

And $x'(t) = -8te^{-4t}$ for all t>0.

Its position at equilibrium rapidly approaches zero as time tends towards infinity. The accompanying graph illustrates this trend:



5. Conclusion

The paper comprehensively analyses the Laplace transform and its role in various fields. It emphasizes the significance and contributions of the Laplace transform in modelling and analyzing complex systems, as well as its impact on the advancement of science and technology. The research delves into the core concepts and mathematical characteristics of the Laplace transform and examines how it can be used in various fields such as physics, engineering, and other related areas.

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